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AUTOCORRELATION OF THE SUNSPOT NUMBERS

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As it is known, one of the fundamental characteristics of random processes is the autocorrelation function [1—4]. As far as solar activity may be considered as a random process (or, more precisely, as time series, since the observations are discreetely distributed along the time axis) the determination of the autocorrelation functions for various indeces of solar activity representing the solar activity, are of certain interest.

In this paper the results of the calculation of three autocorrelation functions for sunspot numbers are given.

1. Autocorrelation function of synodic sunspot numbers. The synodic sunspot numbers, introduced in the present paper, represent arithmetic means of daily Wolf numbers for synodic solar rotations.

A great difficulty in the investigation of solar activity is due to the fact, that the Sun is observed (at least at the present) from the Earth only, i. e. only the one solar semisphere is observed at each moment. The statistical character of solar activity (mostly appearing in great groups of spots), combined with the rotation of the Sun around its axis and the motion of the Earth around the Sun leads to the existence of a certain cycle with duration about 27 days.

A sidereal rotation of the Sun has a duration of 27.35 days (for a point on the solar equator) and it is a function of the heliographic latitude. For the zone, in which the spots are located, it is accepted that a synodic rotation of the Sun has an average duration of 27.2853 days and this value varies with several hundredths, since the Earth's orbit is an elliptic one. The beginning of synodic rotation is given with the coincidence of the beginning meridian with the central meridian. It is assumed that the beginning meridian is the meridian which has passed through the ascending node of the solar equator on the ecliptic on 1854 January 1, Greenwich mean noon (J. D. 239 8220.0).This definition is always made use of [5], although the beginning meridian of Carvington passed the ascending node 12 h earlier. The numbering of the synodic rotations is a continuation of the rotations introduced by Carrington, rotation Nr. 1 having commenced on 1853 November 9.

The effect of the "synodic sycle" in solar activity may be eliminated by averaging of the dialy Wolf numbers for synodic rotations only. Actually, the averaging in this case is equivalent to the operation filtering (or purification) of the solar activity.

As the synodic rotations are determined since 1853, and to obtain more statistical data it is necessary to introduce numbers of solar rotations. We

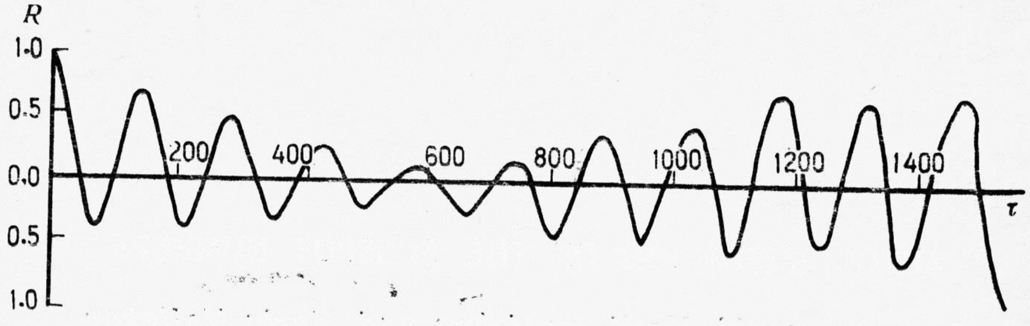

Fig. 1

calculated the moments of beginnings of rotations from -65 Rot. The obtained synodic Wolf numbers, which form a very voluminous table, will be given in a next paper.

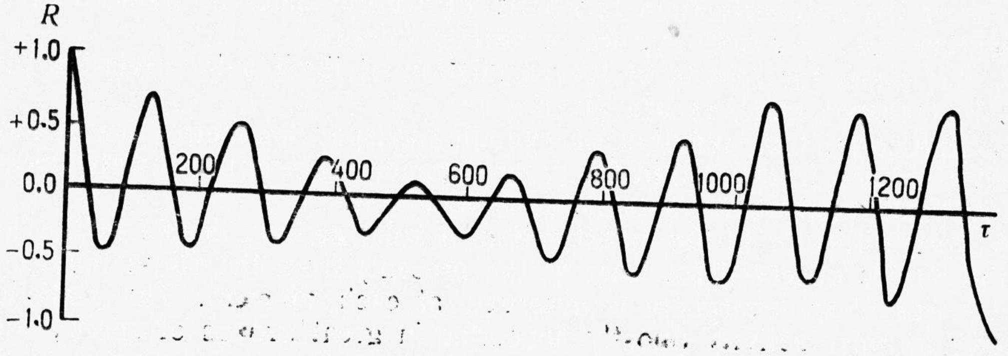

Fig. 2

The "synodic cycles" may also be purified by another method if one determines the so-called global Wolf number, concerning the whole sun surface. But in as far as the series of global Wolf numbers has been determined since hardly about ten years and is still an object of discussion, this series can not be used for statistical purposes. Actually, it is more correct to determine the global Wolf numbers on the basis of empirical distributions — stochastic representation.

Our normalized autocorrelation function of synodic Wolf numbers is calculated for rotations from -65 Rot to $+1488$ Rot and is shown in Fig. 1. Obviously, this autocorrelation function may be approximated only to $\tau \leq 600$. The valuet of $K(\tau)$ by τ are given in Table 1.

2. The autocorrelation function of the monthly Wolf numbers is calculated for time interval 1849 January — 1964 December, which interval approximately corresponds to the one, which we have in case of the synodic rotations. This function is given in Fig. 2. Obviously, it may also be appro-

Table 1

τ	$K(\tau)$	τ	$K(\tau)$	τ	$K(\tau)$	τ	$K(\tau)$	τ	$K(\tau)$
1	2	3	4	5	6	7	8	9	10
0	+0.000	57	—0.368	114	+0.321	171	+0.197	228	—0.365
1	+0.918	58	—0.381	115	+0.342	172	+0.178	229	—0.350
2	+0.888	59	—0.394	116	+0.366	173	+0.157	230	—0.331
3	+0.874	60	—0.405	117	+0.396	174	+0.128	231	—0.314
4	+0,863	61	—0.413	118	+0.414	175	+0.106	232	—0.297
5	+0.854	62	—0.427	119	+0.442	176	+0.079	233	—0.281
6	+0.839	63	—0.436	120	+0.466	177	+0.059	234	—0.266
7	+0.826	64	—0.441	121	+0.491	178	+0.031	235	—0.248
8	+0.815	65	—0.451	122	+0.512	179	—0.001	236	—0.233
9	+0.803	66	—0.458	123	+0.524	180	—0.027	237	—0.224
10	+0.795	67	—0.462	124	+0.546	181	—0.047	238	—0.207
11	+0.778	68	—0.462	125	+0.562	182	—0.070	239	—0.186
12	+0.755	69	—0.462	126	+0.586	183	—0.094	240	—0.167
13	+0.736	70	—0.464	127	+0.597	184	—0.114	241	—0.144
14	+0.720	71	—0.466	128	+0.609	185	—0.135	242	—0.116
15	+0.697	72	—0.467	129	+0.623	186	—0.157	243	—0.097
16	+0.675	73	—0.467	130	+0.644	187	—0.180	244	—0.076
17	+0.654	74	—0.465	131	+0.656	188	—0.200	245	—0.045
18	+0.632	75	—0.458	132	+0.669	189	—0.217	246	—0.017
19	+0.598	76	—0.457	133	+0.676	190	—0.239	247	—0.004
20	+0.571	77	—0.451	134	+0.682	191	—0.255	248	+0.015
21	+0.546	78	—0.441	135	+0.689	192	—0.270	249	+0.032
22	+0.520	79	—0.430	136	+0.698	193	—0.289	250	+0.060
23	+0.487	80	—0.422	137	+0.703	194	—0.306	251	+0.079
24	+0.460	81	—0.412	138	+0.714	195	—0.322	252	+0.103
25	+0.440	82	—0.401	139	+0.713	196	—0.335	253	+0.128
26	+0.406	83	—0.391	140	+0.716	197	—0.346	254	+0.148
27	+0.376	84	—0.376	141	+0.709	198	—0.364	255	+0.170
28	+0.352	85	—0.351	142	+0.708	199	—0.372	256	+0.197
29	+0.319	86	—0.348	143	+0.704	200	—0.384	257	+0.220
30	+0.280	87	—0.337	144	+0.700	201	—0.390	258	+0.238
31	+0.249	88	—0.322	145	+0.698	202	—0.402	259	+0.254
32	+0.220	89	—0.307	146	+0.687	203	—0.412	260	+0.275
33	+0.189	90	—0.288	147	+0.679	204	—0.423	261	+0.292
34	+0.159	91	—0.268	148	+0.672	205	—0.433	262	+0.308
35	+0.136	92	—0.249	149	+0.659	206	—0.434	263	+0.323
36	+0.105	93	—0.227	150	+0.644	207	—0.441	264	+0.341
37	+0.077	94	—0.205	151	+0.629	208	—0.451	265	+0.358
38	+0.045	95	—0.185	152	+0.619	209	—0.454	266	+0.379
39	+0.018	96	—0.160	153	+0.604	210	—0.459	267	+0.391
40	—0.014	97	—0.142	154	+0.590	211	—0.459	268	+0.404
41	—0.043	98	—0.118	155	+0.574	212	—0.461	269	+0.417
42	—0.070	99	—0.093	156	+0.558	213	—0.459	270	+0.432
43	—0.099	100	—0.068	157	+0.535	214	—0.460	271	+0.448
44	—0.124	101	—0.036	158	+0.504	215	—0.458	272	+0.452
45	—0.147	102	—0.008	159	+0.481	216	—0.457	273	+0.462
46	—0.168	103	+0.001	160	+0.465	217	—0.451	274	+0.467
47	—0.193	104	+0.051	161	+0.449	218	—0.447	275	+0.473
48	—0.216	105	+0.073	162	+0.430	219	—0.445	276	+0.481
49	—0.232	106	+0.098	163	+0.408	220	—0.441	277	+0.493
50	—0.250	107	+0.126	164	+0.378	221	—0.436	278	+0.500
51	—0.270	108	+0.154	165	+0.350	222	—0.428	279	+0.511
52	—0.288	109	+0.185	166	+0.334	223	—0.422	280	+0.518
53	—0.301	110	+0.212	167	+0.308	224	—0.410	281	+0.517
54	—0.323	111	+0.239	168	+0.215	225	—0.400	282	+0.522
55	—0.339	112	+0.265	169	+0.248	226	—0.392	283	+0.522
56	—0.353	113	+0.292	170	+0.224	227	—0.381	284	+0.524

Continuation of Table 1

1	2	3	4	5	6	7	8	9	10
285	+0.521	345	—0.329	404	+0.083	463	+0.020	522	—0.210
286	+0.518	346	—0.335	405	+0.099	464	+0.009	523	—0.205
287	+0.515	347	—0.349	406	+0.115	465	—0.002	524	—0.192
288	+0.517	348	—0.357	407	+0.128	466	—0.027	525	—0.191
289	+0.507	349	—0.360	408	+0.140	467	—0.034	526	—0.182
290	+0.498	350	—0.370	409	+0.149	468	—0.045	527	—0.176
291	+0.497	351	—0.375	410	+0.157	469	—0.053	528	—0.166
292	+0.494	352	—0.377	411	+0.167	470	—0.071	529	—0.168
293	+0.480	353	—0.381	412	+0.181	471	—0.089	530	—0.155
294	+0.469	354	—0.383	413	+0.189	472	—0.098	531	—0.153
295	+0.466	355	—0.393	414	+0.205	473	—0.108	532	—0.142
296	+0.455	356	—0.395	415	+0.211	474	—0.124	533	—0.131
297	+0.438	357	—0.396	416	+0.219	475	—0.132	534	—0.121
298	+0.422	358	—0.392	417	+0.227	476	—0.145	535	—0.114
299	+0.407	359	—0.388	418	+0.231	477	—0.151	536	—0.108
300	+0.393	360	—0.389	419	+0.236	478	—0.162	537	—0.105
301	+0.380	361	—0.387	420	+0.239	479	—0.175	538	—0.095
302	+0.364	362	—0.384	421	+0.243	480	—0.184	539	—0.082
303	+0.348	363	—0.380	422	+0.256	481	—0.197	540	—0.072
304	+0.334	364	—0.377	423	+0.259	482	—0.208	541	—0.061
305	+0.325	365	—0.372	424	+0.256	483	—0.217	542	—0.053
306	+0.317	366	—0.366	425	+0.263	484	—0.222	543	—0.048
307	+0.303	367	—0.357	426	+0.275	485	—0.226	544	—0.038
308	+0.289	368	—0.345	427	+0.284	486	—0.230	545	—0.037
309	+0.274	369	—0.337	428	+0.284	487	—0.237	546	—0.040
310	+0.261	370	—0.327	429	+0.275	488	—0.250	547	—0.036
311	+0.241	371	—0.320	430	+0.274	489	—0.265	548	—0.028
312	+0.209	372	—0.309	431	+0.273	490	—0.272	549	—0.021
313	+0.197	373	—0.296	432	+0.275	491	—0.276	550	—0.014
314	+0.173	374	—0.287	433	+0.273	492	—0.274	551	—0.010
315	+0.157	375	—0.280	434	+0.265	493	—0.284	552	+0.002
316	+0.138	376	—0.274	435	+0.259	494	—0.289	553	+0.009
317	+0.118	377	—0.261	436	+0.255	495	—0.293	554	+0.020
318	+0.098	378	—0.248	437	+0.251	496	—0.296	555	+0.026
319	+0.078	379	—0.236	438	+0.240	497	—0.294	556	+0.031
320	+0.056	380	—0.223	439	+0.235	498	—0.298	557	+0.032
321	+0.044	381	—0.215	440	+0.233	499	—0.301	558	+0.037
322	+0.020	382	—0.202	441	+0.224	500	—0.306	559	+0.041
323	+0.009	383	—0.195	442	+0.222	501	—0.298	560	+0.046
324	—0.010	384	—0.181	443	+0.221	502	—0.293	561	+0.050
325	—0.033	385	—0.169	444	+0.212	503	—0.277	562	+0.051
326	—0.056	386	—0.154	445	+0.205	504	—0.306	563	+0.059
327	—0.075	387	—0.138	446	+0.196	505	—0.304	564	+0.060
328	—0.099	388	—0.121	447	+0,192	506	—0.294	565	+0.058
329	—0.111	389	—0.108	448	+0.184	507	—0.294	566	+0.064
330	—0.132	390	—0.092	449	+0.172	508	—0.292	567	+0.070
331	—0.150	391	—0.079	450	+0.166	509	—0.296	568	+0.077
332	—0.164	392	—0.063	451	+0.158	510	—0.290	569	+0.083
333	—0.176	393	—0.054	452	+0.147	511	—0.282	570	+0.086
334	—0.193	394	—0.039	453	+0.143	512	—0.273	571	+0.094
335	—0.206	395	—0.024	454	+0.129	513	—0.266	572	+0.096
336	—0.224	396	—0.011	455	+0.122	514	—0.256	573	+0.085
337	—0.240	397	+0.010	456	+0.099	515	—0.252	574	+0.089
338	—0.252	398	+0.028	457	+0.090	516	—0.245	575	+0.094
339	—0.257	399	+0.036	458	+0.079	517	—0.246	576	+0.099
340	—0.273	400	+0.046	459	+0.066	518	—0.240	577	+0.010
341	—0.284	401	+0.060	460	+0.048	519	—0.231	578	+0.010
342	—0.295	402	+0.076	461	+0.039	520	—0.227	579	+0.010
344	—0.310	403	+0.081	462	+0.028	521	—0.218	580	+0.010

Continuation of Table 1

1	2	3	4	5	6	7	8	9	10
581	+0.011	585	+0.094	589	+0.095	593	+0.086	597	+0.074
582	+0.010	586	+0.089	590	+0.095	594	+0.087	598	+0.068
583	+0.094	587	+0.092	591	+0.089	595	+0.086	599	+0.073
584	+0.093	588	+0.092	592	+0.086	596	+0.079		

ximated to $\tau \leqq 600$, for which τ values of the autocorreletion function $K(\tau)$ are respectively given in Table 1.

3. The autocorelation function for the yearly Wolf numbers is plotted in Fig. 3, and it is calculated for the interval 1700—1964. It is clear, that

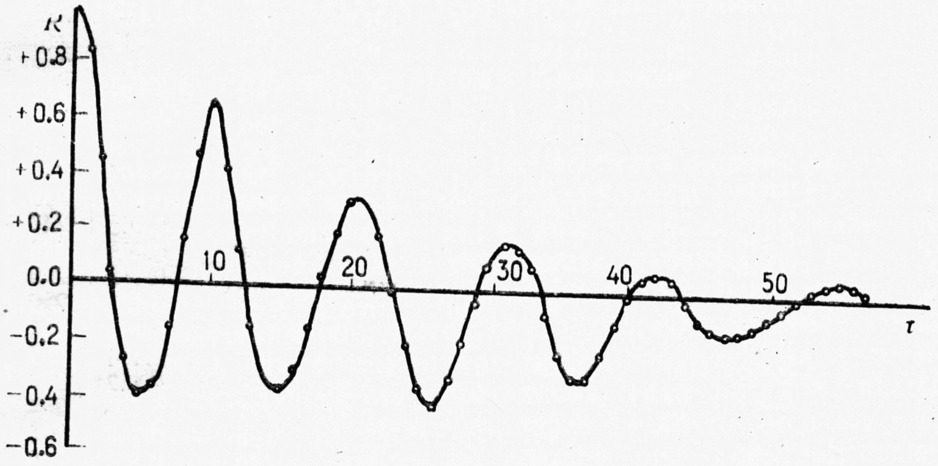

Fig. 3

this function may be approximated to $\tau \leqq 60$, as the values of $K(\tau)$ are respectively given in Tadle 3.

The extreme values of the first two autocorrelation functions are given in Table 4, and the zeroes of these functions — in Table 5, respectively. Analogously the zeroe points for the yearly Wolf numbers have been shown in Table 6, and the extreme values — in Table 7.

The conclusions from the present paper are the following:

I. All examined autocorrelation functions have a negligible dispersion. They are smooth curves and from this point of view they may be comparatively easily be approximated with the help of simple expressions.

II. Two of autocorrelation functions, fnr synodic and monthly Wolf numbers, are different only in detail. It is clear, that this is due to the fact, that the first function is calculated on the basis of a filtering time series.

III. From the first maximuma of the autocorrelation function of synodic and monthly Wolf numbers the average duration of 11-yearly cycle may be obtained. In the first case we obtain $T_{11} = 10.47$ years, and in the second case — $T_{11} = 10.33$ years.

As one can see, these values greatly differ from the generally accepted as they by numerous investigation are comprised in the interval 11.0—11.2 years.

Table 2

τ	$K(\tau)$	τ	$K(\tau)$	τ	$K(\tau)$	τ	$K(\tau)$	τ	$K(\tau)$
1	2	3	4	5	6	7	8	9	10
0	+0.000	57	—0.442	114	+0.604	171	—0.259	228	+0.155
1	+0.928	58	—0.454	115	+0.619	172	—0.273	229	+0.181
2	+0.895	59	—0.461	116	+0.640	173	—0.291	230	+0.211
3	+0.882	60	—0.465	117	+0.658	174	—0.313	231	+0.236
4	+0.868	61	—0.466	118	+0.668	175	—0.332	232	+0.252
5	+0.851	62	—0.466	119	+0.680	176	—0.345	233	+0.277
6	+0.839	63	—0.469	120	+0.688	177	—0.361	234	+0.293
7	+0.825	64	—0.471	121	+0.697	178	—0.373	235	+0.308
8	+0.813	65	—0.473	122	+0.706	179	—0.385	236	+0.331
9	+0.803	66	—0.470	123	+0.714	180	—0.396	237	+0.349
10	+0.784	67	—0.464	124	+0.722	181	—0.406	238	+0.366
11	+0.761	68	—0.461	125	+0.720	182	—0.417	239	+0.389
12	+0.740	69	—0.456	126	+0.718	183	—0.431	240	+0.404
13	+0.715	70	—0.444	127	+0.715	184	—0.440	241	+0.416
14	+0.692	71	—0.435	128	+0.716	185	—0.443	242	+0.436
15	+0.669	72	—0.426	129	+0.710	186	—0.453	243	+0.444
16	+0,640	73	—0.413	130	+0.704	187	—0.459	244	+0.454
17	+0.608	74	—0.404	131	+0.696	188	—0.462	245	+0.468
18	+0.578	75	—0.388	132	+0.688	189	—0.466	246	+0.471
19	+0.546	76	—0.371	133	+0.671	190	—0.465	247	+0.480
20	+0.516	77	—0.357	134	+0.662	191	—0.468	248	+0.493
21	+0.483	78	—0.347	135	+0.640	192	—0.463	249	+0.503
22	+0.454	79	—0.331	136	+0.631	193	—0.463	250	+0.507
23	+0.428	80	—0.311	137	+0.617	194	—0.460	251	+0.520
24	+0.391	81	—0.290	138	+0.596	195	—0.454	252	+0.518
25	+0.362	82	—0.269	139	+0.581	196	—0.452	253	+0.524
26	+0.325	83	—0.246	140	+0.562	197	—0.450	254	+0.529
27	+0.283	84	—0.223	141	+0.533	198	—0.447	255	+0.527
28	+0.247	85	—0.200	142	+0.503	199	—0.434	256	+0.525
29	+0.214	86	—0.176	143	+0.482	200	—0.428	257	+0.520
30	+0.181	87	—0.154	144	+0.460	201	—0.415	258	+0.519
31	+0.149	88	—0.129	145	+0.438	202	—0.403	259	+0.508
32	+0.121	89	—0.100	146	+0.418	203	—0.394	260	+0.507
33	+0.088	90	—0.070	147	+0.448	204	—0.382	261	+0.499
34	+0.052	91	—0.039	148	+0.357	205	—0.361	262	+0.494
35	+0.019	92	—0.003	149	+0.332	206	—0.345	263	+0.480
36	+0.015	93	+0.025	150	+0.300	207	—0.325	264	+0.471
37	—0.048	94	+0.051	151	+0.268	208	—0.306	265	+0.464
38	—0.079	95	+0.078	152	+0.234	209	—0.289	266	+0.445
39	—0.011	96	+0.112	153	+0.209	210	—0.270	267	+0.426
40	—0.135	97	+0.143	154	+0.188	211	—0.254	268	+0.413
41	—0.162	98	+0.176	155	+0.166	212	—0.243	269	+0.393
42	—0.190	99	+0.209	156	+0.132	213	—0.222	270	+0.378
43	—0.211	100	+0.238	157	+0.103	214	—0.202	271	+0.361
44	—0.230	101	+0.268	158	+0.072	215	—0.181	272	+0.348
45	—0.253	102	+0.304	159	+0.046	216	—0.150	273	+0.334
46	—0.277	103	+0.333	160	+0.014	217	—0.124	274	+0.321
47	—0.297	104	+0.361	161	—0.014	218	—0.010	275	+0.310
48	—0.316	105	+0.392	162	—0.041	219	—0.073	276	+0.294
49	—0.334	106	+0.418	163	—0.064	220	—0.039	277	+0.278
50	—0.352	107	+0.448	164	—0.089	221	—0.017	278	+0.260
51	—0.369	108	+0.480	165	—0.116	222	+0.002	279	+0.234
52	—0.384	109	+0.500	166	—0.149	223	+0.023	280	+0.208
53	—0.392	110	+0.521	167	—0.165	224	+0.052	281	+0.190
54	—0.411	111	+0.547	168	—0 192	225	+0.076	282	+0.170
55	—0.424	112	+0.566	169	—0.209	226	+0.106	283	+0.147
56	—0.433	113	+0.587	170	—0.234	227	+0.131	284	+0.122

Continuation of Table 2

1	2	3	4	5	6	7	8	9	10
285	+0.100	344	—0.190	403	+0 166	462	—0.254	521	+0.102
286	+0.078	345	—0.172	404	+0.160	463	—0.251	522	+0.099
287	+0.053	346	—0.157	405	+0.150	464	—0.244	523	+0.094
288	+0.034	347	—0.135	406	+0.142	465	—0.238	524	+0.093
289	+0.017	348	—0.118	407	+0.129	466	—0.235	525	+0.090
290	—0.003	349	—0.101	408	+0.117	467	—0.221	526	+0.094
291	—0.027	350	—0.083	409	+0.099	468	—0.212	527	+0.093
292	—0.055	351	—0.070	410	+0.090	469	—0.206	528	+0.096
293	—0.071	352	—0.056	411	+0.076	470	—0.196	529	+0.092
294	—0.096	353	—0.041	412	+0.056	471	—0.184	530	+0.090
295	—0.110	354	—0.022	413	+0.044	472	—0.178	531	+0.086
296	—0.137	355	—0.008	414	+0.029	473	—0.170	532	+0.086
297	—0.155	356	+0.020	415	+0.019	474	—0.165	533	+0.086
298	—0.170	357	+0.035	416	+0.006	475	—0.156	534	+0.087
299	—0.192	358	+0.046	417	—0.011	476	—0.149	535	+0.079
300	—0.208	359	+0.060	418	—0.031	477	—0.136	536	+0.068
301	—0.224	360	+0.071	419	—0.045	478	—0.122	537	+0.070
302	—0.245	361	+0.083	420	—0.054	479	—0.117	538	+0.062
303	—0.259	362	+0.089	421	—0.065	480	—0.111	539	+0.057
304	—0.265	363	+0.102	422	—0.087	481	—0.103	540	+0.051
305	—0.281	364	+0.119	423	—0.098	482	—0.094	541	+0.041
306	—0.295	365	+0.136	424	—0.115	483	—0.084	542	+0.035
307	—0.308	366	+0.144	425	—0.128	484	—0.068	543	+0.033
308	—0.324	367	+0.155	426	—0.141	485	—0.057	544	+0.021
309	—0.335	368	+0.167	427	—0.144	486	—0.049	545	+0.076
310	—0.343	369	+0.176	428	—0.160	487	—0.040	546	+0.017
311	—0.354	370	+0.188	429	—0.172	488	—0.029	547	—0.013
312	—0.364	371	+0.205	430	—0.188	489	—0.036	548	—0.025
313	—0.371	372	+0.214	431	—0.200	490	—0.030	549	—0.025
314	—0.378	373	+0.222	432	—0.212	491	—0.025	550	—0.030
315	—0.385	374	+0.228	433	—0,222	492	—0.017	551	—0.035
316	—0.388	375	+0.233	434	—0.226	493	—0.008	552	—0.040
317	—0.394	376	+0.236	435	—0.234	494	—0.004	553	—0.045
318	—0.396	377	+0.244	436	—0.236	495	+0.005	554	—0.056
319	—0.400	378	+0.252	437	—0.251	496	+0.019	555	—0.063
320	—0.405	379	+0.260	438	—0.263	497	+0.028	556	—0.076
321	—0.399	380	+0.260	439	—0.273	498	+0.034	557	—0.087
322	—0.396	381	+0.265	440	—0.280	499	+0.037	558	—0.095
323	—0.395	382	+0.277	441	—0.278	500	+0.040	559	—0.098
324	—0.396	383	+0.284	442	—0.288	501	+0.042	560	—0.103
325	—0.390	384	+0.283	443	—0.295	502	+0.054	561	—0.108
326	—0.387	385	+0.278	444	—0.294	503	+0.054	562	—0.121
327	—0.382	386	+0.275	445	—0.297	504	+0.061	563	—0.125
328	—0.378	387	+0.276	446	—0.298	505	+0.059	564	—0.127
329	—0.370	388	+0.277	447	—0.304	506	+0.064	565	—0.131
330	—0.355	389	+0.265	448	—0.306	507	+0.066	566	—0.144
331	—0.342	390	+0.259	449	—0.305	508	+0.071	567	—0.152
332	—0.335	391	+0.251	450	—0.298	509	+0.079	568	—0.162
333	—0.322	392	+0.246	451	—0.307	510	+0.085	569	—0.170
334	—0·307	393	+0.239	452	—0.308	511	+0.092	570	—0.177
335	—0.298	394	+0.239	453	—0.304	512	+0.098	571	—0.190
336	—0.290	395	+0.230	454	—0.298	513	+0.095	572	—0.195
337	—0.281	396	+0.227	455	—0.297	514	+0.093	573	—0.201
338	—0.266	397	+0.225	456	—0.300	515	+0.096	574	—0.205
339	—0.251	398	+0.218	457	—0.292	516	+0.098	575	—0.213
340	—0.238	399	+0.205	458	—0.286	517	+0.102	576	—0.215
341	—0.224	400	+0.197	459	—0.272	518	+0.101	577	—0.225
342	—0.216	401	+0.189	460	—0.264	519	+0.103	578	—0.232
343	—0.203	402	+0.178	461	—0.258	520	+0.106	579	—0.236

Continuation of Table 2

1	2	3	4	5	6	7	8	9	10
580	—0.249	584	—0.274	588	—0.293	592	—0.309	596	—0.309
581	—0.262	585	—0.283	589	—0.291	593	—0.312	597	—0.318
582	—0.264	586	—0.277	590	—0.291	594	—0.312	598	—0.317
583	—0.267	587	—0.283	591	—0.299	595	—0.310	599	—0.315

Table 3

τ	$K(\tau)$	τ	$K(\tau)$	τ	$K(\tau)$
0	+1.000	20	+0.212	40	+0.116
1	+0.821	21	+0.310	41	+0.126
2	+0.451	22	+0.308	42	+0.054
3	+0.044	23	+0.190	43	+0.078
4	—0.263	24	—0.005	44	+0.054
5	—0.406	25	—0.203	45	—0.009
6	—0.362	26	—0.353	46	—0.079
7	—0.153	27	—0.400	47	—0.115
8	+0.161	28	—0.331	48	—0.135
9	+0.473	29	—0.198	49	—0.134
10	+0.639	30	—0.047	50	—0.111
11	+0.619	31	—0.081	51	—0.076
12	+0.424	32	—0.162	52	—0.048
13	+0.134	33	—0.157	53	—0.019
14	+0.140	34	—0.069	54	—0.012
15	+0.325	35	—0.084	55	—0.040
16	+0.382	36	—0.232	56	—0.069
17	+0.316	37	—0.314	57	—0.055
18	+0.160	38	—0.311	58	—0.033
19	+0.042	39	—0.230	59	—0,028

Table 4

No.	τ_{syn}	$K(\tau)_{syn}$	τ	$K(\tau)_{mon}$
1	72.0	—0.467	65	—0.473
2	140.2	+0.716	124	+0.722
3	211.5	—0.461	191	—0.529
4	283.6	+0.522	254	+0.529
5	356.2	—0.396	320	—0.405
6	427.1	+0.284	383	+0.284
7	502.0	—0.300	450	—0.306
8	579.5	+0.105	520	+0.106
9	668.5	—0.322	597	—0.317
10	738.0	+0.188	662	+0.186
11	809.5	—0.491	725	—0.495
12	883.0	+0.386	795	+0.394
13	950.0	—0.556	852	—0.563
14	1027.0	+0.489	921	+0.502
15	1096.0	—0.608	982	—0.607
16	1170.0	+0.764	1049	+0.787
17	1341.0	—0.560	1113	—0.568
18	1314.0	+0.704	1177	+0.722
19	1372.0	—0.677	1235	—0.699
20	1468.0	+0.782	1321	+0.799

Table 5

No.	$\tau_{\circ syn}$	$\tau_{\circ mon}$
1	39.6	35.5
2	102.6	92.1
3	178.8	160
4	274.3	222
5	323.9	290
6	396.5	355
7	465.8	416
8	551.9	495
9	609.8	546
10	708.4	634
11	764.6	686
12	850.2	762
13	912.7	819
14	994.6	892
15	1059.4	950
16	1135.2	1018
17	1205.3	1082
18	1277.5	1146
19	1344.1	1206
20	1415.9	1272
21	1497.3	1344

IV. An approximate value of the duration of 11-yearly cycle may be used and on the basis of the correlation function of yearly Wolf numbers. This value is $T_{11} \approx 10$ years.

V. The autocorrelation function of yearly Wolf numbers shows an exceptional peculiarity — the availability of secular cycle as its duration may be determined immediately

$$T_{\text{sec}} = 169.0 \pm 0.5 \text{ years.}$$

Let us mention that many authors speak about similar cycles — for instance [6—10] and about cycles with various duration and [11—15]. There is no wonder that it was found that this secular cycle is double.

Table 6

No.	τ_0	No.	τ_0	No.	τ_0	No.	τ_0	No.	τ_0
0	3.1	11	64.6	22	130.2	33	191.3	44	260.2
1	7.5	12	69.2	23	122.3	34	196.2		
2	12.5	13	80.6	24	142.8	35	201.8		
3	17.9	14	85.8	25	144.9	36	207.5		
4	23.9	15	92.8	26	154.4	37	212.5		
5	31.2	16	96.7	27	159.6	38	217.7		
6	34.5	17	83.1	28	165.0	39	223.1		
7	41.5	18	107.2	29	171.5	40	228.2		
8	44.9	19	113.5	30	175.0	41	233.1		
9	52.1	20	118.4	31	181.3	42	240.4		
10	58.5	21	123.4	32	184.2	43	242.1		

Table 7

No.	τ	$K(\tau)$	No.	τ	$K(\tau)$	No.	τ	$K(\tau)$	No.	τ	$K(\tau)$
0	5	−0.406	12	72	—0.243	24	138	—0.336	36	205	—0.506
1	10	+0.639	13	78	+0.187	25	144	+0.151	37	210	+0.582
2	15	—0.382	14	84	—0.187	26	151	—0.309	38	215	—0.562
3	21	+0.310	15	89	+0.357	27	156	+0.175	39	220	+0.655
4	27	—0.400	16	94	—0.216	28	162	—0.322	40	226	—0.577
5	32	+0.162	17	100	+0.430	29	169	+0.585	41	231	+0.491
6	37	—0.314	18	105	—0.201	30	174	—0.352	42	237	—0.649
7	43	+0.078	19	110	+0.455	31	179	+0.726	43	241	+0.162
8	48	—0.135	20	116	—0.292	32	184	—0.435	44	246	—0.804
9	54	+0.040	21	121	+0.339	33	189	+0.651			
10	62	—0.187	22	127	—0.336	34	194	—0.453			
11	67	+0.156	23	132	+0.084	35	199	+0.542			

VI. Our autocorrelation functions show that the existence of 11-yearly cycles with double maximuma must be renounced, as they are assumed in [16—18]. If cycles with double maximuma exist, this could give a reflection to the appearance of all autocorrelation functions.

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АВТОКОРЕЛАЦИЯ НА СЛЪНЧЕВАТА АКТИВНОСТ

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(Резюме)

Изложени са резултататите от пресмятането на три автокорелационни функции за волфовите числа. Първата се отнася за синодичните волфови числа за −65 Rot до +1488 Rot. Втората е за месечните волфови числа почти за същия интервал от време (I. 1849—XII. 1964). Третата автокорелационна функция е пресметната върху годишните волфови числа за интервала 1700—1964 г.

Автокорелационните функции са представени графически. Приведени са стойностите на автокорелационните функции до τ, до което има смисъл да бъдат апроксимирани. Направена е оценка на продължителността на 11-годишния цикъл — 10,47 и 10,33 години. Определена е и продължителността на вековния цикъл на слънчевата активност — 169,0±0,5 години. Автокорелационните функции не потвърждават съществуването на двойни 11-годишни цикли (с два максимума).

АВТОКОРРЕЛЯЦИЯ СОЛНЕЧНОЙ АКТИВНОСТИ

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(Резюме)

Излагаются результаты исчисления трех автокорреляционных функций вольфовых чисел. Первая из них относится к синодическим вольфовым числам с 65 Rot по +1488 Rot. Вторая относится к месячным вольфовым числам для почти одного и того же промежутка времени (январь 1849— декабрь 1964). Третья автокорреляционная функция вычислена на основании годичных вольфовых чисел для промежутка времени с 1700 по 1964 г.

Автокорреляционные функции представлены графически. Приведены величины автокорреляционных функций до τ, до которого имеется смысл аппроксимировать их. Произведена оценка продолжительности одиннадцатилетнего цикла — 10,47 и 10,33 лет. Определена также продолжительность векового цикла солнечной активности — 169,0±0,5 года. Автокорреляционные функции не подтверждают существования двойных одиннадцатилетних циклов (с двумя максимумами).