БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ * BULGARIAN ACADEMY OF SCIENCES
ИЗВЕСТИЯ НА СЕКЦИЯТА ПО АСТРОНОМИЯ
BULLETIN OF THE SECTION OF ASTRONOMY
Том (Vol.) II

MULTIVARIATE DISTRIBUTION OF THE ORBITAL ELEMENTS OF ASTEROIDS .I

Ljubomir-Alexander Sadowsky and Marin Kalinkov

1. The distribution of the orbital elements of asteroids has been an object of continuous investigations in the last hundred years. A great number of papers, the purpose of which is to establish the peculiarities in distribution and their possible explanations are published. The study of asteroids has very great importance on account of the following reasons.

The origin of asteroids is unknown. As it is known, a large number of hypotheses concerning cosmogony of minor planets are offered. Some authors, who come out from original ideas of Olbers, support today also the hypothesis that asteroids are originating as products after explosion of a planet with orbit between orbits of Mars and Jupiter. Other hypothesis (which seems to explain the observed distribution of orbital elements of asteroids) supposes their origin from several centers or bodies.

Because of this several peculiarities in orbital elements distribution have not explanations up to now, for example, the Kirkwood gaps in the mean motions.

2. The complete statistical treatment of all orbital elements of asteroids is not yet made. The most important papers, on the results of which we shall base, are [1—15].

We shall investigate the possibilities which discover the following idea. Let us consider the five orbital elements (without the time of perihelion passage) as formed a linear five-space. It is clear the existence of Hirayama families will present as clusters of points. The introducing of distance in this linear five-space will help to distinguish separate groups or families (as one bears in mind the distances between the centres of clusters). Really, it is necessary to make several assumptions about the criterion for clustering in each cluster and about the separation of the groups, whose origin is caused by fluctuations. This may be established by two methods.

According to the first method it is necessary to divide the five-space to equal volume cubs, which are examined as elementary units and their contents will serve as a base for five-space statistics. Analogous methods are discussed in multivariate statistical analysis, particularly in testing of hypotheses (for example [16, 17]).

The second method supposes the determination of all possible distances between the points in five-space. Obviously, the calculations of distances (the number of which is $>10^6$) in our space may be made with the help of electronic computers. The second method is very acceptable, particularly when one forces limits by distances between separate groups and in this case a choice of all families may be easily made.

As a modification of the first method may be pointed out the determination of coefficients (or correlation ratio) between various parameters of asteroids and the determination of serial correlation coefficients.

The first method, applied to the published asteroid orbital elements [18] requests normalizing of the five coordinates, which can not be made without detail consideration of all element distributions. However remains the risk to exclude the elements (points of considered point set), which exceed by some parameters the limit values by the normalization.

Table 1

a	*n*	*a*	*n*	*a*	*n*	*a*	*n*	*a*	*n*	*a*	*n*	*a*	*n*	*a*	*n*	*a*	*n*
1	5	31	3	61	4	91	10	121	15	151	7	181	2	211	2	241	1
2	3	32	5	62	3	92	8	122	12	152	4	182	4	212	4	242	1
3	3	33	4	63	1	93	7	123	11	153	5	183	2	213	0	243	1
4	8	34	4	64	3	94	6	124	10	154	4	184	6	214	2	244	4
5	10	35	3	65	1	95	9	125	9	155	2	185	6	215	1	245	3
6	10	36	5	66	0	96	10	126	8	156	1	186	15	216	1	246	1
7	9	37	6	67	0	97	7	127	11	157	0	187	7	217	0	247	0
8	2	38	6	68	1	98	12	128	5	158	1	188	9	218	0	248	4
9	6	39	15	69	0	99	7	129	2	159	1	189	12	219	1	249	3
10	11	40	7	70	1	100	18	130	2	160	2	190	14	220	0	250	1
11	11	41	9	71	3	101	7	131	0	161	2	191	18	221	0	251	2
12	20	42	11	72	2	102	11	132	0	162	6	192	12	222	0	252	2
13	12	43	7	73	5	103	10	133	1	163	5	193	20	223	1	253	3
14	14	44	4	74	10	104	8	134	1	164	8	194	22	224	0	254	0
15	12	45	11	75	6	105	6	135	2	165	9	195	14	225	0	255	0
16	15	46	3	76	11	106	14	136	2	166	6	196	12	226	0	256	1
17	7	47	8	77	18	107	4	137	9	167	5	197	24	227	0	257	1
18	4	48	2	78	6	108	3	138	5	168	13	198	14	228	0	258	0
19	7	49	6	79	4	109	5	139	8	169	28	199	12	229	0	259	0
20	4	50	8	80	7	110	8	140	10	170	23	200	20	230	0	260	0
21	5	51	6	81	8	111	11	141	11	171	8	201	19	231	0	261	0
22	1	52	7	82	9	112	7	142	2	172	9	202	7	232	0	262	1
23	4	53	11	83	9	113	10	143	7	173	2	203	10	233	1	263	0
24	6	54	7	84	9	114	16	144	9	174	2	204	8	234	1	264	1
25	2	55	8	85	10	115	9	145	8	175	6	205	11	235	0	265	0
26	7	56	4	86	8	116	14	146	6	176	4	206	13	236	1	266	0
27	1	57	4	87	2	117	8	147	6	177	6	207	7	237	0	267	2
28	3	58	5	88	5	118	7	148	7	178	3	208	12	238	1	268	1
29	2	59	5	89	21	119	10	149	10	179	8	209	5	239	1	269	0
30	4	60	2	90	10	120	14	150	3	180	7	210	9	240	2	270	0

Another question must be discussed also — may be a statistical treatment on the usual orbital elements? The proper elements seem to be more representative for our purposes. It is clear that the first elements vary with time and conclusions on this consideration can not be valid for a very removed epoch. But correct statistics may be made as on usual elements [18], so on the proper elements.

In a series of papers we shall made a detail statistics on asteroip orbital elements.

In this paper the results about all one-dimensional distributions and several two-dimensional (or joint) distributions of asteroids orbital elements

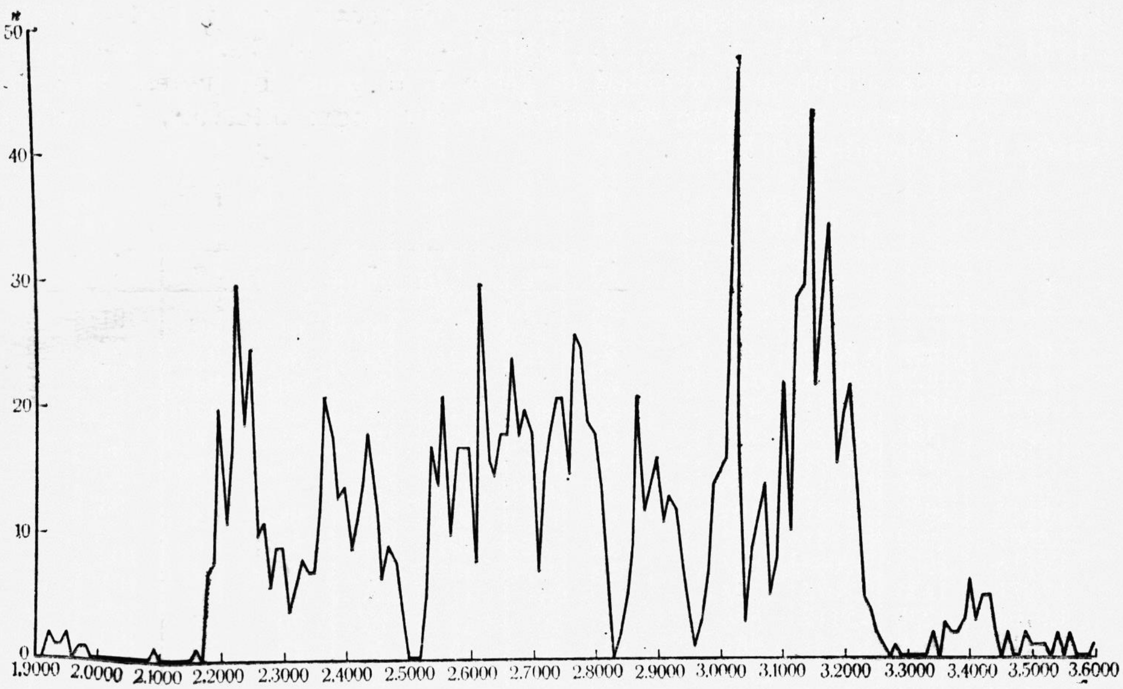

Fig. 1

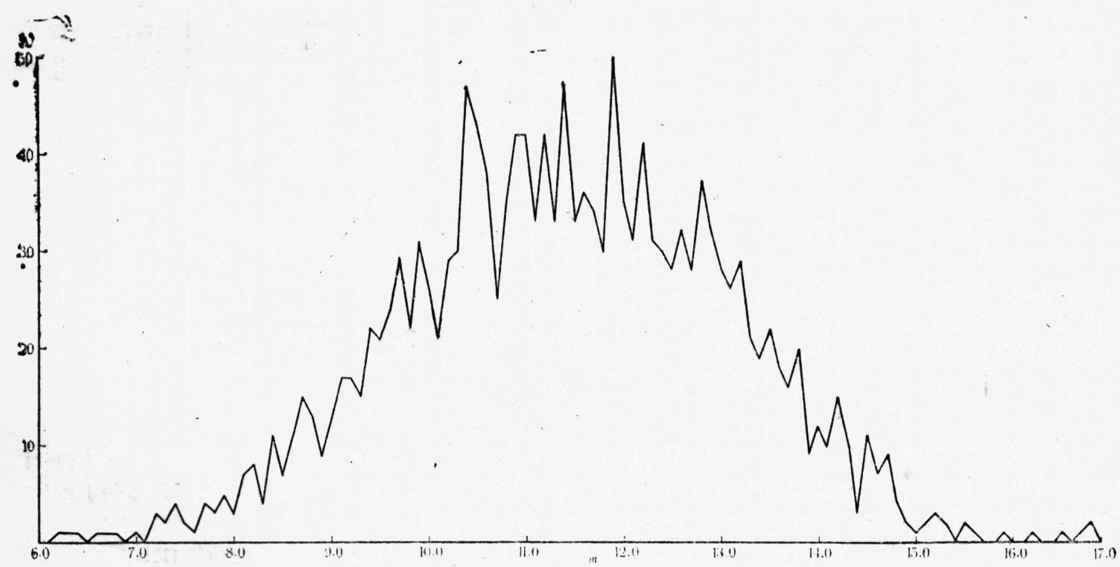

Fig. 2

(by [18]) are given. The one-dimensional distributions are conside*r*ed from many authors. We report these distributions because on them the next analysis is based, In reality we examine not 5, but 6 orbital elements, as added the ab solute magnitude g only. Thus we have 6 one-dimensional and 15 two-joint distributions.

3. Distribution of semi-major axes a is presented in Table 1 for the smallest intervals, for which it has sence of statistical distribution. The intervals 1, 2, 3, . . . , 270 respond to the next intervals by a 2.1701—2.1750, 2.1751—2.1800, 2.1801—2.1850, . . . , 3.5151—3.5200. In Table 2 are given these asteroids, which are excluded in Table 1. It must be noted that the distribution preserves its own characteristics for larger intervals, for which reason it is included in Fig. 1.

Table 2

a	No.	a	No.
1.0777	1566	4.2829	279
1.2442	1620	5.0958	1173
1.3677	1685	5.1211	524
1.4581	433	5.1328	911
1.8490	1600	5.1431	1437
1.8531	1355	5.1613	1208
1.8642	1627	5.1631	1404
1.8664	1509	5.1881	1172
1.8775	1656	5.2068	617
1.8971	1453	5.2076	1143
1.9102	1235	5.2112	588
1.9116	1019	5.2164	884
1.9223	1221	5.2224	1647
1.9337	1103	5.2368	659
1.9443	434	5.2765	1583
1.9472	1139	5.8139	944
1.9792	1025		
2.0893	330		
2.1537	1095		

Table 3

i	n	i	n	i	n	i	n
1	3	31	30	61	19	91	3
2	7	32	24	62	13	92	3
3	3	33	28	63	13	93	2
4	10	34	26	64	17	94	7
5	18	35	32	65	8	95	2
6	10	36	26	66	10	96	4
7	27	37	20	67	10	97	2
8	17	38	30	68	5	98	2
9	25	39	26	69	6	99	3
10	35	40	23	70	4	100	5
11	31	41	38	71	6	101	11
12	20	42	12	72	7	102	5
13	35	43	26	73	10	103	1
14	20	44	35	74	5	104	0
15	25	45	22	75	8	105	3
16	33	46	20	76	8	106	3
17	27	47	22	77	7	107	1
18	38	48	16	78	3	108	3
19	30	49	18	79	5	109	0
20	31	50	12	80	4	110	0
21	29	51	24	81	1	111	0
22	37	52	19	82	4	112	1
23	41	53	22	83	8	113	0
24	28	54	17	84	3	114	1
25	41	55	12	85	5	115	1
26	19	56	19	86	7	116	1
27	24	57	16	87	5	117	1
28	23	58	17	88	4	118	0
29	23	59	14	89	1	119	0
30	22	60	20	90	4	120	0
						121	1

Table 4

No.	i	No.	i	No.	i
594	32.641	1252	33.918	2	34.798
945	32.851	1301	33.933	1373	38.902
1208	33.698	531	34.545	944	42.993
				1580	52.026

As it is known, some peculiarities of n and φ around the gaps exist [19]. According to our results (which will be discussed in a following paper) analogous peculiarities take place by other elements also.

Table 3 contents the distribution of i (as to the tabulated intervals 1, 2, 3, . . . , 121 respond the next intervals by i 0°.001—0°.250, 0°.251—0°.500, 0°.501—0°.750, . . . , 30°.001—30°.250). The asteroids which are excluded out from Table 3, are given in Table 4.

Other distributions are presented in Table 5 — by Ω, Table 6 — by ω (as to the tabulated intervals 1, 2, 3, . . . , 144 respond the next intervals by

Table 5

Ω	n	Ω	n	Ω	n	Ω	n	Ω	n	Ω	n
1	17	25	17	49	14	73	15	97	6	121	9
2	15	26	16	50	10	74	11	98	12	122	11
3	11	27	11	51	13	75	18	99	11	123	11
4	18	28	9	52	14	76	8	100	7	124	12
5	11	29	17	53	13	77	11	101	7	125	5
6	10	30	15	54	18	78	9	102	16	126	8
7	10	31	11	55	13	79	6	103	8	127	9
8	12	32	7	56	9	80	10	104	11	128	9
9	10	33	16	57	15	81	15	105	12	129	11
10	12	34	17	58	15	82	15	106	9	130	14
11	9	35	12	59	16	83	13	107	7	131	14
12	10	36	12	60	11	84	10	108	10	132	15
13	14	37	16	61	9	85	13	109	8	133	18
14	7	38	13	62	11	86	13	110	5	134	13
15	15	39	14	63	15	87	15	111	9	135	13
16	17	40	10	64	13	88	7	112	5	136	7
17	12	41	11	65	14	89	12	113	11	137	7
18	21	42	11	66	11	90	4	114	7	138	17
19	13	43	20	67	10	91	7	115	11	139	11
20	16	44	8	68	7	92	11	116	7	140	11
21	6	45	13	69	11	93	10	117	15	141	10
22	12	46	4	70	14	94	10	118	11	142	18
23	6	47	18	71	13	95	11	119	7	143	19
24	13	48	6	72	16	96	7	120	16	144	17

Table 6

ω	n	ω	n	ω	n	ω	n	ω	n	ω	n
1	16	25	12	49	12	73	10	97	9	121	12
2	9	26	12	50	21	74	11	98	14	122	11
3	6	27	12	51	17	75	16	99	11	123	14
4	9	28	9	52	12	76	10	100	12	124	18
5	15	29	12	53	10	77	7	101	13	125	14
6	11	30	14	54	7	78	18	102	7	126	9
7	16	31	9	55	11	79	12	103	14	127	12
8	9	32	15	56	13	80	7	104	10	128	6
9	9	33	10	57	7	81	9	105	5	129	11
10	10	34	13	58	7	82	11	106	12	130	4
11	10	35	18	59	11	83	14	107	18	131	9
12	11	36	16	60	8	84	9	108	9	132	13
13	19	37	15	61	12	85	11	109	8	133	9
14	9	38	16	62	15	86	6	110	11	134	19
15	9	39	11	63	9	87	9	111	14	135	18
16	11	40	15	64	10	88	12	112	7	136	11
17	16	41	4	65	10	89	7	113	7	137	14
18	20	42	11	66	9	90	7	114	9	138	8
19	15	43	14	67	13	91	13	115	9	139	10
20	17	44	20	68	6	92	13	116	11	140	14
21	13	45	10	69	20	93	9	117	11	141	12
22	20	46	16	70	16	94	11	118	13	142	15
23	15	47	9	71	11	95	9	119	14	143	10
24	15	48	7	72	12	96	8	120	12	144	12

Table 7

φ	n	φ	n	φ	n	φ	n	φ	n	φ	n	φ	n	φ	n
1	5	7	48	13	92	19	75	25	48	31	16	37	4	43	1
2	10	8	60	14	70	20	70	26	46	32	14	38	8	44	1
3	144	9	66	15	67	21	73	27	31	33	13	39	2	45	1
4	24	10	61	16	66	22	59	28	37	34	12	40	7	46	1
5	45	11	71	17	59	23	63	29	30	35	13	41	8	47	1
6	31	12	72	18	73	24	60	30	30	36	10	42	1	48	0

Table 8

g	n	g	n	g	n	g	n	g	n	g	n	g	n
m 7.0	1	m 8.5	7	m 10.0	26	m 11.5	33	m 13.0	28	m 14.5	11	m 16.0	0
1	0	6	11	1	21	6	36	1	26	6	7	1	0
2	2	7	15	2	29	7	34	2	29	7	9	2	1
3	2	8	13	3	30	8	30	3	21	8	4	3	0
4	4	9	9	4	47	9	50	4	19	9	2	4	0
5	2	9.0	13	5	43	12.0	35	5	22	15.0	1	5	1
6	1	1	17	6	38	1	31	6	18	1	2	6	0
7	4	2	17	7	25	2	41	7	16	2	3	7	1
8	3	3	15	8	35	3	31	8	20	3	2	16.8	2
9	5	4	22	9	42	4	30	9	9	4	0	17.7	1
8.0	3	5	21	11.0	42	5	28	14.0	12	5	2	19.1	1
1	7	6	24	1	33	6	32	1	10	6	1		
2	8	7	29	2	42	7	28	2	15	7	0		
3	4	8	22	3	33	8	37	3	10	8	0		
8.4	11	9.9	31	11.4	47	12.9	32	14.4	3	15.9	1		

Ω and ω 0°.001—2°.500, 2°.501—5°.000, 5°.001—7°.500, . . . , 357°.501—360°.000); Table 7 — by φ (to tabulated intervals 1, 2, 3, . . . , 48 responds the next intervals by φ 0°.000—0°.500, 0°.501—1°.000, 1°.001—1°.500,. . ., 23°.501—24°.000) and Table 8 — by g. The excluding asteroids by φ and g are given in Tables 9 and 10 respectively.

Table 9

No.	φ	No.	φ
699	24°.003	1580	29°.497
1508	24.775	887	32.670
1665	24.99	710	32.722
1221	25.336	1036	32.854
1009	27.309	944	41.009
1134	27.777	1566	55.745
1474	29.345		

Table 10

No.	g
1	m 4.0
4	4.2
2	5.1
15	6.2
3	6.3
10	6.4
6	6.6
7	6.7
16	6.8

The distribution of asteroid absolute magnitude is shown on Fig. 2 and may be well approximated with the normal distribution. This fact must be considered as a confirmation that there is a large selection in the discovering and the observations of asteroids, or that all asteroids are discovered already. The last consideration can not be true.

4. The joint distribution of the semi-major axes and the absolute magnitudes is shown on Table 11. As it is shown g is decreasrng with the increasing of a. This dicreasing of g is mostly clear for the limit values of a (Fig. 3, below).

The distribution of g by a in intervals 0.01 demonstrate the same effect (Table 12, where are presented and the distributions of the other elements ω, Ω, i, φ — mean values for given interval by a).

The largest difference between the corresponding maximal values of g for the limiting a (Fig. 3, below) shows that in absolute magnitudes it is approximately equal to 4^m. This difference responds to variation of mean asteroid masses (if the mean density is constant) as function of a.

Table 11

g \ a	2.1701 — 2.2700	2.2701 — 2.3700	2.3701 — 2.4700	2.4701 — 2.5700	2.5701 — 2.6700	2.6701 — 2.7700	2.7701 — 2.8700	2.8701 — 2.9700	2.9701 — 3.0700	3.0701 — 3.1700	3.1701 — 3.2700	3.2701 — 3.3700	3.3701 — 3.4700	3.4701— 3.5700
m m														
16.1—17.0	1	1	0	1	2	0	0	0	0	0	0	0	0	0
15.1—16.0	6	1	0	1	0	1	0	0	0	0	0	0	0	0
14.1—15.0	32	19	7	4	4	1	0	0	0	0	0	0	0	0
13.1—14.0	80	26	25	13	17	6	9	1	1	4	0	0	0	0
12.1—13.0	41	18	35	32	47	39	19	14	23	31	14	0	1	0
11.1—12.0	6	6	18	16	37	37	38	33	55	78	34	1	4	2
10.1—11.0	2	10	13	8	29	39	29	25	51	71	41	5	9	2
9.1—10.0	3	6	15	7	31	35	17	11	17	35	17	1	11	2
8.1— 9.0	1	4	9	1	13	21	3	3	12	18	3	1	1	2
7.1— 8.0	1	1	4	1	3	0	4	4	1	5	2	0	1	0
6.1— 7.0	0	0	2	0	2	0	0	1	0	I	1	0	0	0

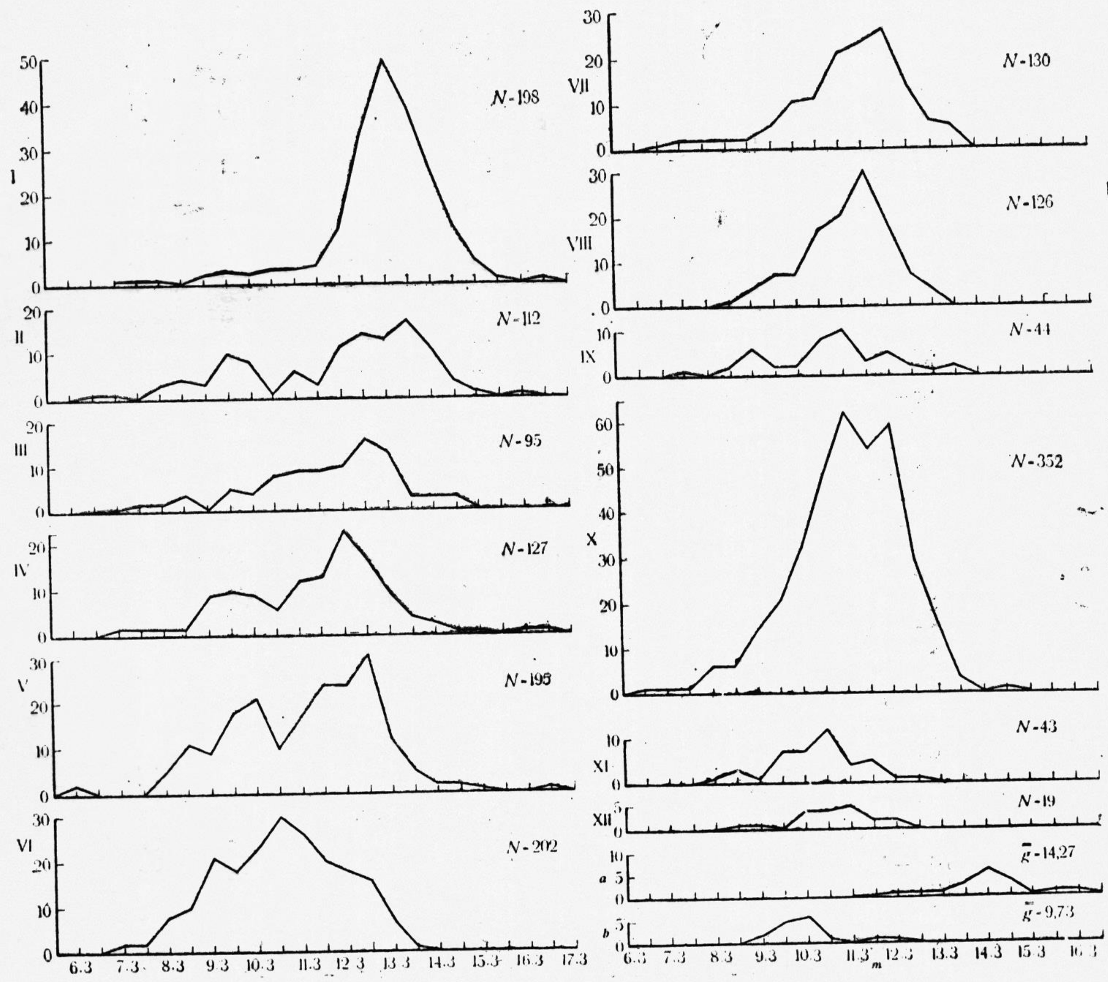

Fig. 3. The distribution of g as a function of a

a — the first 19 ; b — the last 16

Table 12

Intervals by a	$\bar{g}$	$\bar{\omega}$	$\bar{\Omega}$	$\bar{i}$	$\bar{\varphi}$	n
1	2	3	4	5	6	7
	m					
1.0777—2.1537	14.3	174°	185°	19.7	11.6	19
1—2	13.4	192	124	3.8	6.4	8
3—4	13.4	197	167	4.7	8.9	11
5—6	13.2	172	163	6.6	9.5	20
7—8	12.7	147	202	4.0	9.9	11
9—10	12.4	181	166	4.2	7.7	17
11—12	13.6	177	150	4.9	9.0	31
13—14	13.5	159	140	5.6	9.6	26
15—16	13.8	162	172	5.0	9.5	27
17—18	13.5	112	288	5.2	9.1	11
19—20	12.9	162	126	5.0	7.6	11
21—22	13.9	166	105	5.1	9.6	6
23—24	12.5	227	219	7.4	9.5	10
25—26	12.0	147	235	7.6	11.4	9
27—28	13.2	169	198	16.0	11.2	4
29—30	12.9	139	179	9.1	8.7	6
31—32	12.5	120	161	11.4	11.9	8
33—34	12.6	146	181	11.4	9.8	8
35—36	12.4	191	185	10.5	9.8	8
37—38	12.3	162	185	9.9	9.4	12
39—40	12.4	150	194	9.7	10.9	22
41—42	11.5	186	186	12.5	9.1	20
43—44	10.6	157	186	10.6	10.9	11
45—46	12.5	183	211	11.8	12.2	14
47—48	11.2	121	196	8.8	11.4	10
49—50	12.4	167	113	7.5	11.1	14
51—52	11.4	180	238	9.7	9.4	13
53—54	11.8	186	140	6.5	7.3	18
55—56	11.4	189	163	5.4	9.2	12
57—58	10.8	136	196	9.2	7.6	9
59—60	12.0	148	167	5.5	7.9	7
61—62	11.9	131	144	7.9	7.2	7
63—64	12.6	148	166	3.5	8.8	4
65—66	12.9	29	352	4.5	12.1	1
67—68	12.8	18	213	3.9	4.3	1
69—70	11.1	176	187	13.7	4.3	1
71—72	12.1	218	105	11.3	10.5	5
73—74	12.0	221	185	9.6	8.8	15
75—76	12.6	166	191	9.4	9.7	17
77—78	11.8	179	215	10.2	9.5	24
79—80	11.9	121	199	10.8	9.4	11
81—82	11.2	183	160	11.2	9.4	17
83—84	11.0	196	199	11.3	12.0	18
85—86	10.7	177	178	9.9	11.7	18
87—88	11.3	213	165	7.8	8.1	7
89—90	11.8	162	187	12.1	11.3	31
91—92	12.1	123	193	10.7	10.6	18
93—94	11.7	196	168	15.5	12.4	13
95—96	11.1	140	185	9.5	9.7	19
97—98	11.0	183	163	11.0	11.4	18
99—100	11.0	159	194	10.0	9.9	27
101—102	11.1	215	190	8.9	9.2	18
103—104	11.6	164	196	8.9	12.8	18
105—106	11.6	142	188	9.8	9.5	20
107—108	10.5	225	208	10.1	9.0	7
109—110	11.2	153	153	9.4	7.7	13

Continuation of Table 12

1	2	3	4	5	6	7
	m					
111—112	10.8	184°	147°	8.6°	9.2°	18
113—114	10.9	153	191	11.2	12.7	26
115—116	10.7	168	148	9.6	9.8	23
117—118	10.2	176	139	9.8	8.0	15
119—120	10.6	162	132	10.3	9.9	21
121—122	10.3	179	158	9.6	8.9	27
123—124	11.2	184	158	9.4	9.1	21
125—126	10.6	185	167	11.7	9.4	17
127—128	11.1	162	170	12.7	8.8	16
129—130	11.8	217	224	4.9	5.0	4
131—132						0
133—136	11.1	173	222	9.9	6.8	6
137—138	11.4	178	151	7.6	5.9	14
139—140	11.6	178	151	6.6	5.7	18
141—142	11.2	190	170	7.7	8.0	13
143—144	10.7	191	125	7.4	6.5	16
145—146	11.0	141	182	8.7	5.4	14
147—148	11.1	172	185	8.1	5.3	13
149—150	11.2	150	204	10.1	6.3	13
151—152	10.2	197	147	12.6	8.4	11
153—154	10.7	239	122	10.5	7.4	9
155—160	11.1	163	213	8.1	6.5	7
161—162	10.5	138	177	11.0	8.3	8
163—164	10.5	194	143	10.6	8.0	13
165—166	10.6	167	206	12.9	6.2	15
167—168	11.4	190	198	10.7	6.1	18
169—170	11.2	187	180	9.9	4.9	51
171—172	11.5	153	157	9.3	5.9	17
173—174	11.1	122	278	6.7	7.3	4
175—176	10.7	136	210	9.8	9.1	10
177—178	10.6	124	211	8.6	8.6	9
179—180	9.9	199	192	11.6	8.2	15
181—182	11.4	252	112	9.6	9.1	6
183—184	11.0	139	160	10.4	7.6	8
185—186	11.1	160	165	8.2	8.2	21
187—188	11.2	230	135	9.3	8.7	16
189—190	10.9	195	169	10.9	8.2	26
191—192	10.5	190	174	10.7	8.6	30
193—194	10.9	170	168	7.3	8.7	42
195—196	10.8	176	159	10.6	9.0	26
197—198	10.7	206	161	7.9	8.6	38
199—200	10.9	201	158	11.4	9.2	32
201—202	11.2	181	184	8.7	9.0	26
203—204	10.7	174	162	10.2	7.8	18
205—206	10,7	173	174	13.5	7.7	24
207—208	10.7	192	181	11.7	5.8	19
209—210	10.7	207	158	13.6	5.7	14
211—216	10.8	231	161	13.3	6.6	10
219—242	10.2	193	197	15.1	7.8	10
243—248	10.1	101	168	10.1	6.8	13
249—258	10.6	182	164	10.7	6.0	13
261	10.1	256	131	10.6	5.9	

The authors are grateful to comrades Peshev and Traikov of the Computer Centre by Central Statistical Institute for a serial calculation on tabulator computer machine.

It is a pleasure for us to acknowledge the help of our colleagues J. Zlateva, V. Ivanova, V. Kraichev and, specially, J. Kostova.

One of us (M. K.) is grateful to academician V. G. Fessenkov (USSR) and Dr. J. V. Batrakov (ITA — Leningrad) for some interesting discussions.

REFERENCES

1. Kirkwood D., Ann. Smith. Inst., 1876, and 1877.
2. Callandreanu O., Ann. Obs., Paris (mem.), **22**, 1886.
3. Hirayama K., Ann. Obs., Tokyo, **4**, 1918.
4. Hirayama K., Ann. Obs., Tokyo, **5**, 1918.
5. Klose A., AN, **218**, 1923.
6. Robe E., ZfA, **40**, No. 2, 1956.
7. Schütte K., AN, **233**, No. 5580, 1928.
8. Султанов Г. Ф., Труды сектора астроф., **1**, 1959, 22.
9. Чибисов А. Н., Труды ГАИШ, **9**, 1959, 130.
10. Путилин И. И., Малые планеты, Москва, 1953.
11. Самойлова-Яхонтова Н. С., УАН, **5**, 1950, 136.
12. Hagihara Y., Smith. Contr. Astroph., **5**, 1961, 59.
13. Brouwer D., AJ, **55**, 1950, 162.
14. Brouwer D., AJ, **56**, 1951, 9.
15. Леман Э., Проверка статистических гипотез (перев. с англ.), Москва, 1964.
16. Андерсон, Введение в многомерный статистический анализ (перев. с англ.), Москва, 1963.
17. Эфемериды малых планет на 1967 г., Москва — Ленинград, 1966.
18. Батраков Ю. В., Бюллетень ИТА, **6**, 1958.

Presented on February 7, 1967

МНОГОМЕРНО РАЗПРЕДЕЛЕНИЕ НА ОРБИТНИТЕ ЕЛЕМЕНТИ НА АСТЕРОИДИТЕ. I

Л.-А. Садовски и М. Калинков

(Резюме)

Работата представлява първа част от изследване на орбитните елементи на астероидите в многомерно пространство. Изложена е основната идея, която позволява определянето на астероидните фамилии. С помощта на табулаторни машини е намерено разпределението на пет орбитни елемента на астероидите заедно с абсолютната звездна величина. Резултатите са представени таблично. Разпределението на g е апроксимирано с гаусово разпределение. Намерени са и някои съвместни разпределения — на g, ω, Ω, φ и i по голямата полуос. Установена е промяна на средното g с изменението на голямата полуос a. Приведени са и номерата на астероидите, които не са включени в табличните разпределения.

МНОГОМЕРНОЕ РАСПРЕДЕЛЕНИЕ ОРБИТАЛЬНЫХ ЭЛЕМЕНТОВ АСТЕРОИДОВ. I

Л. Садовски и М. Калинков

(Резюме)

Предлагаемая работа является первой частью исследования орбитальных элементов астероидов в многомерном пространстве. Излагается основная идея, позволяющая определить семейства астероидов. При помощи табулирующих машин выведено распределение пяти орбитальных элементов астероидов вместе с абсолютной звездной величиной. Полученные результаты представлены в виде таблиц. Распределение g аппроксимировано с гауссовым. Установлены также некоторые совместные распределения g, ω, Ω, φ и i по большой полуоси. Установлено изменение среднего g с изменением большой полуоси a. Приводятся также номера астероидов, которые не включены в распределение на таблицах.