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MEASUREMENTS OF DIAMETERS OF GALAXIES ON THE PALOMAR OBSERVATORY SKY SURVEY

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1. INTRODUCTION

As it is known it is possible to receive several integral characteristics of the near galaxies from direct observations. As for the distant (and faint) galaxies, the number of the determinable integral characteristics sharply cuts down. Palomar Observatory Sky Survey (POSS) offers the possibilty for the determination of at least one integral characteristic — the apparent magnitude. At that the integral apparent magnitudes of the galaxies are received as secondary product — from the apparent angular diameters.

The angular diameters are secondary when images of galaxies are photometered upon plates and in a homogeneous system they are related to a given isophote (for instance Holmberg [1] — for surface photographic magnitude $26^m.5/\square''$). Evidently the application of a similar procedure on POSS is not correct (in spite of the numerous attempts — for example Denisyuk and Tymakova [2] — the results from the photometry on Palomar Survey prints do not agree with photometry on plates) and besides that it is inapplicable for faint galaxies. The measurement of diameters remains the sole possibility for the determination of magnitudes of galaxies on the POSS. Naturally the received magnitudes will be influenced by accidental and systematic errors, the main sources of which are: a) observed — subjective estimations; b) class (S, E, I, peculiar, close multiple — particularly interacting — systems); c) quality of the print of the POSS, etc.

The errors due to subjective factors are investigated by Holmberg [3] for measurements on plates (major diameters are overestimated). Holmberg's results are used for correction of the major diameters as well as for the sphericities (for example by Rood and Baum [4] with whose values we shall make comparison in a next paper). It is necessary to note that investigations of the subjective effects connected with the measurement of diameters on the POSS have not been carried out.

The errors due to the class or type of galaxies are errors in the true sense of the word only for the faint galaxies, for which the determination of class is merely impossible. The determination of the class (S, E and I) for the comparatively bright galaxies (for Palomar prints up to $18^m.0-18^m.5$) is still possible but about the limiting apparent magnitude only in too rare cases it is possible the distinction of S from E as well as normal from interacting galaxies. Therefore it is necessary to receive sure correlations between diameters of galaxies and magnitudes at least for the spirals and ellipticals up to a given magnitude with the purpose of working at extrapolation with averaged dependences. By the way an account of the influence of the interacting and multiple (indivisible) systems can be given if we know their mean spatial density (for example a great number of papers speak in favour of the statement that the relative spatial density of the interacting systems is $0.06 \leq \delta \leq 0.08$ [5]—[7]).

Another very important problem in that aspect is formulated by Vorontsov-Velyaminov [8] as the problem of the comparison of the dimensions of galaxies. According to Vorontsov-Velyaminov the apparent angular diameters of S and E galaxies are generally incomparable if we do not use for S galaxies the dimensions only for the spiral component (or spiral structure). On the other hand the dimensions of the spiral structure are estimated more uncertainly. Therefore Vorontsov-Velyaminov's suggestion is inapplicable to our case. But in the present paper an attempt is made to avoid the effects discussed in [8].

Concerning the quality of each print of the POSS it influenced the confidence limits of the determined correlations.

The determination of magnitudes requires different accuracy depending on the nature of the treated problems. For example an error of $\pm 0^m.2$ to $\pm 0^m.5$ is admissible when the luminosity function of galaxies is investigated. In many cases an error of $\pm 0^m.5$ is admissible for count of galaxies up to a given limiting magnitude.

The main purposes of the present paper are:

1. Investigation of subjective effects for measurements of diameters of galaxies on the Palomar prints and their influence on the receiving integral magnitudes.
2. On the basis of Zwicky [9]—[11], Vorontsov-Velyaminov's [12]—[15] and G. and A. de Vaucouleurs's [16] catalogues to determine the correlations between the diameters of galaxies, measured on the Palomar prints and the magnitudes (Zwicky) up to $15^m.7$.
3. Extrapolation (nonlinear) of these correlations from $15^m.7$ up to the limiting magnitude for the POSS and determination of the confidence intervals of the extrapolation.
4. To receive the connection between the magnitudes according to Zwicky and the magnitudes $B(0)$ according to [16].

Here are given the results for POSS prints 1398 O and 64 O which are studied in detail, aiming the investigation of the surface and spatial distribution of galaxies in these regions on the celestial sphere.

The method of measurement of diameters of galaxies on the Palomar prints is the same, used in [17]—[19]. As it had been shown the optimal magnification for measurement of diameters of galaxies on the POSS is 15×. All measurements are carried out with 16× magnification with the help of eyepiece scale. One scale division is 0.050 mm which at a scale of the Palomar prints $67''.1$ mm^{-1} corresponds to $3''.555$. Everywhere in the present paper the diameters are given in arc seconds.

All measurements are carried out independently by three authors. Four quantities are measured for every galaxy, if possible: D''_1, D''_2 — major external and minor external diameters and d''_1, d''_2 — major internal and minor internal diameters.

The external diameters are measured to the limits where the peripheral parts of the galaxies pass into the background and the internal diameters — with respect to the maximal gradient of blackening. Measurements show that in spite of the absence of a quantitative criterion for determination of the isophotes, up to which the diameters are measured, the deviations and variances of the mean diameters for given stellar magnitudes are not considerable.

Note. Since the external and internal diameters are measured with different accuracy, for the determination of the stellar magnitudes the introduction of some kinds of combination between D'' and d'' may be applied.

2. PRINT 1398 O MEASUREMENTS

Print 1398 has equatorial coordinates (of the center) $\alpha_{1950}=12^h12^m48$ and $\delta_{1950}=+29°28'18''$ [20] and galactic coordinates $l^{II}=196°.28$ and $b^{II}=+81°.78$ [21]. This print corresponds to F.158 of Zwicky's catalogue [10] and to field 5-29 of Vorontsov-Veljaminov's catalogue [13]. According to [10] the investigated region has $12^h00^m \lesssim \alpha_{1950} \lesssim 12^h26^m$, $+26°.5 \lesssim \delta_{1950} \lesssim +32°.5$, and according to [13] $12^h00^m.0 \leqq \delta_{1950} \leqq 12^h25^m.9$, $+26°30' \leqq \delta_{1950} \leqq +32°29'$.

All galaxies catalogued for F.158 are measured. The galaxies are numbered according to α as they are arranged in [10] (that is why the coordinates of the galaxies and the identification according to NGC or IC are not given here).

All measurements on print 1398 O are given in Tables 1 and 2. Table 1 contains the external diameters D''_1 and D''_2, the corresponding mean values for the three authors ($\bar{D}''$ and $\bar{d}''$) as well as Vorontsov-Velyaminov diameters [13], transformed in arc seconds. The corresponding internal diameters as well as the magnitudes according to Zwicky are included in Table 2. The last column contains the type of the surely classified galaxies.

Zwicky's catalogues contain magnitudes of galaxies up to $15^m.7$. Therefore Tables 1 and 2 do not include all galaxies measured by Vorontsov-Velyaminov in the investigated field. The identification of galaxies from [13] according to [10] shows that some objects cannot be surely identified. For that reason several doubtful cases are excluded from the tables, i. e. Vorontsov-Velyaminov's diameters for these cases are not given.

A small part of the galaxies in the investigated field are classified in [10] and the transition from Vorontsov-Velyaminov's morphological classification into the standard classification is not always definitive [12], [22]. All measured galaxies are by the authors and compared with other sources [1], [23]—[25]. Finally, only the sure classification is given in the last column of Table 2. The class is not indicated in the cases of disagreement between different sources.

Table 1

No.	Kal. (Kalinkov)			M. (Mihnevsky)			Kar. (Karadjov)			V.-V. (Vorontsov-Velyaminov)		
	D_1''	D_2''	$\overline{D}''$	D_1''	D_2''	$\overline{D}''$	D_1''	D_2''	$\overline{D}''$	D_1''	D_2''	$\overline{D}''$
1	2	3	4	5	6	7	8	9	10	11	12	13
1	148	77	112	117	64	90	117	67	92			
2	111	34	72	111	27	69	114	24	69			
3	30	24	27	44	20	32	47	17	32			
4	30	30	30	18	15	17	17	13	15			
5	104	20	62	112	22	67	80	17	48	72	12	42
6	67	24	45	34	13	24	30	13	22			
7	44	34	39	40	28	34	24	24	24	24	24	24
8	268	121	195	262	94	178	235	84	159	258	108	183
9	50	47	49	54	50	52	47	37	42	42	42	42
10	34	17	26	30	17	24	30	12	21			
11	30	24	27	24	20	22	20	17	18			
12	80	47	64	74	40	57	70	34	52	66	30	48
13	74	60	67	80	47	64	60	37	48	90	90	90
14	37	34	36	37	24	30	24	20	22	36	36	36
15	50	30	40	30	20	25	34	20	27	30	18	24
16	141	50	96	74	30	52	67	17	42			
17	40	37	38	37	17	27	37	13	25			
18	50	20	35	44	15	30	44	17	30	42	12	27
19	44	18	31	57	15	36	40	12	26	30	12	21
20	47	30	38	30	18	24	34	17	26			
21	57	20	38	57	17	37	50	13	32			
22	24	24	24	24	20	22	20	17	18			
23	30	30	30	24	20	22	27	24	26			
24	134	77	106	131	47	89	60	34	47			
25	37	20	28	40	18	29	30	12	21			
26	74	47	60	47	30	38	37	34	36	42	42	42
27	40	34	37	20	17	18	17	15	16			
28	40	17	28	44	20	32	40	17	28			
29	80	54	67	77	40	58	44	24	34	72	30	51
30	57	30	44	54	24	39	47	18	32	60	18	39
31	148	60	104	141	40	90	117	40	78	114	48	81
32	87	34	60	97	24	60	60	17	38	72	18	45
33	57	57	57	40	34	37	34	30	32			
34	268	235	252	225	180	203	185	174	180	240	210	225
35	101	64	82	107	74	90	77	64	70	90	60	75
36	87	74	81	64	57	60	65	54	60	54	48	51
37	128	107	117	128	94	111	84	54	69			
38	57	37	47	57	27	42	40	20	30			
39	44	37	40	34	27	30	20	20	20	30	24	27
40	37	20	28	44	24	34	37	17	27	36	12	24
41	117	57	87	94	50	72	60	40	50			
42	47	20	34	54	24	39	37	17	27			
43	319	44	182	252	47	150	158	20	89	270	30	150
44	50	20	35	60	27	44	40	17	28	36	12	24
45	124	30	77	107	27	67	101	20	60	90	18	54
46	67	30	48	57	24	40	50	17	34	42	18	30
47	168	134	151	128	94	111	111	80	96	144	102	123
48	64	24	44	57	17	37	37	10	24	60	24	42
49	40	30	35	40	27	34	27	17	22			
50	60	47	54	77	40	59	37	28	32	48	30	39
51	40	13	26	44	15	30	27	12	20			
52	24	18	21	24	18	21	17	15	16	18	18	18
53	101	—	—	94	47	70	25	24	24	54	54	54

Continuation of Table 1

1	2	3	4	5	6	7	8	9	10	11	12	13
54	50	30	40	47	30	38	40	22	31	36	24	30
55	74	24	49	70	20	45	64	17	40	66	42	54
56	40	30	35	64	47	56	37	24	30	36	24	30
57	84	12	48	74	13	44	67	15	41	72	12	42
58	20	15	18	27	17	22	18	17	18			
59	184	168	176	158	114	136	87	67	77	180	120	150
60	252	117	184	161	84	122	151	57	104	240	180	210
61	74	60	67	60	57	59	50	44	47	48	42	45
62	50	34	42	47	24	36	30	22	26			
63	25	24	24	27	24	26	20	17	18			
64	57	44	50	60	27	44	50	17	34	66	48	57
65	34	24	29	30	18	24	20	17	18			
66	54	50	52	50	30	40	34	30	32	54	54	54
67	34	30	32	27	24	26	20	17	18			
68	30	20	25	24	17	20	17	13	15			
69	47	40	44	44	34	39	27	24	26	24	24	24
70	57	40	48	97	40	68	57	37	47	60	36	48
71	470	168	319	419	128	274	205	80	143	450	150	300
72	50	47	48	64	34	49	34	30	32			
73	40	40	40	57	50	54	44	40	42	48	36	42
74	24	17	20	30	17	24	24	17	20			
75	27	20	24	27	22	24	17	15	16			
76	117	84	100	124	54	89	67	54	60	114	72	93
77	184	168	176	213	173	193	101	94	97			
78	24	20	22	20	17	18	15	13	14			
79	40	24	32	47	24	36	34	20	27	42	24	33
80	60	57	59	70	47	59	47	40	44			
81	40	30	35	40	24	32	30	25	28	30	24	27
82	40	24	32	47	34	40	37	24	30	36	24	30
83	87	57	72	84	50	67	74	27	50	96	48	72
84	60	20	40	70	24	47	34	13	24	48	18	33
85	37	30	34	47	27	37	20	17	18			
86	27	27	27	24	20	22	22	17	20			
87	40	20	30	37	20	28	30	17	24			
88	60	50	55	50	37	44	37	30	34			
89	30	20	25	37	20	28	20	15	18			
90	67	64	66	77	60	68	70	64	67	72	66	69
91	54	30	42	47	40	44	44	40	42	27	27	27
92	111	67	89	117	47	82	80	35	58	120	54	87
93	235	235	235	268	201	235	158	54	106	270	240	255
94	57	30	44	40	27	34	34	20	27			
95	40	20	30	47	27	37	37	20	28			
96	124	17	70	124	13	68	104	10	57	120	12	66
97	37	20	28	44	20	32	44	13	28			
98	40	34	37	47	34	40	64	27	46	36	36	36
99	252	50	151	228	40	134	151	30	91	198	42	120
100	80	54	67	80	47	64	50	44	47	84	84	84
101	101	20	60	117	24	70	67	13	40	72	9	40
102	84	74	79	94	64	79	67	50	58	78	78	78
103	37	13	25	34	17	26	34	13	24			
104	184	184	184	191	154	172	151	124	138	180	180	180
105	77	27	52	70	34	52	54	30	42	60	24	42
106	30	17	24	34	17	26	27	12	20			
107	40	30	35	44	10	27	18	15	16			
108	235	168	201	233	208	223	188	101	144	180	84	132
109	57	27	42	50	24	37	40	17	28			
110	37	24	30	37	22	30	30	17	24			
111	37	30	34	30	20	25	20	17	18			

Continuation of Table 1

1	2	3	4	5	6	7	8	9	10	11	12	13
112	34	30	32	111	64	88	101	60	80	96	66	81
113	252	101	176	218	91	154	195	60	128	192	66	129
114	37	20	28	40	27	34	22	17	20			
115	70	20	45	67	13	40	60	10	35			
116	30	30	30	—	—	—	18	17	18			

The galaxies No. 11, 16, 32, 51, 53, 82, 104, 109, 110 and 116 are excluded from further analysis because they are close double unresolved (and interacting) systems or at least one of the authors has not measured D'' or d''.

Evidently, it must be checked up whether Zwicky magnitudes are in good agreement with magnitudes of other authors. For that purpose a sample is made from the Catalogue of galaxies [16] contained in F. 158 for which photometrical data are given (Table 3). The first column contains NGC number and the second column — the serial number from Tables 1 and 2. m_H is the Harvard photographic apparent magnitude [26]; m_C — Harvard photographic magnitude statistically corrected to the $B(0)$ system ([16], [27]). The mean error of the corrected Harvard magnitude is of the order of $\pm 0^m.2$ to $\pm 0^m.3$ depending on magnitude, diameter and axis ratio. $B(0)$ is the integrated magnitude in the B system within a circle of diameter $D(0)$ derived by interpolation or extrapolation from photoelectric and modern photographic data in (or reduced to) the standard B system [28]. $B'(0)$ is the mean B surface brightness in mag./□′ within a circle of diameter $D(0)$, according to the relation $B'(0)=B(0)+5\lg D(0)-5.26$ and m'_C is the mean B surface brightness derived from the corrected Harvard magnitude $m'_C=m_C+5\lg D(0)-5.26$.

The final column of Table 3 contains Zwicky magnitude.

As it is seen from Tables 1 and 2 definite connections exist between $\bar{D}''$ and $\bar{d}''$, measured by the authors and the magnitudes (or surface brightness) according to different sources (Table 3). Naturally the confidence intervals are rather wide.

Here we are interested only in the connection m_{Zw} — $B(0)$. Obviously that connection is linear with insignificant variance at that. The number of galaxies is only 10 (without NGC 4136). The method of the least squares allows the determination of the parameters of the linear regression

$$m_{Zw}=\underset{\pm 41}{11.810}+\underset{\pm 629}{1.0254}\,[B(0)-12.121]=-0.619+1.025B(0) \tag{1}$$

with equation for the confidence limits of the theoretical regression (for 1σ)

$$s^2=0.00166+0.00395\,[B(0)-12.121]^2. \tag{1'}$$

The correlation coefficient between m_{Zw} and $B(0)$ is $r=0.958$. The second linear regression is given by

$$B(0)=\underset{\pm 39}{12.121}+\underset{\pm 58}{0.9468}\,(m_{Zw}-11.81)=0.939+0.947m_{Zw} \tag{2}$$

Table 2

Na	Kal.			M.			Kar.			V.-V.			m_{Zw}	Type
	d''_1	d''_2	$\overline{d''}$	d''_1	d''_2	$\overline{d''}$	d''_1	$\overline{d''_2}$	d''	d''_1	d''_2	$\overline{d''}$		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	30	24	27	35	22	28	34	20	27				$14^m.3$	S
2	30	13	22	27	15	21	34	17	26				14 .4	S
3	20	10	15	24	12	18	24	10	17				15 .4	E
4	12	12	12	10	8	9	13	10	12				15 .5	E
5	40	10	25	64	12	38	60	10	35				15 .2	S
6	20	12	16	24	10	17	17	12	14				15 .3	S
7	20	13	16	28	15	22	20	17	18				15 .7	
8	50	30	40	116	56	86	151	54	102	90	48	69	11 .9	S
9	24	18	21	27	20	24	30	20	25	24	18	21	14 .0	S
10	27	12	20	27	10	18	28	10	19				15 .2	
11	20	10	15	20	10	15	17	12	14				15 .7	
12	22	18	20	60	24	42	60	24	42	18	18	18	14 .0	S
13	8	8	8	7	5	6	13	10	12				15 .1	S
14	15	15	15	18	18	18	17	13	15	18	18	18	14 .8	
15	17	12	14	22	12	17	20	10	15	24	12	18	15 .2	
16	12 12	10 10	11 11	17	12	14	13	12	12				15 .1	
17	10	10	10	24	20	22	24	10	17				15 .6	
18	13	12	12	18	10	14	17	13	15	18	12	15	15 .6	S
19	22	10	16	35	10	22	20	10	15				15 .6	S
20	15	13	14	17	15	16	17	13	15				15 .4	
21	13	8	10	20	12	16	17	13	15				15 .7	
22	12	12	12	13	12	12	13	13	13				15 .4	
23	12	12	12	15	13	14	17	15	16				15 .3	
24	34	18	26	37	20	28	37	20	28	42	24	33	13 .7	E
25	17	8	12	24	12	18	20	8	14				15 .7	S
26	13	13	13	17	15	16	15	13	14	15	15	15	15 .2	
27	13	13	13	13	12	12	12	10	11				15 .5	
28	17	13	15	24	13	18	17	12	14				15 .7	S
29	28	15	22	30	15	22	30	17	24	30	18	24	14 .1	S
30	24	12	18	27	15	21	27	13	20	27	15	21	14 .6	S
31	44	24	34	54	24	39	30	24	27	30	24	27	13 .8	S
32	—	—	—	—	—	—	30	7	18	30	9	20	15 .7	
33	13	13	13	20	18	19	17	17	17	18	18	18	14 .4	E
34	77	56	63	57	44	50	64	44	54	60	60	60	12 .1	S
35	7	7	7	17	12	14	13	10	12	18	18	18	15 .3	S
36	13	10	12	17	15	16	17	12	14	18	15	16	13 .8	S
37	47	37	42	74	44	59	50	37	44	60	42	51	12 .6	E
38	17	13	15	27	15	21	17	15	16				15 .3	S
39	12	10	11	13	12	12	13	12	12	15	15	15	15 .1	E
40	22	12	17	27	13	20	20	13	16	24	12	18	15 .3	
41	40	27	34	47	24	36	37	25	31	42	24	33	12 .9	E
42	30	13	22	34	17	26	32	15	24	30	15	22	14 .8	E
43	74	10	42	104	17	60	97	17	57				13 .7	S
44	24	12	18	37	17	27	34	13	24				14 .3	S
45	27	15	21	47	20	34	40	13	26				14 .2	S
46	37	13	25	37	13	25	30	13	22	36	18	27	15 .0	
47	17	12	14	24	24	24	20	10	15	24	18	21	13 .5	S
48	34	8	21	37	10	24	34	8	21	36	9	22	15 .7	
49	13	8	10	13	13	13	13	12	12				15 .7	S
50	24	18	21	30	25	28	24	20	22	24	21	22	13 .7	E
51	13	8	10	24	12	18	17	10	14				15 .3	
52	10	10	10	13	12	12	12	10	11	12	12	12	15 .4	

Continuation of Table 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
53	17 7	15 7	16 7	20	15	18	17	15	16	24	18	21	$14^m.4$	
54	20	10	15	37	13	25	37	17	27				14 .6	
55	17	12	14	24	13	18	18	12	15	18	12	15	15 .3	S
56	7	7	7	30	25	28	17	7	12				15 .5	S
57	13	8	10	30	10	20	24	10	17				15 .5	S
58	7	5	6	13	13	13	10	8	9				15 .7	
59	47	30	39	44	30	37	40	34	37	78	60	69	12 .4	S
60	77	37	57	74	44	59	74	37	55	138	60	99	11 .5	S
61	30	15	23	30	13	22	30	20	25	24	18	21	13 .7	S
62	10	8	9	15	10	12	10	7	8				15 .7	S
63	7	7	7	7	7	7	8	7	8				15 .6	S
64	27	13	20	32	15	24	30	15	22	30	12	21	14 .5	S
65	15	12	14	17	12	14	13	10	12				15 .7	E
66	8	7	8	10	10	10	8	7	8	18	15	16	15 .4	
67	10	8	9	15	13	14	13	12	12	12	12	12	15 .5	E
68	12	8	10	13	10	12	10	8	9	12	10	11	15 .7	E
69	10	8	9	15	13	14	12	10	11	12	10	11	15 .4	E
70	8	5	6	10	8	9	10	7	8				15 .3	
71	60	47	54	77	54	66	70	54	62	66	48	57	11 .1	S
72	20	17	18	24	20	22	20	18	19	24	18	21	14 .2	E
73	34	34	34	37	34	36	35	34	34				13 .4	
74	13	10	12	17	12	14	12	10	11				15 .6	
75	13	10	12	15	12	14	12	10	11				15 .6	E
76	13	7	10	27	13	20	13	10	12	30	30	30	14 .9	S
77	67	54	60	77	57	67	67	64	65	75	75	75	11 .2	E
78	10	8	9	12	12	12	12	10	11				15 .7	E
79	7	7	7	12	10	11	12	7	10				15 .7	S
80	34	30	32	30	28	29	30	27	28	36	36	36	13 .1	E
81	24	20	22	24	18	21	22	18	20				14 .5	
82	—	—	—	27	20	24	27	13	20				15 .0	
83	15	10	18	17	13	15	17	15	16	18	12	15	14 .7	S
84	20	10	15	27	10	18	22	10	16	24	12	18	15 .7	
85	15	12	14	18	15	16	13	13	13	18	15	16	15 .0	
86	10	10	10	24	10	17	12	10	11				15 .7	E
87	10	8	9	37	12	24	13	10	12				15 .7	
88	20	17	18	24	20	22	18	17	18	24	24	24	14 .3	E
89	13	10	12	18	10	14	15	10	12	18	12	15	15 .5	S
90	17	13	15	30	18	24	17	13	15	24	18	21	14 .3	
91	7	7	7	17	15	16	10	7	8	18	18	18	15 .7	
92	34	17	25	40	18	29	34	20	27	36	18	27	13 .5	E
93	54	37	45	144	50	97	97	47	72	126	48	87	11 .5	S
94	7	7	7	27	17	22	24	17	20				15 .3	
95	5	3	4	27	12	20	10	7	8				15 .7	
96	34	7	20	44	10	27	44	7	26				15 .6	S
97	20	10	15	22	12	17	17	12	14				15 .3	
98	13	13	13	22	15	18	17	15	16	18	18	18	15 .1	S
99	60	20	40	80	17	49	74	17	45	72	18	45	13 .9	S
100	34	27	30	32	28	30	34	30	32	12	12	12	13 .9	S
101	24	8	16	20	13	22	30	10	20				15 .4	S
102	10	5	8	13	10	12	10	8	9	12	12	12	15 .7	S
103	8	8	8	10	10	10	10	10	10				15 .7	S
104	—	—	—	30	24	27	30	20	25	60	24	42	13 .8	S
105	17	12	15	27	17	22	—	—	—	27	12	20	15 .1	S
106	12	10	11	24	13	18	13	10	12				15 .5	S
107	13	10	12	24	7	16	15	12	14				15 .2	S
108	94	54	74	161	77	119	134	77	106				10 .9	E
109	10 5	10 3	10 4	10	10	10	10	8	9				15 .7	S

Continuation of Table 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
110	—	—	—	20	15	18	17	10	14				$15^m.7$	
111	15	13	14	18	13	16	13	12	12				15 .3	
112	17	15	16	15	15	15	15	13	14	33	24	28	14 .4	S
113	84	30	57	91	40	65	94	35	64	84	42	63	11 .9	S
114	7	7	7	7	7	7	7	5	6				15 .7	S
115	8	5	6	47	10	28	50	8	29				15 .7	S
116	15	15	15	—	—	—	12	12	12				15 .1	

Table 3

NGC	No.	m_H	m_C	$B(0)$	$B'(0)$	m'_C	m_{Zw}
4062	8	12.1	12.10	12.17	14.01	13.94	11.9
4136	34	12.1	11.76			14.35	12.1
4150	37	12.6	12.65	12.83	13.37	13.19	12.6
4245	59	12.3	12.51	12.57	13.46	13.40	12.4
4251	60	11.6	11.89	12.03	12.87	12.73	11.5
4274	71	11.7	11.52	11.50	13.99	14.01	17.1
4278	77	11.6	11.74	11.58	12.62	12.78	11.2
4283	80	12.8	13.09	13.42	12.81	12.48	13.1
4314	93	11.7	11.52	11.61	14.05	13.96	11.5
4414	108	11.1	11.59	11.21	13.00	13.38	10.9
4448	113	11.9	12.07	12.29	13.83	13.61	11.9

with confidence limits

$$s^2=0.00153+0.00337\,(m_{Zw}-11.81)^2. \tag{2'}$$

The regression (1) together with the observations is given on Fig. 1 where the confidence limits (internal) refer to 1σ and are calculated from (1'). The 95% confidence limits (external lines) for t (2.5%, 10−2)=2.31 are given on Fig. 1 too.

The mean square errors in the regression coefficients (1) show that we may use in first approximation

$$m_{Zw}=B(0)-0.62 \tag{3}$$

and for (2)

$$B(0)=0.95m_{Zw}+0.94. \tag{4}$$

In this way the transition between m_{Zw} and $B(0)$ is already established

The agreement between the measured mean diameters on the Palomar print and $B(0)$ for the 10 galaxies from Table 3 is shown on Fig. 2 for Vorontsov-Velyaminov (exception! — only 7 galaxies for $\overline{D}''$ and 9 galaxies for $\overline{d}''$), Fig. 3 for Kalinkov, Fig. 4 for Mihnevsky and Fig. 5 for Karadjov. The method of least squares gives for the linear regressions and the confidence limits (Fig. 2—5):

$$\overline{D}''_{V.V}=\underset{\pm 24.8}{194.1}-\underset{\pm 55.29}{51.80}\,[B(0)-11.91]=811.1-51.80B(0), \tag{5}$$

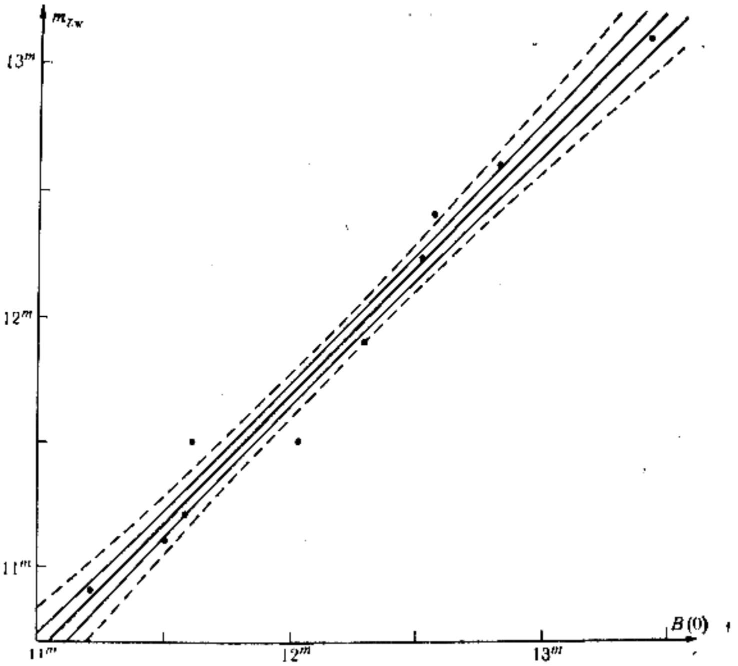


Fig. 1

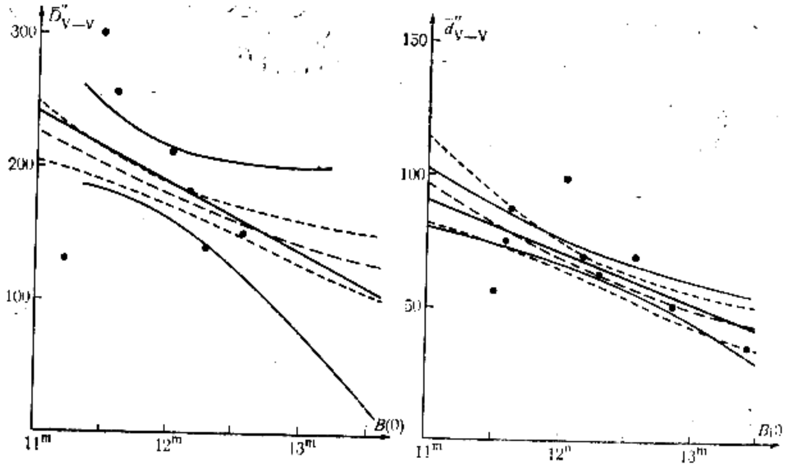


Fig. 2

(5′) $$s^2 = 616 + 3057\,[B(0) - 11.91]^2;$$

(6) $$\underset{\pm 5.1}{\overline{d}''_{V\text{-}V}} = \underset{\pm 8.41}{67.3 - 19.17}\,[B(0) - 12.22] = 301.6 - 19.17\,B(0)$$

(6′) $$s^2 = 25.76 + 70.73\,[B(0) - 12.22]^2;$$

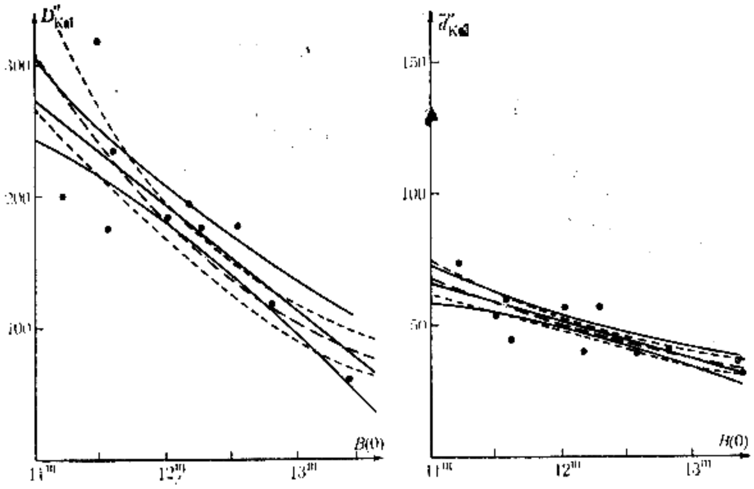


Fig. 3

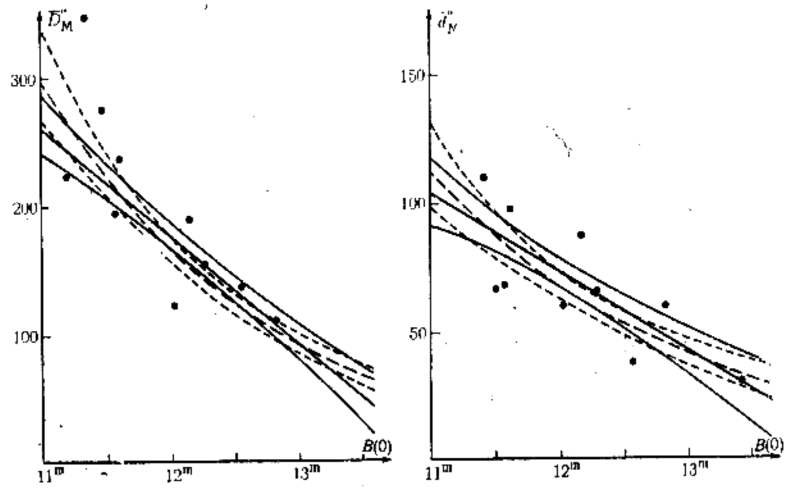


Fig. 4

$$\bar{D}''_{\mathrm{Kal}} = \underset{\pm 13.6}{183.8} - \underset{\pm 20.96}{79.84}\,[B(0) - 12.12] = 1151.5 - 79.84B(0), \tag{7}$$

$$s^2 = 184.62 + 439.53\,[B(0) - 12.12]^2; \tag{7'}$$

$$\bar{d}''_{\mathrm{Kal}} = \underset{\pm 2.6}{50.0} - \underset{\pm 3.99}{14.56}\,[B(0) - 12.12] = 226.4 - 14.56B(0), \tag{8}$$

$$s^2 = 6.68 + 15.89\,[B(0) - 12.12]^2; \tag{8'}$$

$$\bar{D}''_{\mathrm{M}} = \underset{\pm 9.7}{168.5} - \underset{\pm 14.91}{85.18}\,[B(0) - 12.12] = 1200.9 - 85.18B(0), \tag{9}$$

(9′) $$s^2=93.34+222.22\,[B(0)-12.12]^2;$$

(10) $$\underset{\pm 5.5\ \ \pm 8.53}{\overline{d}''_{\mathrm{M}}=68.4-30.77}\,[B(0)-12.12]=441.3-30.77B(0),$$

(10′) $$s^2=30.56+72.75\,[B(0)-12.12]^2;$$

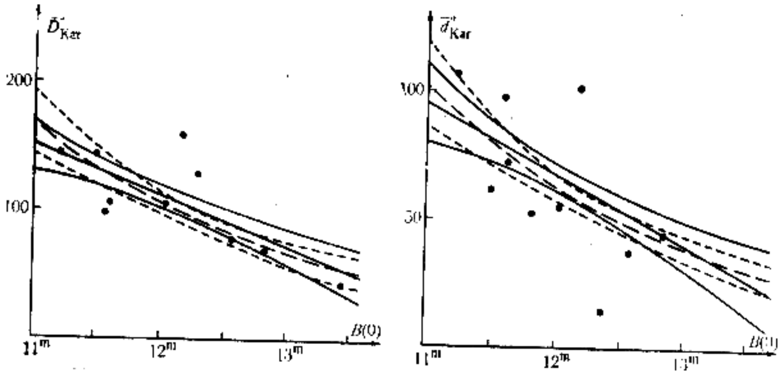


Fig. 5

(11) $$\underset{\pm 8.4\pm 13.00}{\overline{D}''_{\mathrm{Kar}}=107.1-39.51}\,[B(0)-12.12]=586.0-39.51B(0),$$

(11′) $$s^2=70.96+168.95\,[B(0)-12.12]^2;$$

(12) $$\underset{\pm 5.9\ \ \pm 9.12}{\overline{d}''_{\mathrm{Kar}}=63.5-26.67}\,[B(0)-12.12]=386.7-26.67B(0),$$

(12′) $$s^2=34.90+83.10\,[B(0)-12.12]^2.$$

The wide confidence intervals for Vorontsov-Velyaminov (Fig. 2) are due to the considerable variance as well as the small number of diameters, moreover located in an interval $\Delta m<1^m.5$.

The corresponding correlation coefficients between $B(0)$ and the mean diameters (according to the order of the regressions) are -0.384, -0.653; -0.803, -0.719; -0.896, -0.787; -0.732, -0.719.

The regressions (5)—(12) can be used for interpolation within the interval $11^m.2\leqq B(0)\leqq 13^m.5$, but they are inapplicable for extrapolation or prediction because from (5), (7), (9) and (11) it follows that for $\overline{D}''=0$, $14^m.1<B(0)<15^m.7$, and from (6), (8), (10) and (12) for $\overline{d}''=0$, $14^m.3<B(0)<$ $<15^m.7$.

To use the regressions for a wider range of $B(0)$ (suitable for prediction) it is necessary to apply the logarithmic transformation (i. e. to introduce $D''=a_1\exp[-b_1B(0)]$, or $\lg D''=\lg a_1-b_2B(0)$). The results got by the method of least squares are:

(13) $$\underset{\pm 20\quad \pm 44}{\lg\overline{D}''_{\mathrm{V.V}}=2.268-0.099}\,[B(0)-11.911]=3.447-0.099B(0),$$

$$\lg s^2 = 0.00040 + 0.00196\,[B(0) - 11.911]^2; \tag{13'}$$

$$\lg \bar{d}''_{\text{V-V}} = \underset{\pm 32}{1.812} - \underset{\pm 54}{0.145}\,[B(0) - 12.222] = 3.584 - 0.145B(0), \tag{14}$$

$$\lg s^2 = 0.00105 + 0.00288\,[B(0) - 12.222]^2; \tag{14'}$$

$$\lg \bar{D}''_{\text{Kal}} = \underset{\pm 34}{2.231} - \underset{\pm 53}{0.243}\,[B(0) - 12.121] = 5.176 - 0.243B(0), \tag{15}$$

$$\lg s^2 = 0.00120 + 0.00285\,[B(0) - 12.121]^2; \tag{15'}$$

$$\lg \bar{d}''_{\text{Kal}} = \underset{\pm 21}{1.686} - \underset{\pm 33}{0.131}\,[B(0) - 12.121] = 3.274 - 0.131B(0), \tag{16}$$

$$\lg s^2 = 0.00045 + 0.00107\,[B(0) - 12.121]^2; \tag{16'}$$

$$\lg \bar{D}''_{\text{M}} = \underset{\pm 26}{2.192} - \underset{\pm 40}{0.262}\,[B(0) - 12.121] = 5.368 - 0.262B(0), \tag{17}$$

$$\lg s^2 = 0.00066 + 0.00158\,[B(0) - 12.121]^2; \tag{17'}$$

$$\lg \bar{d}''_{\text{M}} = \underset{\pm 35}{1.803} - \underset{\pm 53}{0.220}\,[B(0) - 12.121] = 4.470 - 0.220B(0), \tag{18}$$

$$\lg s^2 = 0.00120 + 0.00286\,[B(0) - 12.121]^2; \tag{18'}$$

$$\lg \bar{D}''_{\text{Kar}} = \underset{\pm 35}{2.002} - \underset{\pm 53}{0.202}\,[B(0) - 12.121] = 4.450 - 0.202B(0), \tag{19}$$

$$\lg s^2 = 0.00120 + 0.00286\,[B(0) - 12.121]^2; \tag{19'}$$

$$\lg \bar{d}''_{\text{Kar}} = \underset{\pm 36}{1.770} - \underset{\pm 56}{0.213}\,[B(0) - 12.121] = 4.352 - 0.213B(0), \tag{20}$$

$$\lg s^2 = 0.00132 + 0.00314\,[B(0) - 12.121]^2. \tag{20'}$$

The regressions (13)—(20) together with the confidence limits are given also on Fig. 2—5 (with dotted line). The correlation coefficients between $\lg \bar{D}''$, $\lg \bar{d}''$ and $B(0)$ (according to the order of the regressions) are -0.339, -0.715; -0.849, -0.816; -0.919, -0.825 and -0.799, -0.802.

As it is seen the agreement between the diameters of different authors, measured by different methods (except $\bar{D}''_{\text{V-V}}$) for the galaxies, chosen as standard, is quite good.

3. ANALYSIS OF THE PRINT 1398 O MEASUREMENT

Only 17 among the measured by the authors galaxies on the Palomar Print 1398 O are included in the catalogue [16]. These galaxies allow to determine the agreement between our measurements and the standard galaxies. Table 4 contains the NGC-number, the corresponding number from Table 1 (or 2) and Vorontsov-Velyaminov number. The next columns contain D''_1 — mean major diameters, D''_2 — mean minor diameter (these all are external diameters!) and $\bar{D}''_{\text{Vauc}} = (D''_1 + D''_2)/2$. The last 4 columns are calculated from the data of the Catalogue [16], where $\lg D$, $\lg R$ and $\lg D(0)$ are given. D in [16] is the

Table 4

NGC	No.	No. V.–V.	D''_1	D''_2	$\overline{D}''_{\mathrm{Vauc}}$	$D''(0)$
4062	8	4	222	95	158	157
4136	34	25	228	217	222	222
4146	36	28	91	91	91	91
4150	37	29	102	70	86	89
4245	59	49	114	86	100	102
4251	60	50	151	54	102	100
4253	61	51	56	41	48	49
4274	71	60	322	120	221	213
4278	77	62	114	104	109	109
4283	80	63	52	49	50	51
4286	83	65	83	63	73	74
4310	92	74	85	50	68	67
4314	93	75	222	128	205	213
4359	99	79	203	40	122	107
4375	100	80	83	75	79	79
4414	108	85	194	111	152	154
4448	113	89	208	76	142	137

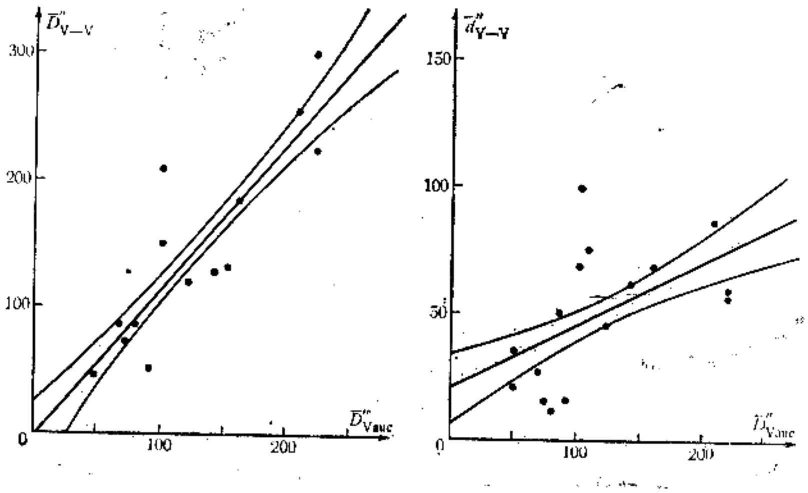


Fig. 6

mean major diameter measured visually on photographs and reduced to the standard "corrected" Heidelberg system following [29], [30] (unit of D is 0′.1). R is the mean ratio of major diameter to minor diameter, derived from visual measurements on photographs, corrected for subjective systematic errors following [3] and [29], [30]. $D(0)$ is the major diameter (in 0′.1), statistically corrected to "face on" by the relation $\lg D(0) = \lg D - 0.4 \lg R$ [29], [30].

The differences $\overline{D}''_{\mathrm{Vauc}} - D''(0)$ are insignificant (except for NGC 4359).

Let us compare the mean diameters $\overline{D}''_{\mathrm{Vauc}}$ from Table 4 with the corresponding mean diameters from Tables 1 and 2. The results are given on

Fig. 6 — 9 together with the regressions and the confidence limits of the theoretical regressions, which are the confidence limits of the theoretical regressions, which are given with:

(21) $$\underset{\pm 10.9\ \ \pm 20}{\bar{D}''_{\text{V-V}} = 145.9 + 1.18\,(D''_{\text{Vauc}} - 127.36)} = 4.4 + 1.8\bar{D}''_{\text{Vauc}},$$

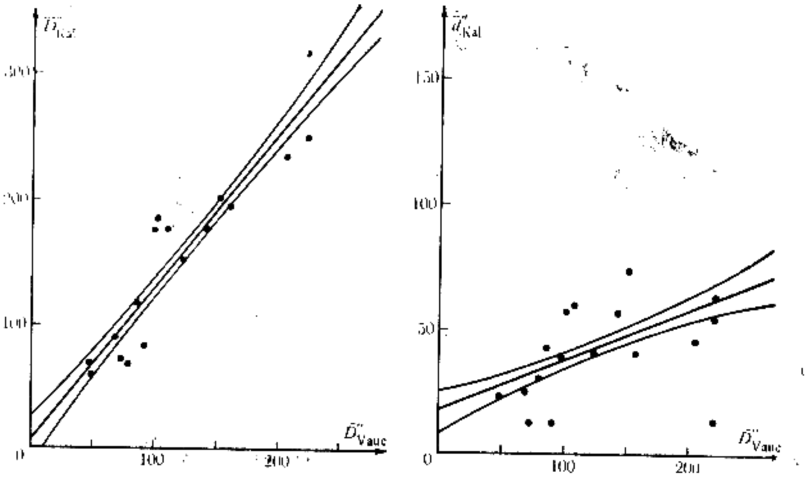


Fig. 7

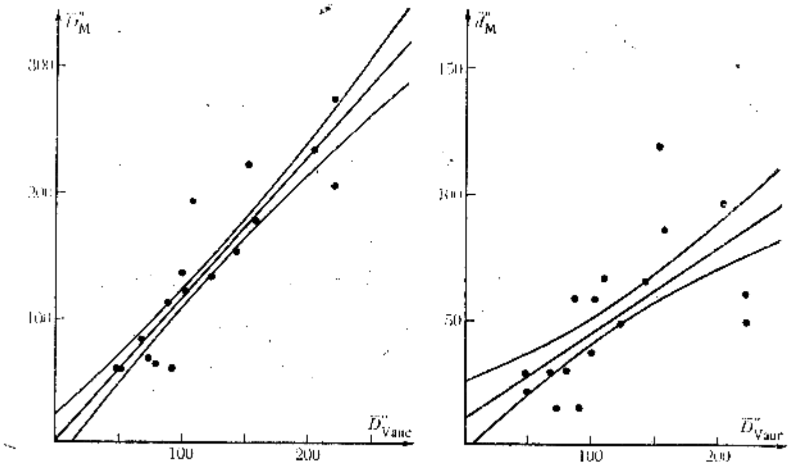


Fig. 8

(21′) $$s^2 = 118.04 + 0.038(\bar{D}''_{\text{Vauc}} - 127.36)^2;$$

(22) $$\underset{\pm 5.8\ \ \pm 10}{\bar{d}''_{\text{V-V}} = 50.1 + 0.26(\bar{D}''_{\text{Vauc}} - 117.25)} = 19.6 + 0.26\bar{D}''_{\text{Vauc}},$$

$$s^2=33.67+0.011(\bar{D}''_{\mathrm{Vauc}}-117.25)^2; \tag{22'}$$

$$\begin{aligned}\bar{D}''_{\mathrm{Kal}}&=153.9+1.25(\bar{D}''_{\mathrm{Vauc}}-119.29)=4.8+1.25\bar{D}''_{\mathrm{Vauc}},\\ &\pm7.0\quad\pm13\end{aligned} \tag{23}$$

$$s^2=49.64+0.017(\bar{D}''_{\mathrm{Vauc}}-119.29)^2; \tag{23'}$$

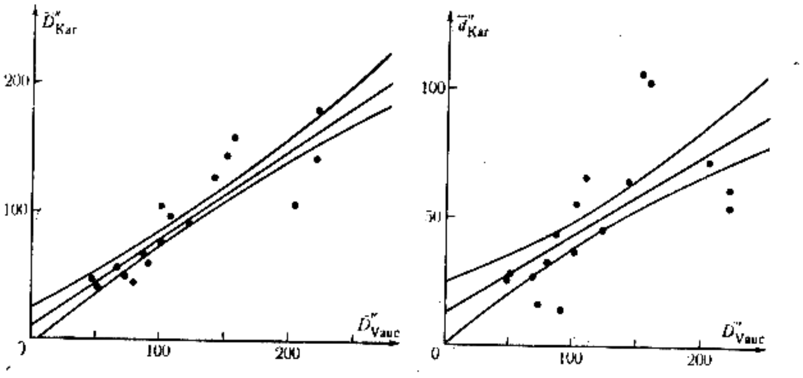


Fig. 9

$$\begin{aligned}\bar{d}''_{\mathrm{Kal}}&=41.5+0.20(\bar{D}''_{\mathrm{Vauc}}-119.29)=17.6+0.20\bar{D}''_{\mathrm{Vauc}},\\ &\pm3.4\quad\pm6\end{aligned} \tag{24}$$

$$s^2=11.74+0.004(\bar{D}''_{\mathrm{Vauc}}-119.29)^2; \tag{24'}$$

$$\begin{aligned}\bar{D}''_{\mathrm{M}}&=138.5+1.12(\bar{D}''_{\mathrm{Vauc}}-119.29)=4.9+1.12\bar{D}''_{\mathrm{Vauc}},\\ &\pm7.3\quad\pm14\end{aligned} \tag{25}$$

$$s^2=53.94+0.018(\bar{D}''_{\mathrm{Vauc}}-119.29)^2; \tag{25'}$$

$$\begin{aligned}\bar{d}''_{\mathrm{M}}&=52.6+0.34(\bar{D}''_{\mathrm{Vauc}}-119.29)=12.0+0.34\bar{D}''_{\mathrm{Vauc}},\\ &\pm5.5\quad\pm10\end{aligned} \tag{26}$$

$$s^2=30.70+0.010(\bar{D}''_{\mathrm{Vauc}}-119.29)^2; \tag{26'}$$

$$\begin{aligned}\bar{D}''_{\mathrm{Kar}}&=94.4+0.68(\bar{D}''_{\mathrm{Vauc}}-119.29)=13.3+0.68\bar{D}''_{\mathrm{Vauc}},\\ &\pm5.2\quad\pm9\end{aligned} \tag{27}$$

$$s^2=26.69+0.009(\bar{D}''_{\mathrm{Vauc}}-119.29)^2; \tag{27'}$$

$$\begin{aligned}\bar{d}''_{\mathrm{Kar}}&=49.9+0.31(\bar{D}''_{\mathrm{Vauc}}-119.29)=12.9+0.31\bar{D}''_{\mathrm{Vauc}},\\ &\pm5.1\quad\pm9\end{aligned} \tag{28}$$

$$s^2=26.22+0.009(\bar{D}''_{\mathrm{Vauc}}-119.29)^2. \tag{28'}$$

As it is seen the coefficient b in the above relations for the mean external diameters is about $+1$, which must be expected if the measurements are not effected by considerable systematic errors. But the corresponding coefficient for the internal diameters is considerably smaller, which is a real observational effect since internal diameters are compared with external (according to [16]) diameters. The mean value for b from (21)—(28) for the external diameters is 1.06, and for the internal diameters 0.28 respectively.

The galaxies, cosyidered as standard in Table 4, allow the investigation of the relation between the sphericities for which according to Holmberg [1],

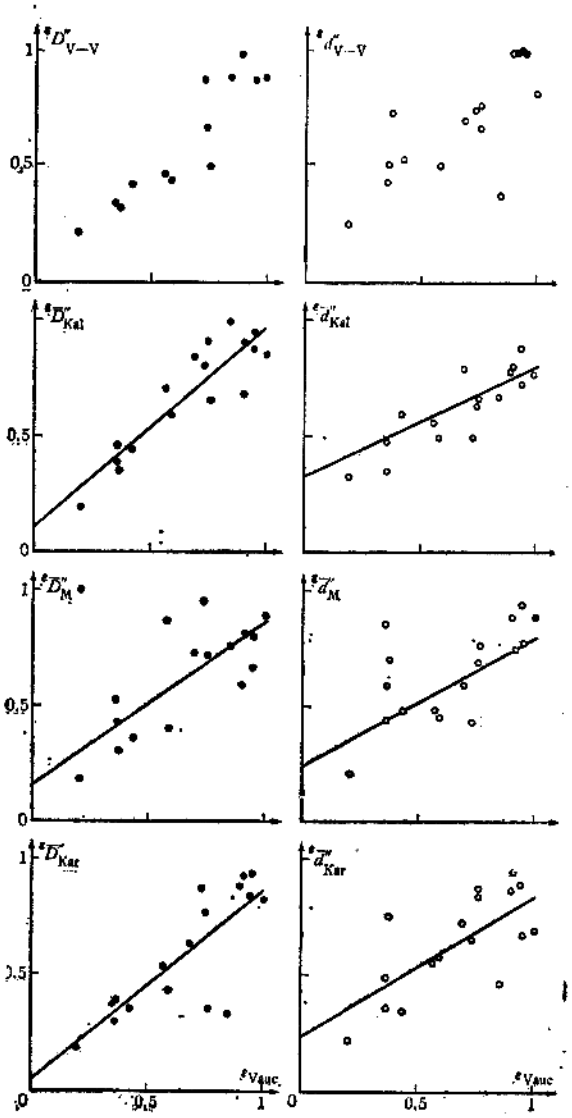


Fig. 10

[3] the measurements on plates must be corrected. The sphericities of the standard galaxies are given in Table 5 for the external as well as the internal diameters $\varepsilon_D = D''_2/D''_1$ and $\varepsilon_d = d''_2/d''_1$ respectively. The connections

between the Palomar print measurement and $\varepsilon_{\text{Vauc}}$ are given on Fig. 10. The regressions are not given here because they are not necessary. It is enough to know that between the sphericities obtained from our measurements show a good agreement.

Table 5

No.	$\varepsilon_{D_{\text{Kal}}}$	$\varepsilon_{D_{\text{M}}}$	$\varepsilon_{D_{\text{Kar}}}$	$\varepsilon_{D_{\text{V-V}}}$	$\varepsilon_{d_{\text{Kal}}}$	$\varepsilon_{d_{\text{M}}}$	$\varepsilon_{d_{\text{Kar}}}$	$\varepsilon_{d_{\text{V-V}}}$	$\varepsilon_{d_{\text{Vauc}}}$
8	0.45	0.36	0.36	0.42	0.60	0.48	0.36	0.53	0.43
34	0.88	0 80	0.94	0.88	0.73	0.77	0.69	1.00	0.95
36	0.85	0.89	0.83	0.89	0.77	0.88	0.70	0.83	1.00
37	0.84	0.73	0.64	—	0.79	0.59	0.71	0.70	0.69
59	0.91	0.72	0.77	0.67	0.64	0.68	0.85	0.77	0.75
60	0.46	0.52	0.38	0.75	0.48	0.59	0.50	0.43	0.36
61	0.81	0.95	0.88	0.88	0.50	0.43	0.67	0.75	0.73
71	0.36	0.30	0.39	0.33	0.78	0.70	0.77	0.73	0.37
77	0.91	0.81	0.93	—	0.80	0.74	0.96	1.00	0.91
80	0.95	0.67	0.85	—	0.88	0.93	0.90	1.00	0.94
83	0.66	0.60	0.36	0.50	0.67	0.76	0.88	0.67	0.76
92	0.60	0.40	0.44	0.45	0.50	0.45	0.59	0.50	0.59
93	1.00	0.75	0.34	0.89	0.68	0.35	0.48	0.38	0.85
99	0.20	0.18	0.20	0.21	0.33	0.21	0.23	0.25	0.25
100	0.68	0.59	0.88	1.00	0.79	0.88	0.88	1.00	0.90
108	0.71	0.87	0.54	0.47	0.57	0.48	0.57	—	0.57
113	0.40	0.42	0.31	0.34	0.36	0.44	0.37	0.50	0.36

Let us denote

$$x_1=\varepsilon_{D''_{\text{Kal}}},\quad x_2=\varepsilon_{D''_{\text{M}}},\quad x_3=\varepsilon_{D''_{\text{Kar}}},\tag{29}$$

$$y_1=\varepsilon_{d''_{\text{Kal}}},\quad y_2=\varepsilon_{d''_{\text{M}}},\quad y_3=\varepsilon_{d''_{\text{Kar}}},\quad z=\varepsilon_{\text{Vauc}}.$$

The corresponding correlation matrix is

$$\begin{Vmatrix} 1.0000 & 0.9058 & 0.7173 & 0.6385 & 0.4480 & 0.6153 & 0.9020 \\ 0.9058 & 1.0000 & 0.7265 & 0.4285 & 0.3534 & 0.4927 & 0.7682 \\ 0.7173 & 0.7265 & 1.0000 & 0.6120 & 0.6652 & 0.7090 & 0.8015 \\ 0.6385 & 0.4285 & 0.6120 & 1.0000 & 0.8026 & 0.7931 & 0.7374 \\ 0.4480 & 0.3534 & 0.6652 & 0.8026 & 1.0000 & 0.8209 & 0.6717 \\ 0.6153 & 0.4927 & 0.7090 & 0.7931 & 0.8209 & 1.0000 & 0.7126 \\ 0.9020 & 0.7682 & 0.8015 & 0.7374 & 0.6717 & 0.7126 & 1.0000 \end{Vmatrix}.\tag{30}$$

The matrix (30) gives the statistical structure of our measurement. As it is seen, the correlation between ε_D and $\varepsilon_{\text{Vauc}}$ is greater than the correlation between ε_d and $\varepsilon_{\text{Vauc}}$. Besides that the correlation coefficients between ε_D are greater than these between ε_d.

Let us apply the methods of multivariate statistical analysis (for example [31]—[34]).

The matrix for x_1, x_2, y_1, y_2 is (block matrices are denoted)

$$\left\Vert\begin{array}{cc|cc} 1.0000 & 0.9058 & 0.6385 & 0.4480 \\ 0.9058 & 1.0000 & 0.4285 & 0.3534 \\ \hline 0.6385 & 0.4285 & 1.0000 & 0.8026 \\ 0.4480 & 0.3534 & 0.8026 & 1.0000 \end{array}\right\Vert = \begin{Vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{Vmatrix},\tag{31}$$

whence

$$R_{22}^{-1}R_{21}=\begin{Vmatrix} 0.7840 & 0.4071 \\ -0.1812 & 0.0267 \end{Vmatrix} \tag{32}$$

and

$$R_{12}R_{22}^{-1}R_{21}=\begin{Vmatrix} 0.4194 & 0.2719 \\ 0.2719 & 0.1839 \end{Vmatrix}. \tag{33}$$

Then we have the determinantal equation

$$0=\begin{Vmatrix} 0.4194-\mu & 0.2719-0.9058\mu \\ 0.2719-0.9058\mu & 0.1839-\mu \end{Vmatrix}, \tag{34}$$

whence for the canonical correlations we have

$$\sqrt{\mu_1}=0.7619, \quad \sqrt{\mu_2}=0.1752. \tag{35}$$

The first canonical correlation is larger than all correlation coefficients determined for both different sets. Consequently the correlation between the first pair is more important because the second pair is more weakly correlated (the second canonical correlation is significant too).

In this way from the matrix for x_1, x_3, y_1, y_3 we have

$$\left\Vert\begin{array}{cc|cc} 1.0000 & 0.7173 & 0.6385 & 0.6153 \\ 0.7173 & 1.0000 & 0.6120 & 0.7090 \\ \hline 0.6385 & 0.6120 & 1.0000 & 0.7931 \\ 0.6153 & 0.7090 & 0.7931 & 1.0000 \end{array}\right\Vert=\begin{Vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{Vmatrix}, \tag{36}$$

$$R_{22}^{-1}R_{21}=\begin{vmatrix} 0.4057 & 0.1340 \\ 0.2936 & 0.6028 \end{vmatrix}, \tag{37}$$

$$R_{12}R_{22}^{-1}R_{21}=\begin{Vmatrix} 0.4397 & 0.4565 \\ 0.4565 & 0.5094 \end{Vmatrix}, \tag{38}$$

with the corresponding determinantal equation

$$0=\begin{vmatrix} 0.4397-\nu & 0.4565-0.7173\nu \\ 0.4565-0.7173\nu & 0.5094-\nu \end{vmatrix} \tag{39}$$

and for the canonical correlations we have

$$\sqrt{\nu_1}=0.7398, \quad \sqrt{\nu_2}=0.2423. \tag{40}$$

In this case the previous conclusion remains valid too — the first canonical correlation is larger than the correlation coefficients between the different sets. The second canonical correlation is significant too but it is much smaller than the first one.

The mean measured diameters of the galaxies are statistically connected with the corresponding apparant magnitudes. However, the relation is non-linear. The least squares method shows that observations cannot be presented as $\overline{D}=a=bm_{\mathrm{Zw}}+cm^2_{\mathrm{Zw}}$, because $\overline{D}$ (or $\overline{d}$)=min for $16^{\mathrm{m}}<m_{\mathrm{Zw}}<17^{\mathrm{m}}$ and in some cases $\overline{D}(\overline{d})<0$.

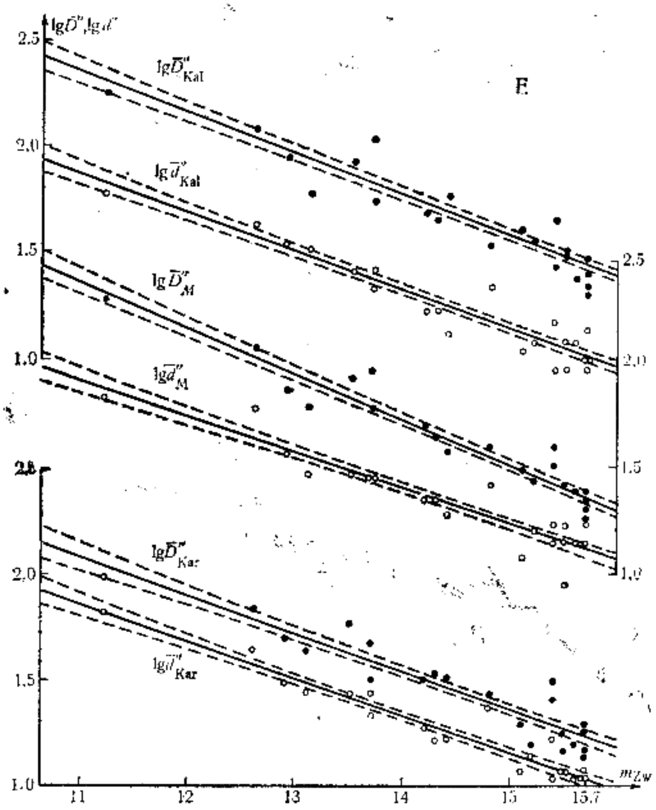


Fig. 11

A good presentation is again received with the logarithmic transformation $D=a_1\exp(-b_1m_{\mathrm{Zw}})$ or $\lg D=\lg a_1-b_2m_{\mathrm{Zw}}$, as the number of parameters being reduced with 1.

Let us use the classification from Table 2. The relations between the mean diameters and m_{Zw} for ellipticals (22 galaxies) are plotted on Fig. 11. The regressions, confidence limits and correlation coefficients are:

$$\lg\overline{D}''_{\mathrm{Kal}}=\underset{\pm 18}{1.666}-\underset{\pm 15}{0.188}(m_{\mathrm{Zw}}-14.495)=4.391-0.188m_{\mathrm{Zw}},\quad r=-0.941, \tag{41}$$

$$\lg s^2=0.00034+0.00023(m_{\mathrm{Zw}}-14.495)^2, \tag{41'}$$

$$\lg\overline{d}''_{\mathrm{Kal}}=\underset{\pm 17}{1.227}-\underset{\pm 14}{0.178}(m_{\mathrm{Zw}}-14.495)=3.807-0.178m_{\mathrm{Zw}},\quad r=-0.946, \tag{42}$$

(42′) $$\lg s^2 = 0.00028 + 0.00018(m_{\mathrm{Zw}} - 14.495)^2;$$

(43) $$\lg \overline{D}''_{\mathrm{M}} = \underset{\pm 19}{1.609} - \underset{\pm 15}{0.211}(m_{\mathrm{Zw}} - 14.495) = 4.667 - 0.211 m_{\mathrm{Fw}}, \quad r = -0.952,$$

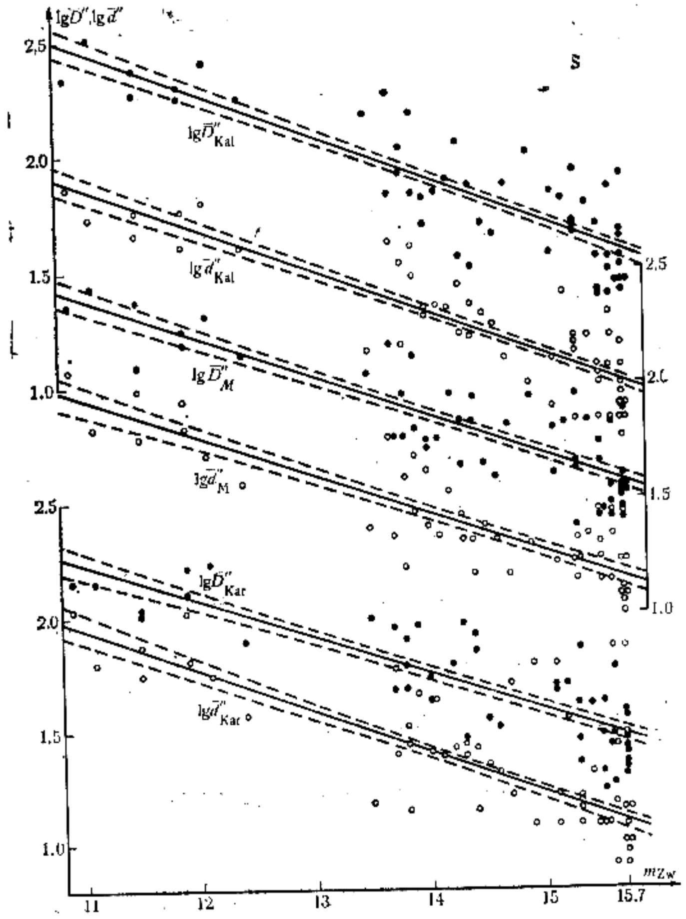


Fig. 12

(43′) $$\lg s^2 = 0.00035 + 0.00023(m_{\mathrm{Zw}} - 14.495)^2;$$

(44) $$\lg \overline{d}''_{\mathrm{M}} = \underset{\pm 18}{1.310} - \underset{\pm 14}{0.167}(m_{\mathrm{Zw}} - 14.495) = 3.731 - 0.167 m_{\mathrm{Zw}}, \quad r = -0.933,$$

(44′) $$\lg s^2 = 0.00031 + 0.00021(m_{\mathrm{Zw}} - 14.495)^2;$$

(45) $$\lg \overline{D}''_{\mathrm{Kar}} = \underset{\pm 19}{1.455} - \underset{\pm 15}{0.177}(m_{\mathrm{Zw}} - 14.495) = 4.021 - 0.177 m_{\mathrm{Zw}}, \quad r = -0.934,$$

45') $$\lg s^2 = 0.00035 + 0.00023(m_{\mathrm{Zw}} - 14.495)^2;$$

(46) $$\lg \overline{d}''_{\mathrm{Kar}} = \underset{\pm 14}{1.253} - \underset{\pm 11}{0.172}(m_{\mathrm{Zw}} - 14.495) = 3.746 - 0.172 m_{\mathrm{Zw}}, \quad r = -0.959,$$

(46') $$\lg s^2 = 0.00019 + 0.00013(m_{\mathrm{Zw}} - 14.495)^2.$$

The analogous relations for the spirals (52 galaxies) are plotted on Fig. 12. The regressions (in the same form) are

(47) $$\lg \overline{D}''_{\mathrm{Kal}} = \underset{\pm 22}{1.813} - \underset{\pm 16}{0.190}(m_{\mathrm{Zw}} - 14.417) = 4.552 - 0.190 m_{\mathrm{Zw}}, \quad r = -0.856,$$

(47') $$\lg s^2 = 0.00049 + 0.00026(m_{\mathrm{Zw}} - 14.417)^2;$$

(48) $$\lg \overline{d}''_{\mathrm{Kal}} = \underset{\pm 20}{1.243} - \underset{\pm 15}{0.181}(m_{\mathrm{Zw}} - 14.417) = 3.852 - 0.181 m_{\mathrm{Zw}}, \quad r = -0.866,$$

(48') $$\lg s^2 = 0.00041 + 0.00022(m_{\mathrm{Zw}} - 14.417)^2;$$

(49) $$\lg \overline{D}''_{\mathrm{M}} = \underset{\pm 20}{1.798} - \underset{\pm 15}{0.171}(m_{\mathrm{Zw}} - 14.417) = 4.263 - 0.171 m_{\mathrm{Zw}}, \quad r = -0.850,$$

(49') $$\lg s^2 = 0.00042 + 0.00022(m_{\mathrm{Zw}} - 14.417)^2;$$

(50) $$\lg \overline{d}''_{\mathrm{M}} = \underset{\pm 23}{1.375} - \underset{\pm 17}{0.167}(m_{\mathrm{Zw}} - 14.417) = 3.783 - 0.167 m_{\mathrm{Zw}}, \quad r = -0.812,$$

(50') $$\lg s^2 = 0.00054 + 0.00029(m_{\mathrm{Zw}} - 14.417)^2;$$

(51) $$\lg \overline{D}''_{\mathrm{Kar}} = \underset{\pm 21}{1.681} - \underset{\pm 16}{0.160}(m_{\mathrm{Zw}} - 14.417) = 3.988 - 0.160 m_{\mathrm{Zw}}, \quad r = -0.823,$$

(51') $$\lg s^2 = 0.00046 + 0.00024(m_{\mathrm{Zw}} - 14.417)^2,$$

(52) $$\lg \overline{d}''_{\mathrm{Kar}} = \underset{\pm 22}{1.328} - \underset{\pm 16}{0.180}(m_{\mathrm{Zw}} - 14.417) = 3.923 - 0.180 m_{\mathrm{Zw}}, \quad r = -0.848,$$

(52') $$\lg s^2 - 0.00047 + 0.00025(m_{\mathrm{Zw}} - 14.417)^2.$$

For nonclassified 31 galaxies (Fig. 13) we have

(53) $$\lg \overline{D}''_{\mathrm{Kal}} = \underset{\pm 23}{1.550} - \underset{\pm 46}{0.077}(m_{\mathrm{Zw}} - 15.255) = 2.725 - 0.077 m_{\mathrm{Zw}}, \quad r = -0.300,$$

(53') $$\lg s^2 = 0.00052 + 0.00209(m_{\mathrm{Zw}} - 15.255)^2;$$

(54) $$\lg \overline{d}''_{\mathrm{Kal}} = \underset{\pm 29}{1.096} - \underset{\pm 59}{0.237}(m_{\mathrm{Zw}} - 15.255) = 4.559 - 0.227 m_{\mathrm{Zw}}, \quad r = -0.583,$$

(54') $$\lg s^2 = 0.00086 + 0.00345(m_{\mathrm{Zw}} - 15.255)^2;$$

(55) $$\lg \overline{D}''_{\mathrm{M}} = \underset{\pm 24}{1.511} - \underset{\pm 48}{0.111}(m_{\mathrm{Zw}} - 15.255) = 3.204 - 0.111 m_{\mathrm{Zw}}, \quad r = -0.391,$$

(55') $$\lg s^2 = 0.00058 + 0.00234(m_{\mathrm{Zw}} - 15.255)^2;$$

$$\text{(56)}\quad \lg \overline{d}''_{\mathrm{M}} = \underset{\pm 21}{1.242} - \underset{\pm 42}{0.123}(m_{\mathrm{Zw}} - 15.255) = 3.118 - 0.123 m_{\mathrm{Zw}}, \quad r = -0.474,$$

$$\text{(56')}\quad \lg s^2 = 0.00044 + 0.00179(m_{\mathrm{Zw}} - 15.255)^2;$$

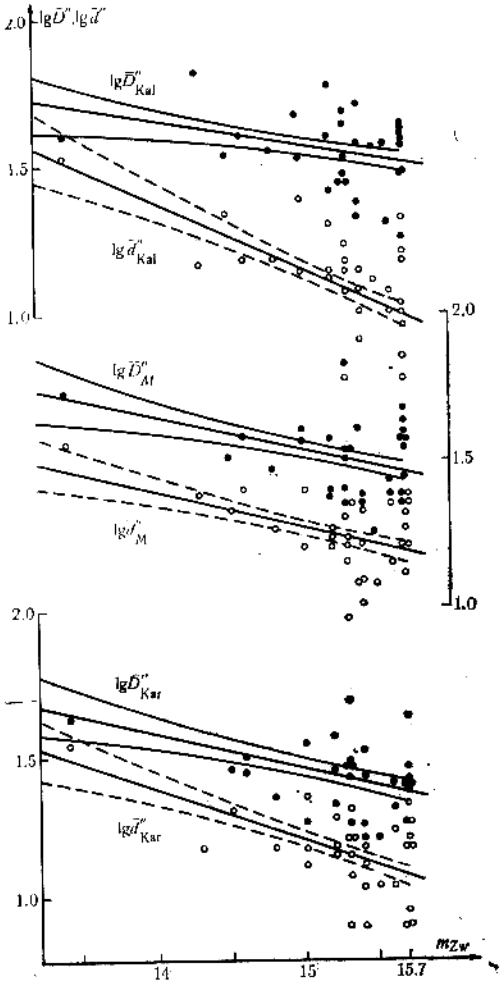


Fig. 13

$$\text{(57)}\quad \lg \overline{D}''_{\mathrm{Kar}} = \underset{\pm 23}{1.423} - \underset{\pm 47}{0.121}(m_{\mathrm{Zw}} - 15.255) = 3.269 - 0.121 m_{\mathrm{Zw}}, \quad r = -0.432,$$

$$\text{(57')}\quad \lg s^2 = 0.00054 = 0.00220(m_{\mathrm{Zw}} - 15.255)^2;$$

$$\text{(58)}\quad \lg \overline{d}''_{\mathrm{Kar}} = \underset{\pm 23}{1.156} - \underset{\pm 46}{0.177}(m_{\mathrm{Zw}} - 15.255) = 3.856 - 0.177 m_{\mathrm{Zw}}, \quad r = -0.583,$$

$$\text{(58')}\quad \lg s^2 = 0.00052 + 0.00211(m_{\mathrm{Zw}} - 15.255)^2.$$

We may make the conclusion from (41)—(58′) that the diameters of ellipticals more certainly are measured, these of spirals — a little less certainly and for nonclassified galaxies the confidence limits of the theoretical regressions are already too wide. For the last case the correlation coefficient is low, moreover $r(\lg D'', m_{Zw}) < r(\lg d'', m_{Zw})$. The confidence limits fo nonclassified galaxies are wider because of the fact that for these galaxie m_{Zw} are situated in a more narrow range.

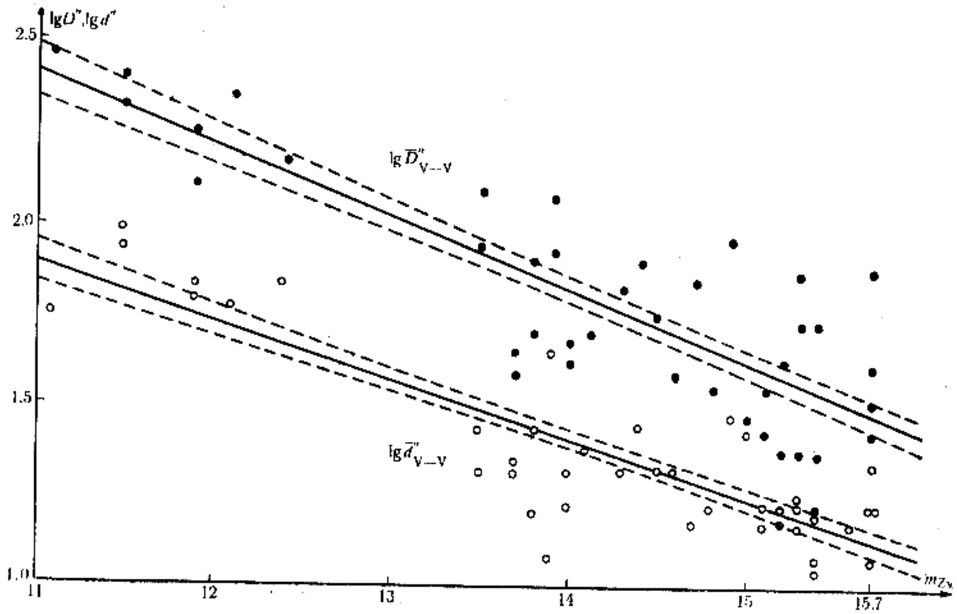


Fig. 14

The analogous relations for galaxies measured by Vorontsov-Velyaminov are given on Fig. 14 and regressions are

59) $$\lg \overline{D}''_{V-V} = \underset{\pm 27}{1.785} - \underset{\pm 21}{0.199}(m_{Zw} - 14.210) = 4.613 - 0.199 m_{Zw}, \quad r = -0.837,$$

59′) $$\lg s^2 = 0.00073 + 0.00043(m_{Zw} - 14.210)^2;$$

(60) $$\lg \overline{d}''_{V-V} = \underset{\pm 20}{1.376} - \underset{\pm 16}{0.163}(m_{Zw} - 14.210) = 3.692 - 0.163 m_{Zw}, \quad r = -0.858,$$

(60′) $$\lg s^2 = 0.00041 + 0.00024(m_{Zw} - 14.210)^2$$

for 41 galaxies, 24 of which are spirals and 4 — ellipticals.

All regressions (41)—(60′) are suitable for prediction (or extrapolation) at least for the next three magnitudes. Practically, when it is impossible to determine the class of galaxies, a common regression must be used.

The common regressions for all processed galaxies (105) with the corresponding correlation coefficients are

(61) $\lg \overline{D}''_{\mathrm{Kal}} = 1.705 - 0.192(m_{\mathrm{Zw}} - 14.681) = 4.524 - 0.192 m_{\mathrm{Zw}}, \quad r = -0.840,$
$\qquad\qquad \pm 15 \quad \pm 12$

(61′) $\lg s^2 = 0.00022 + 0.00015(m_{\mathrm{Zw}} - 14.681)^2;$

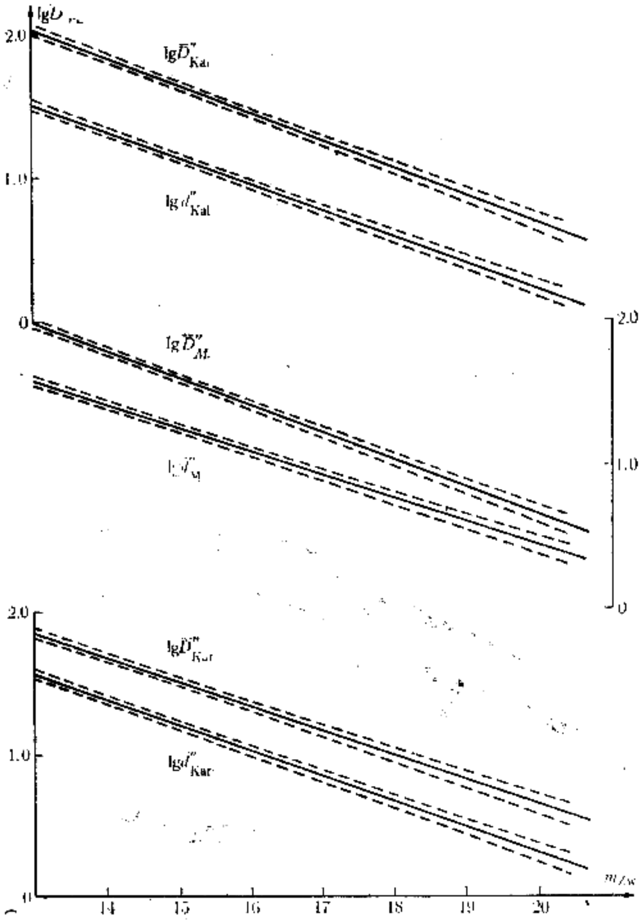


Fig. 15

(62) $\lg \overline{d}''_{\mathrm{Kal}} = 1.674 - 0.188(m_{\mathrm{Zw}} - 14.681) = 4.434 - 0.188 m_{\mathrm{Zw}}, \quad r = -0.830,$
$\qquad\qquad \pm 15 \quad \pm 12$

(62′) $\lg s^2 = 0.00023 + 0.00016(m_{\mathrm{Zw}} - 14.681)^2;$

(63) $\lg \overline{D}''_{\mathrm{M}} = 1.557 - 0.170(m_{\mathrm{Zw}} - 14.681) = 4.053 - 0.170 m_{\mathrm{Zw}}, \quad r = -0.793,$
$\qquad\qquad \pm 16 \quad \pm 13$

(63′) $\lg s^2 = 0.00024 + 0.00017(m_{\mathrm{Zw}} - 14.681)^2;$

(64) $$\lg \overline{d}''_{\mathrm{M}} = \underset{\pm 13}{1.196} - \underset{\pm 11}{0.182}(m_{\mathrm{Zw}} - 14.681) = 3.868 - 14.681\, m_{\mathrm{Zw}}, \quad r = -0.849,$$

(64′) $$\lg s^2 = 0.00018 + 0.00012(m_{\mathrm{Zw}} - 14.681)^2;$$

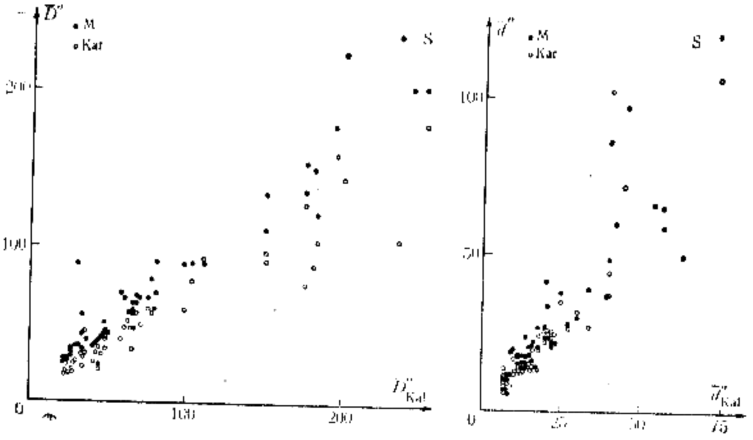


Fig. 16

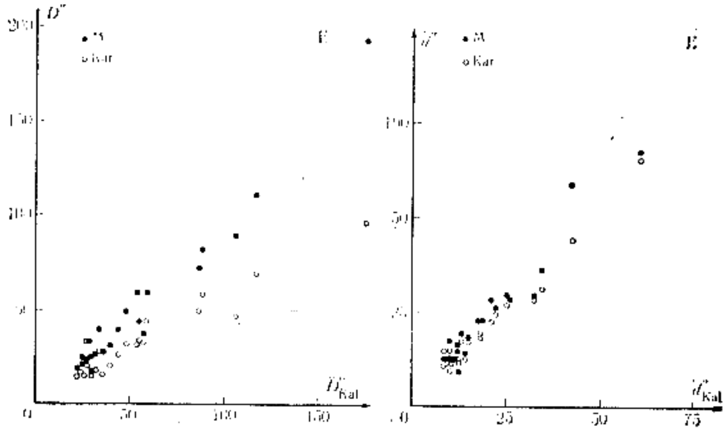


Fig. 17

(65) $$\lg \overline{D}''_{\mathrm{Kar}} = \underset{\pm 14}{1.322} - \underset{\pm 11}{0.163}(m_{\mathrm{Zw}} - 14.681) = 3.715 - 0.163\, m_{\mathrm{Zw}}, \quad r = -0.817.$$

(65′) $$\lg s^2 = 0.00019 + 0.00013(m_{\mathrm{Zw}} - 14.681)^2;$$

(66) $$\lg \overline{d}''_{\mathrm{Kar}} = \underset{\pm 13}{1.262} - \underset{\pm 11}{0.178}(m_{\mathrm{Zw}} - 14.681) = 3.875 - 0.178\, m_{\mathrm{Zw}}, \quad r = -0.852,$$

(66′) $$\lg s^2 = 0.00017 + 0.00012(m_{\mathrm{Zw}} - 14.681)^2.$$

As the nonrandom variable here is m_{Zw} then the determination of the corresponding magnitudes from the diameters has to be done namely from (61)—(66) and not from the regressions $m_{Zw} - \lg \overline{D}''$ or $\lg \overline{d}''$. The results (61)—(66') are given on Fig. 15. These relations are suitable for prediction in a quite wide range along to m_{Zw}, as it is seen from confidence limits.

Most interesting in this case are the limiting magnitudes, determined from (61)—(66). It is possible to measure diameters on the Palomar print down to 0.05 mm (corresponding to 1 division of our eyepiece scale, or 3''.355) and counting of galaxies may be carried out even down to 0.03 mm. We suppose that certain measurement of mean diameters (without danger of counting of stars or defects instead of galaxies) is 0.1 mm (6''.71).

Let us determine the limiting magnitudes of galaxies for $\overline{D}, \overline{d} = 3''.355$ and 6''.71 from (61)—(66). The results are given in Table 6.

Table 6

$\overline{D}, \overline{d}$	$m_{Zw,\ \lim}$ from relation					
	(61)	(62)	(63)	(64)	(65)	(66)
$\overline{D} = 3''.355$	$20^m.8$		$20^m.8$		$20^m.7$	
$\overline{D} = 6''.71$	$19^m.2$		$19^m.2$		$19^m.0$	
$\overline{d} = 3''.355$		$18^m.4$		$19^m.6$		$18^m.8$
$\overline{d} = 6''.71$		$16^m.7$		$17^m.7$		$17^m.1$

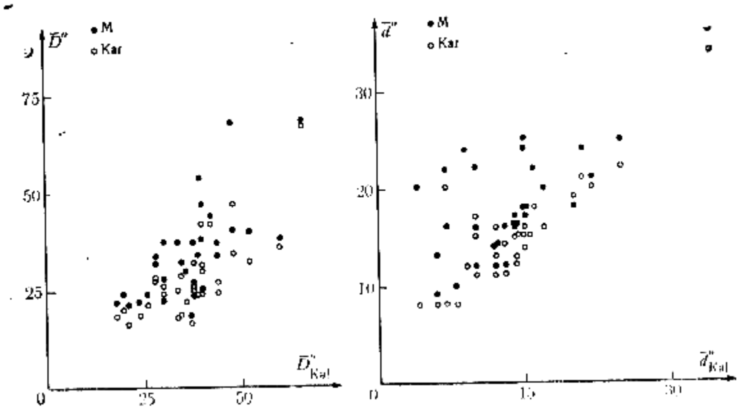


Fig. 18

The confidence limits for determination of m_{Zw} in these cases cannot be found out from (61')—(66'). But surely confidence limits are not necessary because determination of external and internal diameters is impossible for the faintest galaxies. In the final reckoning certain counting of galaxies over investigated Palomar print 1398 O can be carried out up to

$20^m.7-20^m.8$ and measurement of diameters — up to $19^m.2$. As it is known the limiting magnitude for blue Palomar prints is about $21^m.1$ [35].

An idea about the internal convergence between measurements of the three authors may be got from Figs. 16—18 for all processed galaxies (divided into S, E and unclassified). The linear regressions (by pairs) are

$$\begin{aligned} &\overline{D}''_{\mathrm{M}}=3.6+0.86\,\overline{D}''_{\mathrm{Kal}},\quad \overline{D}''_{\mathrm{Kal}}=0.98+1.08\overline{D}''_{\mathrm{M}},\quad r=0.962; \\ (67)\qquad &\overline{D}''_{\mathrm{Kal}}=8.4+0.55\,\overline{D}''_{\mathrm{Kal}},\quad \overline{D}''_{\mathrm{Kal}}=-3.1+1.53\overline{D}''_{\mathrm{Kar}},\quad r=0.920; \end{aligned}$$

$$\begin{aligned} &\overline{D}''_{\mathrm{M}}=-2.3+1.39\,\overline{D}''_{\mathrm{Kar}},\quad \overline{D}''_{\mathrm{Kar}}=7.0+0.63\overline{D}''_{\mathrm{M}},\quad r=0.935; \\ (68)\qquad &\overline{d}''_{\mathrm{M}}=1.3+1.26\overline{d}''_{\mathrm{Kal}},\quad \overline{d}''_{\mathrm{Kal}}=3.1+0.62\overline{d}''_{\mathrm{M}},\quad r=0.887; \end{aligned}$$

$$\begin{aligned} &\overline{d}''_{\mathrm{Kar}}=-0.1+1.20\overline{d}''_{\mathrm{Kal}},\quad \overline{d}''_{\mathrm{Kal}}=4.3+0.64\overline{d}''_{\mathrm{Kar}},\quad r=0.879; \\ (69)\qquad &\overline{d}''_{\mathrm{M}}=2.4+1.00\overline{d}''_{\mathrm{Kar}},\quad \overline{d}''_{\mathrm{Kar}}=-0.9+0.93\overline{d}''_{\mathrm{M}},\quad r=0.970, \end{aligned}$$

from 112 galaxies for $\overline{D}$ and 106 galaxies for $\bar{d}''$.

4. PRINT 64 O MEASUREMENTS

Palomar print 64 O has equatorial coordinates of the center $\alpha_{1950}=12^h38^m40^s$ and $\delta_{1950}=+29°28'42''$ [20] and galactic coordinates $l^{\mathrm{II}}=169°.9$d and $b^{\mathrm{II}}=+86°.92$ [21]. The original plate of the print 64 O is expose3 45 min against 50 min for the plate of the print 1398 O.

Palomar print 64 corresponds to F. 159 from the Catalogue [10] and to field 5—30 from the Catalogue [13]. The investigated region has $12^h26^m\lesssim\alpha_{1950}\lesssim12^h52^m$, $+26°.5\lesssim\delta_{1950}\lesssim+32°.5$ according to the field in [10] $12^h26^m.0\leqq\alpha_{1950}\leqq12^h51^m.9$, $+26°30'\leqq\delta_{1950}\leqq+32°29'$ according to [13].

All galaxies catalogued in F. 159 are measureed. The galaxies are numbered according to α as they are arranged in F. 159. All measurements here are carried out by three authors (Kalinkov, Mihnevsky and Stavrev (St.). The results are given in Tables 7 and 8. Table 7 contains external diameters D''_1 (major) and D''_2 (minor), corresponding mean values $\overline{D}''$ — mean external diameters, as well as external diameters measured from Vorontsov-Velyaminov [13]. Table 8 contains besides corresponding diameters (d''_1, d'_2 and $\bar{d}''$) also magnitudes according to [10] and galaxy types (S, E or unclassified).

As in Tables 1 and 2, here, in Tables 7 and 8, several doubtful cases in identification of the galaxies were found also. Galaxies No. 13, 16, 19, 48, 51, 53, 60, 65, 72, 84, 91, 107 and 109 are excluded out of further analysis for the same reason as for Tables 1 and 2.

It is necessary to check up the agreement between m_{Zw} and magnitudes from other authors. A sample was made from Catalogue [16] for galaxies with detailed photometrical data located in investigated field (Table 9). Explanation of the columns in Table 8 are the same as Table 3.

An inspection of magnitudes (Table 9) and our measured diameters for the corresponding galaxies (from Tables 7 and 8) shows that definite connections exist in spite of the small number of galaxies.

Table 7

No.	Kal.			M.			St.			V.-V.		
	D''_1	D''_2	$\overline{D}''$	D''_1	D''_2	$\overline{D}''$	D''_1	D''_2	$\overline{D}''$	D''_1	D''_2	$\overline{D}''$
1	2	3	4	5	6	7	8	9	10	11	12	13
1	37	34	36	37	28	32	27	24	26			
2	235	57	146	215	80	148	211	74	142			
3	37	27	32	57	47	52	47	34	40	48	48	48
4	37	22	30	40	27	34	37	20	28	30	18	24
5	64	37	50	74	40	57	70	27	48	48	30	39
6	64	10	37	70	12	41	64	7	36			
7	40	34	37	30	24	27	20	17	18	15	15	15
8	101	70	86	121	67	94	84	57	70	108	60	84
9	87	44	66	94	44	69	84	37	60	72	36	54
10	87	13	50	94	15	54	77	13	45	66	12	39
11	80	47	64	81	64	72	77	74	76	78	60	69
12	30	20	25	34	27	30	24	24	24			
13	—	—	—	262	218	240	—	—	—	360	240	300
14	27	24	26	37	30	34	37	34	36	42	36	39
15	47	47	47	57	37	47	37	34	36	—	—	—
16	218	87	152	175	80	128	151	67	109	150	72	111
17	34	30	32	57	24	40	34	20	27	42	42	42
18	80	30	55	74	34	54	80	20	50	84	30	57
19	60	20	40	74	27	50	64	17	40	66	24	45
20	57	34	46	57	30	44	47	27	37			
21	94	67	80	128	57	92	67	50	58	—	—	—
22	60	44	52	91	47	69	54	34	44	—	—	—
23	30	30	30	47	27	37	34	24	29	—	—	—
24	570	235	402	772	285	528	704	184	444	720	300	510
25	44	24	34	47	24	36	40	20	30	48	30	39
26	40	17	28	34	17	26	27	13	20	—	—	—
27	64	30	47	50	27	38	50	17	34	54	24	39
28	27	27	27	67	27	47	37	34	36	54	54	54
29	42	17	30	44	20	32	37	13	25			
30	27	20	24	34	24	29	24	17	20	—	—	—
31	44	24	34	47	24	36	27	13	20	30	24	27
32	27	17	22	34	24	29	27	17	22			
33	94	27	60	84	27	56	80	20	50	84	24	54
34	40	17	28	30	20	25	37	15	26			
35	20	20	20	30	17	24	20	13	16	30	18	24
36	47	30	38	60	44	52	50	27	38	48	42	45
37	64	27	46	77	40	58	40	27	34	36	24	30
38	64	34	49	70	40	55	50	24	37	72	30	51
39	54	47	50	64	47	56	54	40	47	60	48	54
40	47	37	42	70	20	45	40	13	26	42	15	28
41	20	20	20	24	24	24	24	20	22			
42	20	17	18	27	24	26	17	17	17			
43	37	20	28	40	27	34	37	24	30	36	27	32
44	50	13	32	50	17	34	40	10	25			
45	30	27	28	37	24	30	27	18	22			
45	47	47	47	27	27	27	34	27	30			
47	50	37	44	40	30	35	37	27	32	36	30	33
48	37	17	27	37	22	30	30	15	22			
49	44	20	32	44	22	33	37	17	27			
50	47	44	46	47	44	46	50	50	50	—	—	—
51	30	27	28	34	27	30	27	27	27	30	30	30
52	77	20	48	87	30	58	84	20	52	84	24	54

Continuation of Table 7

1	2	3	4	5	6	7	8	9	10	11	12	13
53	25	18	22	30	20	25	24	17	20			
54	34	30	32	47	37	42	40	37	38	48	30	39
55	80	10	45	80	20	50	77	13	45	72	15	44
56	47	37	42	64	34	49	47	20	34	48	36	42
57	30	27	28	34	27	30	27	24	26	30	24	27
58	50	15	32	60	20	40	54	15	34			
59	47	25	36	47	30	38	34	27	30	36	24	30
60	—	—	—	87	37	62	67	24	46	72	42	57
61	50	44	47	57	47	52	47	47	47	48	48	48
62	57	20	38	60	24	42	57	15	36	60	24	42
63	24	20	22	30	24	27	27	20	24			
64	40	17	28	50	20	35	44	12	28	36	12	24
65	604	184	394	906	151	528	755	107	431	1200	150	675
66	40	27	34	50	30	40	37	20	28	42	27	34
67	44	24	34	57	40	48	44	24	34	39	21	30
68	40	27	34	54	34	44	40	24	32	48	30	39
69	94	57	76	94	67	80	84	54	69	96	72	84
70	47	30	38	57	44	50	37	34	36	—	—	—
71	54	37	46	50	47	48	47	34	40	48	48	48
72	—	—	—	262	64	163	252	50	151			
73	40	20	30	60	18	39	50	13	32	48	12	30
74	47	30	38	50	24	37	27	17	22	—	—	—
75	40	27	34	47	20	34	34	17	26	—	—	—
76	60	54	57	67	67	67	64	57	60	72	72	72
77	70	20	45	91	18	54	50	13	32	78	18	48
78	67	57	62	101	60	80	44	30	37	—	—	—
79	44	13	28	54	17	36	47	13	30	54	12	33
80	57	15	36	60	17	38	54	13	34	48	12	30
81	44	37	40	50	47	48	47	37	42	42	42	42
82	60	37	48	80	47	64	67	37	52	72	42	57
83	57	40	48	50	37	44	34	20	27	—	—	—
84	40	34	37	47	40	44	37	37	37	42	42	42
85	94	67	80	117	77	97	80	60	70	90	90	90
86	47	17	32	44	15	30	37	10	24	36	12	24
87	64	20	42	47	27	37	24	20	22			
88	34	34	34	47	44	46	37	30	34			
89	50	37	44	47	30	38	34	34	34			
90	60	34	47	54	27	40	50	24	37			
91	37	34	36	40	30	35	34	27	30	36	18	27
92	164	20	92	151	27	89	114	13	64	120	18	69
93	37	34	36	44	30	37	30	27	28	33	27	30
94	34	34	34	37	34	36	34	24	29	36	36	36
95	37	34	36	50	40	45	37	30	34	36	36	36
96	101	30	66	124	54	89	101	34	68	108	36	72
97	27	17	22	27	17	22	27	13	20			
98	37	24	30	50	30	40	24	20	22	—	—	—
99	94	13	54	94	24	59	101	24	62	90	24	57
100	37	15	26	40	20	30	30	13	22	30	15	22
101	24	24	24	27	24	26	20	17	18			
102	57	24	40	64	30	47	54	20	37	—	—	—
103	37	27	32	47	34	40	40	30	35	36	27	32
104	40	27	34	47	34	40	44	30	37	48	36	42
105	37	37	37	47	34	40	24	20	22	—	—	—
106	37	20	28	47	20	34	37	13	25			
107	44	40	42	44	37	40	40	34	37	36	36	36
108	57	27	42	64	30	47	30	24	27	42	30	36
109	117	84	100	201	151	176	101	60	80	180	120	150
110	131	17	74	128	24	76	128	13	70	114	15	64

Continuation of Table 7

1	2	3	4	5	6	7	8	9	10	11	12	13
111	54	15	34	67	17	42	50	13	32	72	15	44
112	50	24	37	50	25	38	37	13	25	—	—	—
113	84	50	67	107	57	82	67	40	54	—	—	—
114	44	27	36	54	30	42	40	24	32			
115	54	15	34	54	17	36	37	10	24			
116	168	77	122	191	101	146	141	67	104			
117	44	20	32	44	24	34	30	12	21			
118	87	54	70	117	67	92	44	37	40			
119	30	20	25	34	24	29	27	17	22			

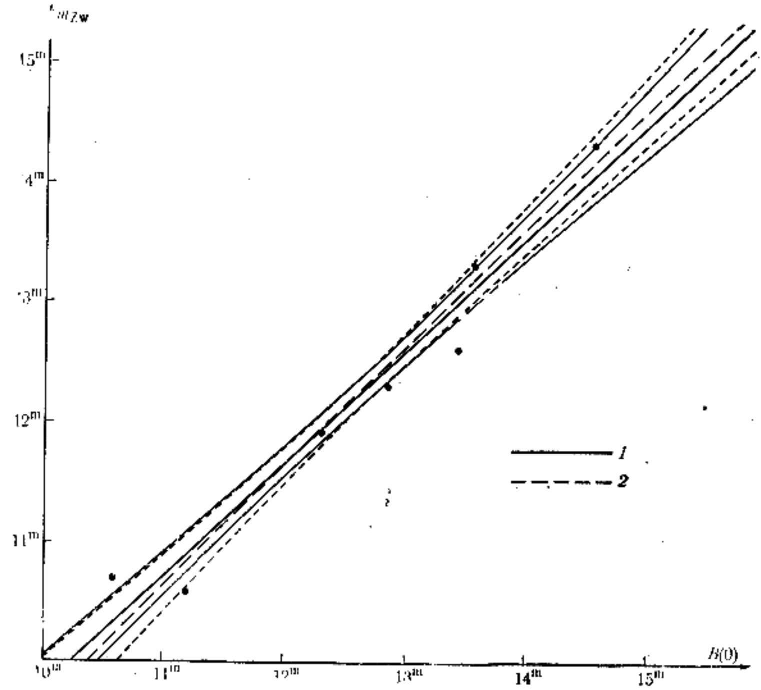


Fig. 19

1 — regression (70); *2* — regression (71)

The relation between m_{Zw} and $B(0)$ is linear. The least squares method gives

$$(70)\qquad m_{Zw}=\underset{\pm 12}{12.24}+\underset{\pm 94}{0.927}[B(0)-12.64]=0.526+0.927B(0)$$

as confidence limits of the theoretical regression are

$$(70')\qquad s^2=0.01477+0.00874[B(0)-12.74]^2.$$

Table 8

No.	Kal.			M.			St.			V.-V.			m_{zw}	Type
	d''_1	d''_2	$\overline{d''}$	d''_1	d''_2	$\overline{d''}$	d''_1	d''_2	$\overline{d''}$	d''_1	d''_2	$\overline{d''}$		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	12	12	12	17	15	16	17	13	15				$15^m.3$	S
2	101	37	69	101	24	62	87	37	62				11 .9	S
3	7	7	7	7	7	7	7	7	7	18	18	18	15 .7	S
4	17	13	15	15	15	15	13	10	12	21	15	18	15 .7	S
5	37	13	25	35	17	26	20	10	15	42	15	28	14 .7	S
6	54	10	32	40	7	24	7	7	7				15 .7	S
7	7	7	7	13	12	12	13	13	13				15 .1	S
8	12	10	11	10	10	10	10	10	10	18	15	16	14 .6	S
9	44	17	30	40	15	28	40	17	28	48	18	33	14 .1	S
10	30	8	19	30	8	19	24	10	17				15 .7	S
11	24	17	20	24	17	20	20	13	16	27	18	22	14 .2	S
12	15	12	14	17	12	14	13	10	12				15 .7	
13				10	5	8				180	60	120	15 .6	
14	13	13	13	12	10	11	13	10	12	15	15	15	15 .6	E
15	30	27	28	34	24	29	27	20	24	30	24	27	14 .1	E
16	40	34	37	47	37	42				60	36	48	13 .0	S
17	12	8	10	12	8	10	13	10	12	18	15	16	15 .4	
18	30	12	21	40	13	26	24	10	17	42	18	30	15 .1	S
19	34	13	24	34	15	24	27	13	20	30	18	24	14 .9	S
20	24	12	18	20	13	16	20	13	16				15 .3	S
21	34	20	27	34	22	28	27	20	24	30	24	27	13 .5	E
22	20	17	18	24	15	20	20	17	18	30	18	24	14 .4	E
23	15	13	14	15	10	12	13	12	12	18	18	18	15 .1	
24	134	67	100	235	67	151	101	60	80	240	120	180	10 .7	S
25	13	10	12	13	10	12	13	10	12	15	12	14	15 .6	E
26	15	10	12	13	10	12	13	10	12	12	12	12	15 .7	
27	22	15	18	25	15	20	20	13	16	27	18	22	15 .4	E
28	17	17	17	15	13	14	17	17	17	18	18	18	15 .0	E
29	34	12	23	30	12	21	13	10	12				15 .4	E
30	17	13	15	20	13	16	17	13	15	24	15	20	15 .4	E
31	27	13	20	22	13	18	20	13	16	24	15	20	15 .3	E
32	13	10	12	17	12	14	13	10	12				15 .7	S
33	30	13	22	27	17	22	17	13	15	27	18	22	15 .0	S
34	17	12	14	15	13	14	13	10	12				15 .7	E
35	13	12	12	13	12	12	17	13	15	15	12	14	15 .5	E
36	12	5	8	20	17	18	13	7	10	18	9	14	15 .4	S
37	37	17	27	24	20	22	30	17	24				14 .6	S
38	24	17	20	24	15	20	20	13	16	30	18	24	14 .6	S
39	37	30	34	40	25	32	27	20	24				14 .0	S
40	17	8	12	20	10	15	20	10	15	27	12	20	15 .2	S
41	10	10	10	12	12	12	12	10	11				15 .5	E
42	15	13	14	13	12	12	15	13	14				15 .4	E
43	15	10	12	15	12	14	13	12	12	18	12	15	15 .3	E
44	30	12	21	30	10	20	27	10	18				15 .6	S
45	13	13	13	12	10	11	13	13	13				15 .6	S
46	13	13	13	15	13	14	13	13	13				15 .2	
47	8	7	8	8	8	8	7	7	7	12	9	10	15 .6	S
48	30	13	22	22	15	18							15 .5	
49	24	13	18	17	15	16	20	13	16				15 .4	S
50	13	13	13	17	15	16	17	17	17	18	18	18	14 .4	
51	7	7	7	7	5	6	7	7	7	12	9	10	15 .4	S
52	12	7	10	17	10	14	20	8	14				14 .9	S

Continuation of Table 8

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
53	13	10	12	10 7	7 7	8 7	17	10	14				$15^m.7$	
54	15	13	14	12	10	11	12	10	11	12	12	12	15.5	S
55	12	8	10	17	10	14	17	10	14	15	12	14	15.6	S
56	37	13	25	24	15	20	24	13	18	30	15	22	15.4	S
57	10	7	8	13	8	10	13	10	12	18	9	14	15.6	
58	24	13	18	20	10	15	20	10	15				15.5	S
59	27	20	24	30	22	26	27	17	22				14.5	S
60				30 13	8 8	19 10							15.5	
61	15	10	12	17	12	14	13	13	13	24	15	20	14.8	S
62	22	10	16	24	10	17	27	10	18	30	9	20	15.7	S
63	10	10	10	12	10	11	13	10	12				15.7	E
64	20	12	16	15	8	12	17	10	14				15.6	S
65	218	47	132	235	50	142	200?	40?	120?	270	60	165	10.6	
66	30	13	22	25	13	19	24	12	18	30	12	21	15.3	
67	20	17	18	24	15	20	27	17	22				14.7	
68	13	10	12	24	12	18	13	13	13	15	12	14	15.7	S
69	40	27	34	50	30	40	40	20	30	45	36	40	12.6	S
70	27	17	22	24	18	21	20	20	20	27	24	26	13.7	E
71	8	7	8	8	7	8	7	7	7	9	6	8	15.5	S
72				30	13	22							14.1	
73	30	10	20	30	12	21	27	10	18				15.1	S
74	20	17	18	18	13	16	17	13	15	18	15	16	15.0	
75	27	13	20	20	12	16	20	10	15	30	15	22	15.2	
76	12	8	10	10	7	8	10	10	10	48	36	42	14.5	S
77	27	13	20	27	13	20	30	13	22	30	15	22	14.7	S
78	30	24	27	25	20	22	20	17	18	33	27	30	14.0	E
79	27	13	20	15	12	14	24	12	18	18	12	15	15.2	S
80	30	10	20	34	10	22	34	10	22				15.7	S
81	7	3	5	7	3	5	7	3	5	12	9	10	15.5	S
82	30	13	22	17	13	15	24	13	18	24	18	21	14.8	S
83	17	13	15	17	12	14	17	13	15	24	18	21	14.9	S
84	10	7	8	17	10	14				12	12	12	15.2	S
85	13	13	13	13	12	12	13	13	13	18	18	18	15.4	S
86	27	12	20	24	10	17	27	10	18				15.2	S
87	18	12	15	15	13	14	17	13	15				15.6	S
88	10	10	10	13	12	12	13	12	12				15.4	S
89	13	13	13	13	13	13	13	13	13				14.8	
90	7	3	5	5	3	4	7	7	7				15.5	S
91	30	17	24	27	13	20							15.1	S
92	64	13	38	67	12	40	30	10	20	90	12	51	14.9	S
93	13	10	12	13	8	10	17	10	14	15	12	14	15.3	S
94	20	17	18	15	13	14	17	13	15	9	9	9	15.4	E
95	24	20	22	17	15	16	13	13	13				14.9	S
96	7	5	6	5	3	4	7	3	5				15.1	S
97	20	13	16	17	12	14	20	12	16				15.4	E
98	17	13	15	13	10	12	13	12	12	18	15	16	15.5	E
99	10	8	9	7	7	7	10	7	8	9	8	8	15.7	S
100	15	12	14	12	10	11	17	10	14	18	12	15	15.5	
101	8	7	8	15	13	14	17	13	15				15.3	
102	44	17	30	44	15	30	40	13	26	45	18	32	14.5	
103	13	10	12	13	10	12	13	10	12	15	12	14	14.8	
104	15	12	14	13	12	12	17	12	14	15	15	15	15.0	
105	15	15	15	13	10	12	13	13	13	18	18	18	15.0	E
106	24	10	17	24	8	16	27	10	18				15.6	
107	24	13	18	15	7	11				21	15	18	15.3	S
108	24	12	18	18	12	15	17	10	14	24	12	18	15.3	

Continuation of Table 8

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
109				3	3	3				72	48	60	$14^m.9$	
110	50	13	32	60	13	36	27	10	18				14.8	S
111	18	12	15	17	12	14	15	10	12	15	12	14	15.5	S
112	24	13	18	24	12	18	20	10	15	24	18	21	15.4	
113	34	27	30	25	22	24	27	20	24	42	30	36	13.3	E
114	22	13	18	8	8	8	10	10	10				15.5	
115	24	10	17	17	10	14	13	8	10				15.6	S
116	91	34	62	74	32	53	67	30	48				12.3	S
117	24	10	17	24	8	16	20	8	14				15.7	E
118	37	27	32	24	18	21	20	20	20				14.3	E
119	18	12	15	15	10	12	17	10	14				15.7	E

The second linear regression is

$$B(0)=\underset{\pm 13}{12.64}+\underset{\pm 104}{1.026}(m_{\mathrm{Zw}}-12.24)=0.071+1.026m_{\mathrm{Zw}}, \tag{71}$$

$$s^2=0.01635+0.01072(m_{\mathrm{Zw}}-12.24)^2 \tag{71'}$$

and $r=+0.975$.

Table 9

NGC	No.	m_{H}	m_{C}	$B(0)$	$B'(0)$	m'_{C}	m_{Zw}
4448	2	$11^m.9$	$12^m.07$	$12^m.29$	$13^m.83$	$13^m.61$	$11^m.9$
4559	24	10.7	10.47	10.56	14.40	14.31	10.7
4656	65	11.3	11.12	11.18	14.72	14.66	10.6
4670	69	12.7	13.26	13.44	13.18	13.00	12.6
4789	113			13.57	13.46		13.3
4793	116	12.5	12.71	12.86	13.85	13.70	12.3
4798	118			14.56	14.50		14.3

Regressions (70) and (71) together with the confidence limits (70′) and (71′) of the theoretical regressions are plotted on Fig. 19. In spite of the small number of galaxies only 7, these regressions are quite close to (1) and (2). For print 64 O the approximated relations

$$m_{\mathrm{Zw}}=B(0)+0.5, \tag{72}$$

$$B(0)=m_{\mathrm{Zw}} \tag{73}$$

can be used.

Unfortunately, relations between mean external diameters $\overline{D''}$ and mean internal diameters $\overline{d''}$ for measured galaxies on one hand and $B(0)$ on the other hand cannot be established for here investigated print 64 O because of the insufficient statistics, i. e. regressions (5)—(20) cannot be received for this print.

5. ANALYSIS OF THE PRINT 64 O MEASUREMENTS

Only 13 among measured by authors galaxies on print 64 are included in the Catalogue [16]. Besides 4 of them refer to neighbouring fields in the Catalogue [13]. In this case we are compelled to use these 4 galaxies too in spite of the fact that they have been measured by Vorontsov-Velyaminov over other Palomar prints. Beside that internal diameters of No. 65 for one of the authors are uncertain and they are only here included in the analysis.

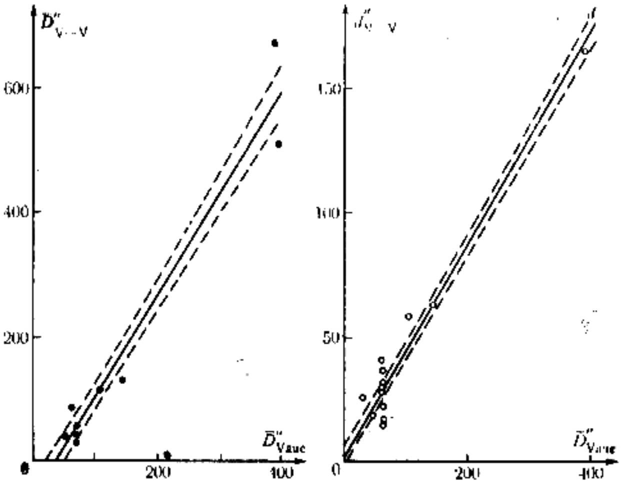


Fig. 20

Galaxies, for which in [16] are given diameters, are adopted as standard in order to study the convergence between our measurements and these of other authors. In this way we compare directly diameters with diameters and not diameters with magnitudes.

Table 10

NGC or IC*	No.	No. V-V	D''_1	D''_2	$\bar{D}''_{Vauc}$	$D''(0)$
4448	2	89 (5-29 ; PA 1398)	208	76	142	137
3546*	20	5 (4-30 ; PA 1435)	97	40	68	67
4555	21	26	64	60	62	61
4559	24	30	560	233	396	397
3598*	33	40	102	31	66	64
4656	65	66	630	141	386	345
4670	69	72	72	49	60	61
4673	70	73	36	30	33	34
4692=823*	78	86	66	66	66	66
4728	87	98	48	48	48	48
4789	113	124	72	54	63	64
4793	116	3(5-31 ; PA 1393)	137	75	106	109
4798	118	4(5-31 ; PA 1393)	66	66	66	66

The data for standard galaxies are given in Table 10, which contains NGC or IC (with asterisk), No. from Tables 7 or 8, No. according to [13] (for the neighbouring fields are given also the corresponding numbers, fields and Palomar prints), mean major diameter D_1'', mean minor diameter D_2'', mean diameter $\bar{D}''_{\mathrm{Vauc}}$ and $D(0)$. All diameters in Table 10 are according to [16] and they are external diameters.

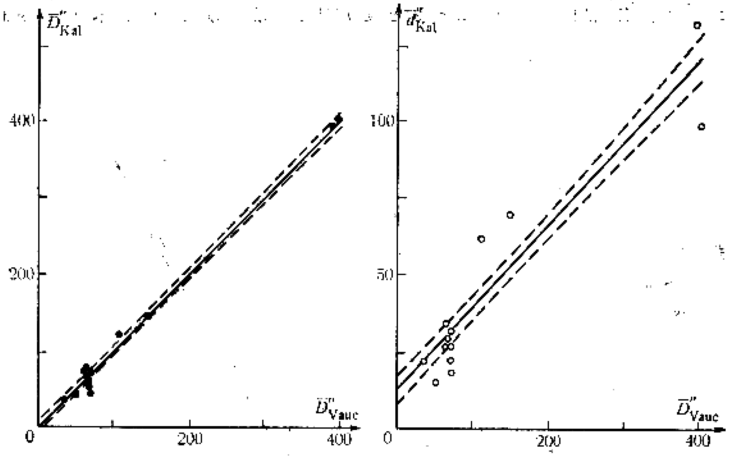


Fig. 21

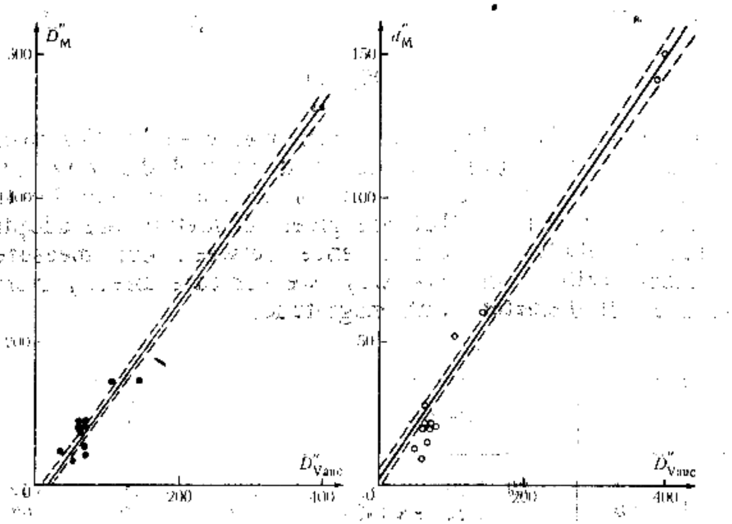


Fig. 22

The difference $\bar{D}''_{\mathrm{Vauc}}-D''(0)$ is significant only for NGC 4656.

Comparisons between diameters, measured by authors (and also by Vorontsov-Velyaminov, according to [13]) and $\bar{D}''_{\mathrm{Vauc}}$ are shown on Fig. 20—23, where theoretical linear regressions with confidence limits are given too.

The regressions are

(74) $\overline{D}''_{\mathrm{V-V}}=\underset{\pm 18.5}{186.0}+\underset{\pm 14}{1.65}(\overline{D}''_{\mathrm{Vauc}}-148.67)=-58.7+1.65\overline{D}''_{\mathrm{Vauc}}$, $r=0.976$,

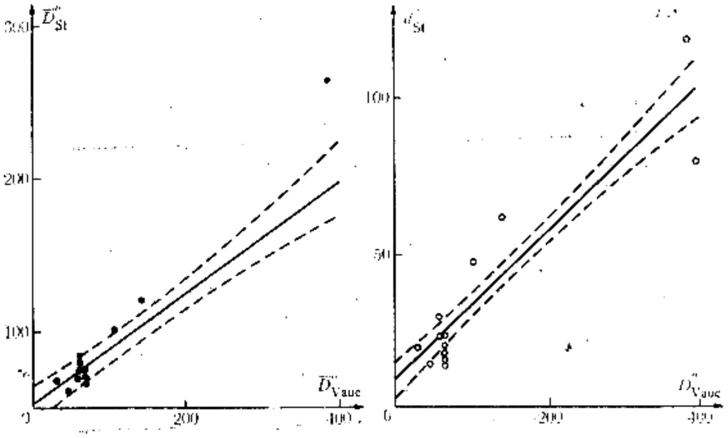


Fig. 23

(74′) $s^2=341.22+0.0195(\overline{D}''_{\mathrm{Vauc}}-148.67)^2$;

(75) $\overline{d}''_{\mathrm{V-V}}=\underset{\pm 2.7}{53.5}+\underset{\pm 2}{0.44}(\overline{D}''_{\mathrm{Vauc}}-120.15)=0.7+0.44\overline{D}''_{\mathrm{Vauc}}$, $r=0.986$,

(75′) $s^2=7.12+0.0005(\overline{D}''_{\mathrm{Vauc}}-120.15)^2$;

(76) $\overline{D}''_{\mathrm{Kal}}=\underset{\pm 3.1}{123.5}+\underset{\pm 3}{1.02}(\overline{D}''_{\mathrm{Vauc}}-120.15)=1.4+1.02\overline{D}''_{\mathrm{Vauc}}$, $r=0.996$,

(76′) $s^2=9.74+0.0007(\overline{D}''_{\mathrm{Vauc}}-120.15)^2$;

(77) $\overline{d}''_{\mathrm{Kal}}=\underset{\pm 3.5}{45.4}+\underset{\pm 3}{0.27}(\overline{D}''_{\mathrm{Vauc}}-120.15)=12.6+0.27\overline{D}''_{\mathrm{Vauc}}$, $r=0.941$,

(77′) $s^2=12.34+0.0009(\overline{D}''_{\mathrm{Vauc}}-120.15)^2$;

(78) $\overline{D}''_{\mathrm{M}}=\underset{\pm 5.7}{151.0}+\underset{\pm 5}{1.38}(\overline{D}''_{\mathrm{Vauc}}-120.15)=-14.5+1.37\overline{D}''_{\mathrm{Vauc}}$, $r=0.993$,

(78′) $s^2=32.14+0.0023(\overline{D}''_{\mathrm{Vauc}}-120.15)^2$;

(79) $\overline{d}''_{\mathrm{M}}=\underset{\pm 2.3}{47.4}+\underset{\pm 2}{0.37}(\overline{D}''_{\mathrm{Vauc}}-120.15)=2.9+0.37\overline{D}''_{\mathrm{Vauc}}$, $r=0.986$,

(79′) $s^2=5.11+0.0004(\overline{D}''_{\mathrm{Vauc}}-120.15)^2$;

(80) $\overline{D}''_{\mathrm{St}}=\underset{\pm 18.0}{94.2}+\underset{\pm 15}{0.72}(\overline{D}''_{\mathrm{Vauc}}-120.15)=7.2+0.72\overline{D}''_{\mathrm{Vauc}}$, $r=0.820$,

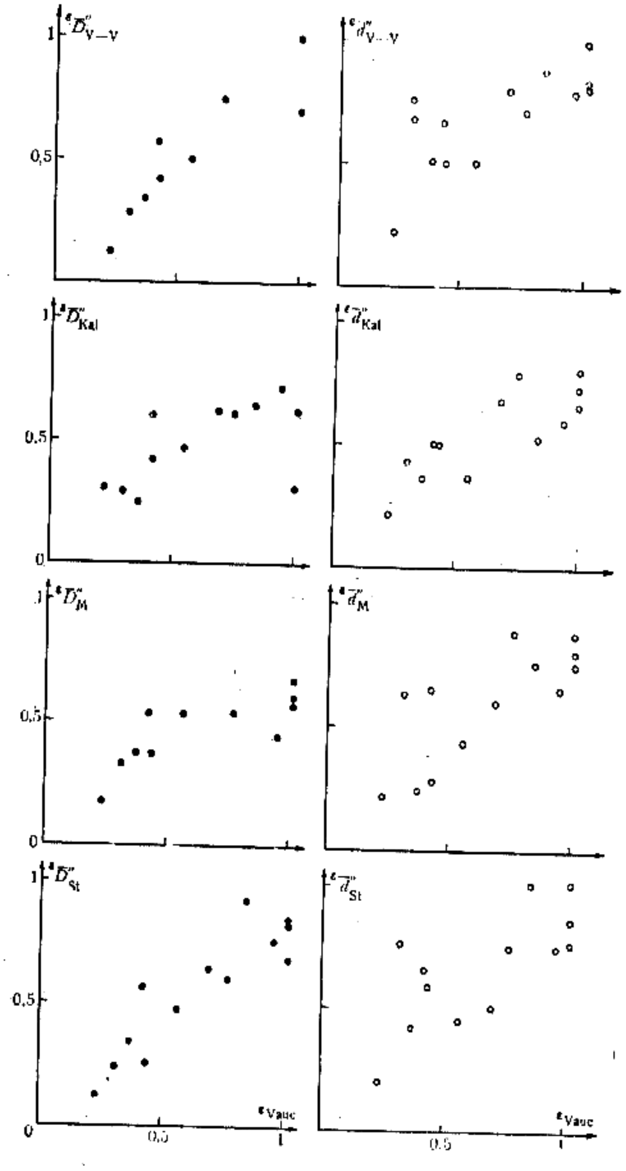


Fig. 24

(80′) $$s^2 = 325.00 + 0.0232(\overline{D}''_{\mathrm{Vauc}} - 120.15)^2;$$

(81) $$\overline{d}''_{\mathrm{St}} = \underset{\pm 3.5}{37.8} + \underset{\pm 3}{0.24}(\overline{D}''_{\mathrm{Vauc}} - 120.15) = 9.1 + 0.24\,\overline{D}''_{\mathrm{Vauc}}, \quad r = 0.924,$$

(81′) $$s^2 = 12.55 + 0.0009(\overline{D}''_{\mathrm{Vauc}} - 120.15)^2.$$

The regression (74) and (74′) are obtained from 9 galaxies and all other regressions — from 13 galaxies.

The mean value of coefficient b in regressions for external diameters is 1.19, while that for internal diameters is 0.33. If we exclude b from (74), respectively (75), then $b=1.04$ for external diameters and $b=0.29$ for internal diameters, against 1.06 and 0.28 respectively, for print 1398 O.

Table 9 contains diameters of the standard galaxies for which sphericity $\varepsilon_{\text{Vauc}} = D''_2/D''_1$ can be determined. Sphericities according to measurements from Tables 7 and 8 for the standard galaxies, together with $\varepsilon_{\text{Vauc}}$ are given in Table 11.

Table 11

No.	$\varepsilon_{D_{\text{Kal}}}$	$\varepsilon_{D_{\text{M}}}$	$\varepsilon_{D_{\text{St}}}$	$\varepsilon_{D_{\text{V-V}}}$	$\varepsilon_{d_{\text{Kal}}}$	$\varepsilon_{d_{\text{M}}}$	$\varepsilon_{d_{\text{St}}}$	$\varepsilon_{d_{\text{V-V}}}$	$\varepsilon_{\text{Vauc}}$
2	0.24	0.37	0.35	0.34	0.37	0.24	0.42	0.50	0.36
20	0.60	0.53	0.57	0.57	0.50	0.65	0.65	0.67	0.41
21	0.71	0.44	0.75		0.59	0.65	0.74	0.80	0.94
24	0.41	0.37	0.26	0.42	0.50	0.28	0.59	0.50	0.42
33	0.29	0.32	0.25	0.28	0.43	0.63	0.76	0.67	0.30
65	0.30	0.17	0.14	0.12	0.22	0.21	0.20	0.22	0.22
69	0.61	0.71	0.64	0.75	0.68	0.60	0.50	0.80	0.68
70	0.64	0.77	0.92		0.63	0.75	1.00	0.89	0.83
78	0.85	0.59	0.68		0.80	0.80	0.85	0.82	1.00
87	0.31	0.57	0.83	1.00	0.67	0.87	0.76	1.00	1.00
113	0.60	0.53	0.60		0.79	0.88	0.74	0.71	0.75
116	0.46	0.53	0.48	0.50	0.37	0.43	0.45	0.50	0.55
118	0.62	0.57	0.84	0.71	0.73	0.75	1.00	0.83	1.00

Table 10 is used to construct Fig. 24 — sphericities for external and internal diameters depending on $\varepsilon_{\text{Vauc}}$. According to Holmberg [1], [3] large systematic deviations should be expected. Of course deviations exist but they are not great. Besides, sphericities received from our measurements are in good agreement. It must be borne in mind that in the right side of Fig. 24 sphericities, determined from internal diameters are compared with sphericities received from external diameters.

Let us denote

$$\begin{gathered} x_1=\varepsilon_{D_{\text{Kal}}},\quad x_2=\varepsilon_{D_{\text{M}}},\quad x_3=\varepsilon_{D_{\text{St}}}, \\ y_1=\varepsilon_{d_{\text{Kal}}},\quad y_2=\varepsilon_{d_{\text{M}}},\quad y_3=\varepsilon_{d_{\text{St}}},\quad z=\varepsilon_{\text{Vauc}}. \end{gathered} \tag{82}$$

The corresponding normalized correlation matrix is

$$\begin{Vmatrix} 1.0000 & 0.6228 & 0.6374 & 0.7090 & 0.5713 & 0.5508 & 0.6589 \\ 0.6228 & 1.0000 & 0.8443 & 0.7207 & 0.6621 & 0.6017 & 0.6793 \\ 0.6374 & 0.8443 & 1.0000 & 0.7658 & 0.7891 & 0.7518 & 0.9023 \\ 0.7090 & 0.7207 & 0.7658 & 1.0000 & 0.8483 & 0.7518 & 0.8430 \\ 0.5713 & 0.6621 & 0.7891 & 0.8483 & 1.0000 & 0.8013 & 0.7718 \\ 0.5508 & 0.6017 & 0.7518 & 0.7518 & 0.8013 & 1.0000 & 0.7204 \\ 0.6589 & 0.6793 & 0.9023 & 0.8430 & 0.7718 & 0.7204 & 1.0000 \end{Vmatrix} \tag{83}$$

To use the methods of multivariate statistical analysis. The matrix (83) gives the statistical structure of our measurements and as it is seen it has considerably more complicated appearance than (30). Here the correlation between ε_D and $\varepsilon_{\mathrm{Vauc}}$ is lower than the correlation between ε_d and $\varepsilon_{\mathrm{Vauc}}$, with the exception for $\varepsilon_{D_{\mathrm{St}}}$.

The matrix for x_1, x_2, y_1, y_2 (divided into block matrices) is

$$\left\|\begin{array}{cc|cc} 1.0000 & 0.6228 & 0.7090 & 0.5713 \\ 0.6228 & 1.0000 & 0.7207 & 0.6621 \\ \hline 0.7090 & 0.7207 & 1.0000 & 0.8483 \\ 0.5713 & 0.6621 & 0.8483 & 1.0000 \end{array}\right\| = \left\|\begin{array}{cc} R_{11} & R_{12} \\ R_{21} & R_{22} \end{array}\right\|. \tag{84}$$

Then

$$R_{22}^{-1} = \left\|\begin{array}{rr} 3.5665 & -3.0255 \\ -3.0255 & 3.5665 \end{array}\right\|, \tag{85}$$

$$R_{22}^{-1} R_{21} = \left\|\begin{array}{rr} 0.8002 & 0.5672 \\ -0.1075 & 0.1809 \end{array}\right\|, \tag{86}$$

$$R_{12} R_{22}^{-1} R_{21} = \left\|\begin{array}{cc} 0.5059 & 0.5055 \\ 0.5055 & 0.5286 \end{array}\right\| \tag{87}$$

whence we receive the determinantal equation

$$\left|\begin{array}{cc} 0.5059-\lambda & 0.5055-0.6228\,\lambda \\ 0.5055-0.6228\,\lambda & 0.5286-\lambda \end{array}\right| = 0 \tag{88}$$

with roots

$$\lambda_1 = 0.630588, \quad \lambda_2 = 0.030801. \tag{89}$$

The canonical correlations are

$$\sqrt{\lambda_1} = 0.7941, \quad \sqrt{\lambda_2} = 0.1755. \tag{90}$$

For x_1, x_3, y_1, y_3 we have

$$\left\|\begin{array}{cc|cc} 1.0000 & 0.6374 & 0.7090 & 0.5508 \\ 0.6374 & 1.0000 & 0.7658 & 0.7518 \\ \hline 0.7090 & 0.7653 & 1.0000 & 0.7518 \\ 0.5508 & 0.7518 & 0.7518 & 1.0000 \end{array}\right\| = \left\|\begin{array}{cc} R_{11} & R_{12} \\ R_{21} & R_{22} \end{array}\right\|, \tag{91}$$

$$R_{22}^{-1} = \left\|\begin{array}{rr} 2.2999 & -1.7291 \\ -1.7291 & 2.2999 \end{array}\right\|, \tag{92}$$

$$R_{22}^{-1} R_{21} = \left\|\begin{array}{cc} 0.6782 & 0.4613 \\ 0.0409 & 0.4049 \end{array}\right\|, \tag{93}$$

$$R_{12} R_{22}^{-1} R_{21} = \left\|\begin{array}{cc} 0.5034 & 0.5501 \\ 0.5501 & 0.6577 \end{array}\right\|, \tag{94}$$

$$(95)\qquad \begin{vmatrix} 0.5034-\xi & 0.5501-0.6374\,\xi \\ 0.5501-0.6374\,\xi & 0.6577-\xi \end{vmatrix}=0$$

and canonical correlations are

$$(96)\qquad \sqrt{\xi_1}=0.8406,\quad \sqrt{\xi_2}=0.2605.$$

For x_2, x_3, y_2, y_3 the result is analogous.

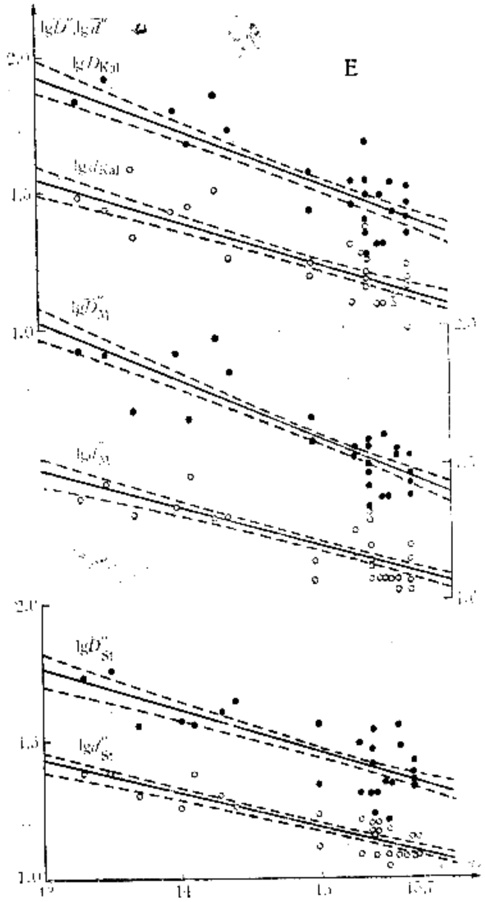
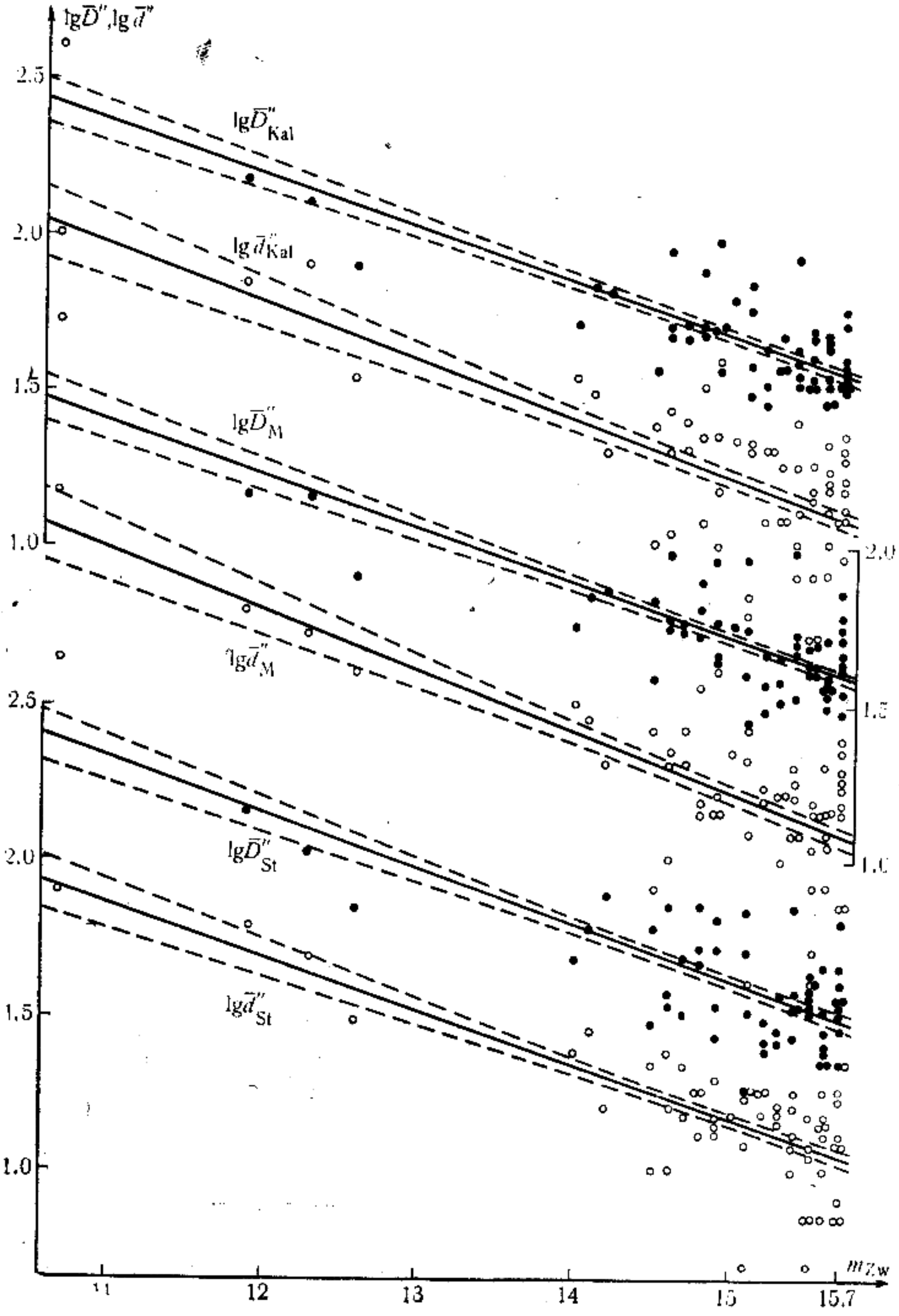


Fig. 25

All above canonical correlations show that the measurements of external diameters are in considerably better agreement than measurements of internal diameters.

We may determine the linear regressions between measured mean diameters and m_{Zw} using the classification of galaxies in Table 8. Here a logarithmic transformation is necessary too. The results for ellipticals are given on Fig. 25, together with the confidence limits, 26 galaxies being used. Regressions, correlation coefficients and confidence limits are



$\lg\bar{D}'', \lg\bar{d}''$
2.5
2.0
1.5
1.0
$\lg\bar{D}''_{\mathrm{Kal}}$
$\lg\bar{d}''_{\mathrm{Kal}}$
$\lg\bar{D}''_{\mathrm{M}}$
$\lg\bar{d}''_{\mathrm{M}}$
2.0
1.5
1.0
2.5
2.0
1.5
1.0
$\lg\bar{D}''_{\mathrm{St}}$
$\lg\bar{d}''_{\mathrm{St}}$
$m_{Z,\mathrm{w}}$
11
12
13
14
15
15,7

Fig. 26

(97) $\lg \bar{D}''_{\mathrm{Kal}} = \underset{\pm 21}{1.526} - \underset{\pm 29}{0.193}(m^{\mathrm{Zw}} - 15.031) = 4.422 - 0.193 m_{\mathrm{Zw}}, \quad r = -0.803,$

(97′) $\lg s^2 = 0.00046 + 0.00085(m_{\mathrm{Zw}} - 15.031)^2;$

(98) $\lg \bar{d}''_{\mathrm{Kal}} = \underset{\pm 18}{1.234} - \underset{\pm 24}{0.151}(m_{\mathrm{Zw}} - 15.031) = 3.510 - 0.151 m_{\mathrm{Zw}}, \quad r = -0.791,$

(98′) $\lg s^2 = 0.00031 + 0.00057(m_{\mathrm{Zw}} - 15.031)^2;$

(99) $\lg \bar{D}''_{\mathrm{M}} = \underset{\pm 19}{1.594} - \underset{\pm 26}{0.213}(m_{\mathrm{Zw}} - 15.031) = 4.797 - 0.213 m_{\mathrm{Zw}}, \quad r = -0.858,$

(99′) $\lg s^2 = 0.00037 - 0.00068(m_{\mathrm{Zw}} - 15.031)^2;$

(100) $\lg \bar{d}''_{\mathrm{M}} = \underset{\pm 15}{1.201} - \underset{\pm 21}{0.138}(m_{\mathrm{Zw}} - 15.031) = 3.279 - 0.138 m_{\mathrm{Zw}}, \quad r = -0.801,$

(100′) $\lg s^2 = 0.00024 + 0.00044(m_{\mathrm{Zw}} - 15.031)^2.$

(101) $\lg \bar{D}''_{\mathrm{St}} = \underset{\pm 19}{1.450} - \underset{\pm 26}{0.152}(m_{\mathrm{Zw}} - 15.031) = 3.741 - 0.152 m_{\mathrm{Zw}}, \quad r = -0.768,$

(101′) $\lg s^2 = 0.00036 + 0.00067(m_{\mathrm{Zw}} - 15.031)^2;$

(102) $\lg \bar{d}''_{\mathrm{St}} = \underset{\pm 10}{1.183} - \underset{\pm 14}{0.120}(m_{\mathrm{Zw}} - 15.031) = 2.980 - 0.120 m_{\mathrm{Zw}}, \quad r = -0.873,$

(102′) $\lg s^2 = 0.00010 + 0.00018 \ (m_{\mathrm{Zw}} - 15.031)^2.$

The results for spirals (Fig. 26) 59 galaxies, are

(103) $\lg \bar{D}''_{\mathrm{Kal}} = \underset{\pm 15}{1.666} - \underset{\pm 16}{0.172}(m_{\mathrm{Zw}} - 14.973) = 4.239 - 0.172\, m_{\mathrm{Zw}}, \quad r = -0.825,$

(103′) $\lg s^2 = 0.00022 + 0.00024(m_{\mathrm{Zw}} - 14.973)^2;$

(104) $\lg \bar{d}''_{\mathrm{Kal}} = \underset{\pm 24}{1.219} - \underset{\pm 25}{0.185}(m_{\mathrm{Zw}} - 14.973) = 3.995 - 0.185 m_{\mathrm{Zw}}, \quad r = 0.694,$

(104′) $\lg s^2 = 0.00060 + 0.00065(m_{\mathrm{Zw}} - 14.973)^2;$

(105) $\lg \bar{D}''_{\mathrm{M}} = \underset{\pm 16}{1.712} - \underset{\pm 16}{0.175}(m_{\mathrm{Zw}} - 14.973) = 4.330 - 0.175 m_{\mathrm{Zw}}, \quad r = -0.815,$

(105′) $\lg s^2 = 0.00025 + 0.00027(m_{\mathrm{Zw}} - 14.973);$

(106) $\lg \bar{d}''_{\mathrm{M}} = \underset{\pm 25}{1.215} - \underset{\pm 26}{0.196}(m_{\mathrm{Zw}} - 14.973) = 4.156 - 0.196 m_{\mathrm{Zw}}, \quad r = -0.709,$

(106′) $\lg s^2 = 0.00062 + 0.00067(m_{\mathrm{Zw}} - 14.973)^2;$

(107) $\lg \bar{D}''_{\mathrm{St}} = \underset{\pm 18}{1.613} - \underset{\pm 19}{0.181}(m_{\mathrm{Zw}} - 14.973) = 4.318 - 0.181 m_{\mathrm{Zw}}, \quad r = -0.782,$

(107′) $\lg s^2 = 0.00034 + 0.00036(m_{\mathrm{Zw}} - 14.973)^2;$

(108) $\lg \bar{d}''_{\mathrm{St}} = \underset{\pm 19}{1.171} - \underset{\pm 20}{0.173}(m_{\mathrm{Zw}} - 14.973) = 3.760 - 0.173 m_{\mathrm{Zw}}, \quad r = -0.756,$

(108′) $\lg s^2 = 0.00036 + 0.00039(m_{\mathrm{Zw}} - 14.973)^2.$

Regressions for 21 galaxies which are unclassified are given on Fig. 27:

(109) $\lg \bar{D}''_{\mathrm{Kal}} = \underset{\pm 15}{1.526} - \underset{\pm 41}{0.139}(m_{\mathrm{Zw}} - 15.190) = 3.632 - 0.139 m_{\mathrm{Zw}}, \quad r = -0.616,$

(109′) $\lg s^2 = 0.00023 + 0.00165(m_{\mathrm{Zw}} - 15.190)^2$;

(110) $\lg \bar{d}''_{\mathrm{Kal}} = 1.167 - 0.105(m_{\mathrm{Zw}} - 15.190) = 2.761 - 0.105 m_{\mathrm{Zw}}, \quad r = -0.290,$

$\pm 30 \quad \pm 80$

(110′) $\lg s^2 = 0.00087 + 0.00633(m_{\mathrm{Zw}} - 15.290)^2$;

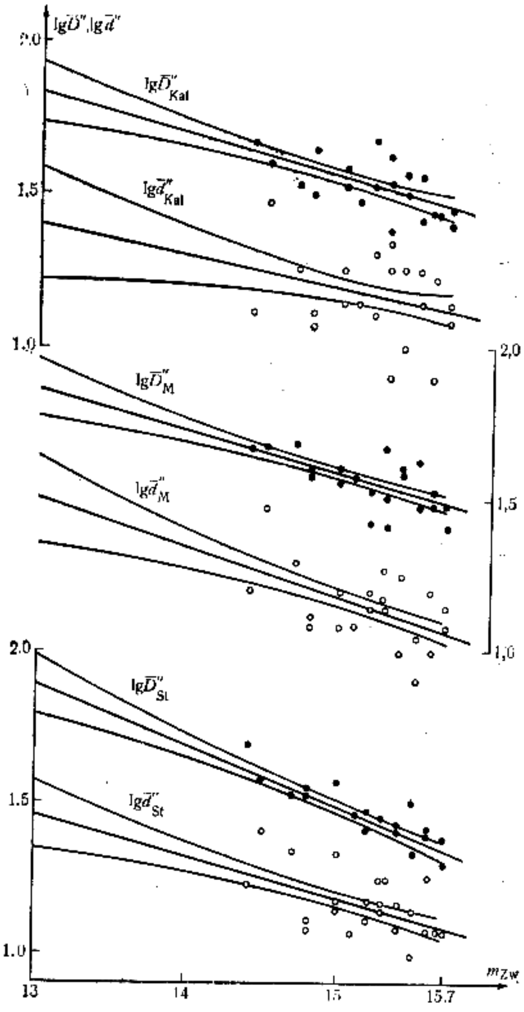


Fig. 27

(111) $\lg \bar{D}''_{\mathrm{M}} = 1.560 - 0.141(m_{\mathrm{Zw}} - 15.190) = 3.701 - 0.141 m_{\mathrm{Zw}}, \quad r = -0.634,$

$\pm 15 \quad \pm 39$

(111′) $\lg s^2 = 0.00022 + 0.00156(m_{\mathrm{Zw}} - 15.190)^2$;

(112) $\lg \bar{d}''_{\mathrm{M}} = 1.148 - 0.162(m_{\mathrm{Zw}} - 15.190) = 3.618 - 0.162 m_{\mathrm{Zw}}, \quad r = -0.498,$

$\pm 24 \quad \pm 65$

(112′) $\lg s^2 = 0.00058 + 0.00422(m_{\mathrm{Zw}} - 15.190)^2;$

(113) $\lg \overline{D}''_{\mathrm{St}} = \underset{\pm 15}{1.450} - \underset{\pm 41}{0.204}(m_{\mathrm{Zw}} - 15.190) = 4.551 - 0.204 m_{\mathrm{Zw}}, \quad r = -0.754,$

(113′) $\lg s^2 = 0.00023 + 0.00167(m_{\mathrm{Zw}} - 15.190)^2;$

(114) $\lg \overline{d}''_{\mathrm{St}} = \underset{\pm 18}{1.159} - \underset{\pm 50}{0.137}(m_{\mathrm{Zw}} - 15.190) = 3.246 - 0.137 m_{\mathrm{Zw}}, \quad r = -0.534,$

(114′) $\lg s^2 = 0.00034 + 0.00249(m_{\mathrm{Zw}} = 15.190)^2.$

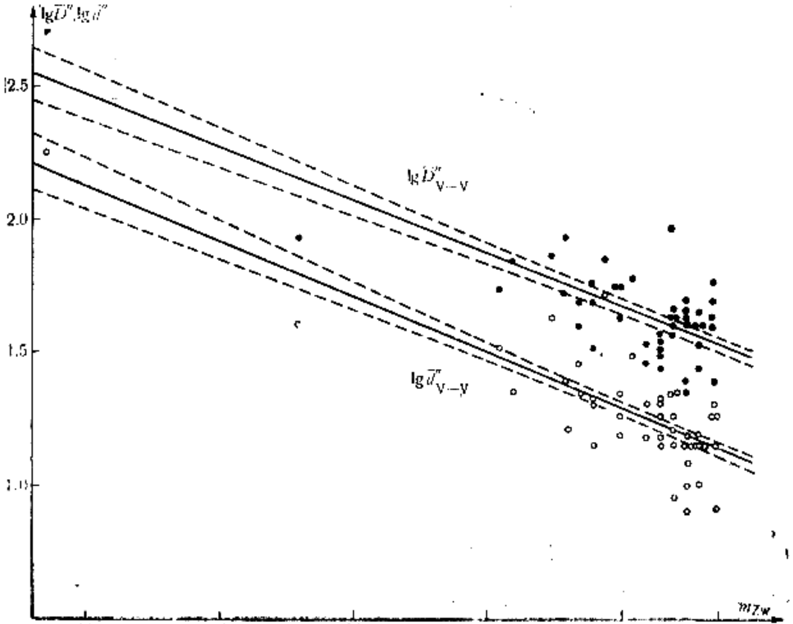


Fig. 28

Regressions for measured by Vorontsov-Velyaminov galaxies in investigated field are given on Fig. 28 (46 galaxies):

(115) $\lg \overline{D}''_{\mathrm{V-V}} = \underset{\pm 19}{1.655} - \underset{\pm 22}{0.199}(m_{\mathrm{Zw}} - 15.063) = 4.650 - 0.199 m_{\mathrm{Zw}}, \quad r = -0.800,$

(115′) $\lg s^2 = 0.00037 + 0.00050(m_{\mathrm{Zw}} - 15.063)^2;$

(116) $\lg \overline{d}''_{\mathrm{V-V}} = \underset{\pm 20}{1.267} - \underset{\pm 23}{0.213}(m_{\mathrm{Zw}} - 15.063) = 4.476 - 0.213 m_{\mathrm{Zw}}, \quad r = -0.810,$

(116′) $\lg s^2 = 0.00039 + 0.00054(m_{\mathrm{Zw}} - 15.063)^2.$

Let us examine in detail measurement of ellipticals. Let us denote

(117)
$$x_1 = \overline{D}''_{\mathrm{Kal}}, \quad x_2 = \overline{D}''_{\mathrm{M}}, \quad x_3 = \overline{D}''_{\mathrm{St}},$$
$$y_1 = \overline{d}''_{\mathrm{Kal}}, \quad y_2 = \overline{d}''_{\mathrm{M}}, \quad y_3 = \overline{d}''_{\mathrm{St}}.$$

The normalized correlation matrix is

$$(118)\qquad \left\|\begin{matrix} 1.0000 & 0.9523 & 0.8538 & 0.8455 & 0.8048 & 0.7794 \\ 0.9523 & 1.0000 & 0.8565 & 0.8212 & 0.7161 & 0.7603 \\ 0.8538 & 0.8565 & 1.0000 & 0.6712 & 0.7060 & 0.7377 \\ 0.8455 & 0.8212 & 0.6712 & 1.0000 & 0.8815 & 0.8381 \\ 0.8048 & 0.7161 & 0.7060 & 0.8815 & 1.0000 & 0.8599 \\ 0.7794 & 0.7603 & 0.7377 & 0.8381 & 0.8599 & 1.0000 \end{matrix}\right\| .$$

The matrix for x_1, x_2, y_1, y_2 is

$$(119)\qquad \left\|\begin{array}{cc|cc} 1.0000 & 0.9523 & 0.8455 & 0.8048 \\ 0.9523 & 1.0000 & 0.8242 & 0.7161 \\ \hdashline 0.8455 & 0.8242 & 1.0000 & 0.8815 \\ 0.8048 & 0.7161 & 0.8815 & 1.0000 \end{array}\right\| = \left\|\begin{matrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{matrix}\right\| .$$

Then

$$(120)\qquad R_{22}^{-1} = \left\|\begin{matrix} 4.4852 & -3.9537 \\ -3.9537 & 4.4852 \end{matrix}\right\| ,$$

$$(121)\qquad R_{22}^{-1}R_{21} = \left\|\begin{matrix} 0.6103 & 0.8520 \\ 0.2668 & -0.0349 \end{matrix}\right\| ,$$

$$(122)\qquad R_{12}R_{22}^{-1}R_{21} = \left\|\begin{matrix} 0.7307 & 0.6923 \\ 0.6922 & 0.6747 \end{matrix}\right\| .$$

The determinantal equation is

$$(123)\qquad \begin{vmatrix} 0.7307-\eta & 0.6923-0.9523\,\eta \\ 0.6922-0.9523\,\eta & 0.6747-\eta \end{vmatrix} = 0$$

whence for canonical correlations we have

$$(124)\qquad \sqrt{\eta_1} = 0.8550, \quad \sqrt{\eta_2} = 0.4501.$$

For matrix determined for x_1, x_3, y_1, y_3 we have

$$(125)\qquad \left\|\begin{array}{cc|cc} 1.0000 & 0.8533 & 0.8455 & 0.7794 \\ 0.8533 & 1.0000 & 0.6712 & 0.7377 \\ \hdashline 0.8455 & 0.8712 & 1.0000 & 0.8381 \\ 0.7794 & 0.7377 & 0.9381 & 1.0000 \end{array}\right\| = \left\|\begin{matrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{matrix}\right\| ,$$

$$(126)\qquad R_{22}^{-1} = \left\|\begin{matrix} 3.3603 & -2.8163 \\ -2.8163 & 3.3603 \end{matrix}\right\| ,$$

$$(127)\qquad R_{22}^{-1}R_{21} = \left\|\begin{matrix} 0.6461 & 0.1778 \\ 0.2378 & 0.5886 \end{matrix}\right\| ,$$

$$(128)\qquad R_{12}R_{22}^{-1}R_{21} = \left\|\begin{matrix} 0.7316 & 0.6091 \\ 0.6091 & 0.5535 \end{matrix}\right\| ,$$

(129) $$\begin{vmatrix} 0.7316-\theta & 0.6091-0.8538\,\theta \\ 0.6091-0.8538\,\theta & 0.5535-\theta \end{vmatrix}=0$$

and canonical correlations are

(130) $$\sqrt{\theta_1}=0.8563, \quad \sqrt{\theta_2}=0.4133.$$

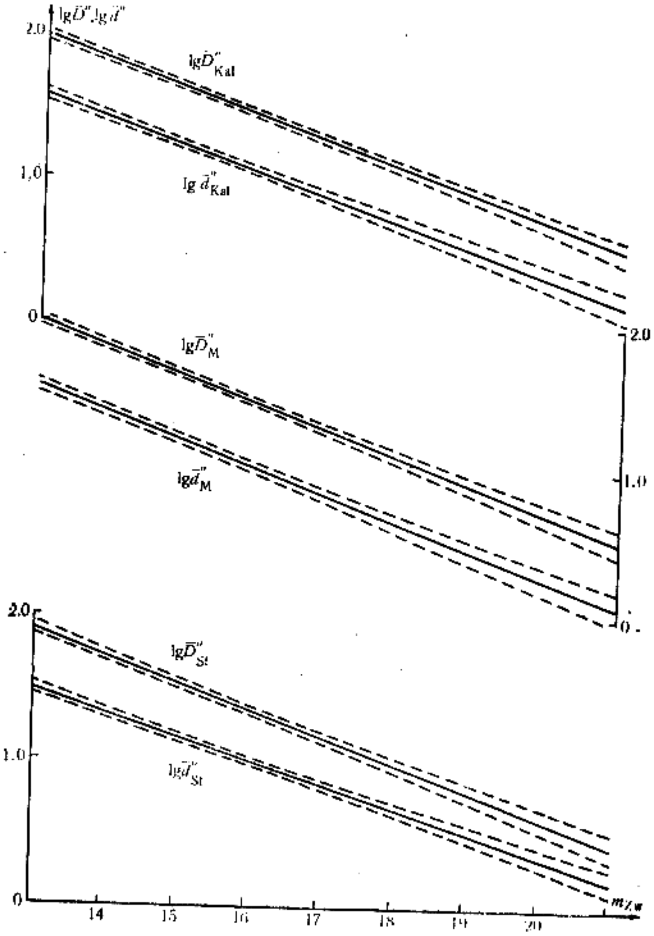


Fig. 29

First canonical correlations in both examined cases are greater than correlation coefficients between different sets. Consequently, external diameters of ellipticals are representative quantities. But second canonical correlations are not near to zero — moreover they are significant and therefore they cannot be completely neglected.

The final theoretical regressions between measured diameters of galaxies and m_{Zw} for Palomar print 64 O, from 106 galaxies, are given on Fig. 29:

$$(131)\ \lg \bar{D}''_{\mathrm{Kal}} = \underset{\pm 12}{1.604} - \underset{\pm 14}{0.179}(m_{\mathrm{Zw}} - 15.030) = 4.300 - 0.179 m_{\mathrm{Zw}}, \quad r = -0.799,$$

$$(131')\qquad \lg s^2 = 0.00014 + 0.00020(m_{\mathrm{Zw}} - 15.030)^2;$$

$$(132)\ \lg \bar{d}''_{\mathrm{Kal}} = \underset{\pm 15}{1.212} - \underset{\pm 19}{0.176}(m_{\mathrm{Zw}} - 15.030) = 3.860 - 0.176 m_{\mathrm{Zw}}, \quad r = -0.680,$$

$$(132')\qquad \lg s^2 = 0.00024 + 0.00035(m_{\mathrm{Zw}} - 15.030)^2;$$

$$(133)\ \lg \bar{D}''_{\mathrm{M}} = \underset{\pm 12}{1.653} - \underset{\pm 14}{0.186}(m_{\mathrm{Zw}} - 15.030) = 4.453 - 0.186 m_{\mathrm{Zw}}, \quad r = -0.792,$$

$$133')\qquad \lg s^2 = 0.00014 + 0.00020(m_{\mathrm{Zw}} - 15.030)^2;$$

$$(134)\ \lg \bar{d}''_{\mathrm{M}} = \underset{\pm 15}{1.198} - \underset{\pm 18}{0.185}(m_{\mathrm{Zw}} - 15.030) = 3.977 - 0.185 m_{\mathrm{Zw}}, \quad r = -0.704,$$

$$134')\qquad \lg s^2 = 0.00023 + 0.00033(m_{\mathrm{Zw}} - 15.030)^2;$$

$$(135)\ \lg \bar{D}''_{\mathrm{St}} = \underset{\pm 13}{1.540} - \underset{\pm 16}{0.182}(m_{\mathrm{Zw}} - 15.030) = 4.275 - 0.182 m_{\mathrm{Zw}}, \quad r = -0.739,$$

$$(135')\qquad \lg s^2 = 0.00018 + 0.00026(m_{\mathrm{Zw}} - 15.030);$$

$$(136)\ \lg \bar{d}''_{\mathrm{St}} = \underset{\pm 12}{1.172} - \underset{\pm 14}{0.160}(m_{\mathrm{Zw}} - 15.030) = 3.575 - 0.160 m_{\mathrm{Zw}}, \quad r = -0.747,$$

$$(136')\qquad \lg s^2 = 0.00013 + 0.00020(m_{\mathrm{Zw}} - 15.030)^2.$$

Regressions (131)—(136) allow the determination of limiting magnitudes up to which counting and measurement of diameters of galaxies is possible. As far as the nonrandom variable here is m_{Zw}, limiting magnitudes must be determined namely from (131)—(136) and not from $m_{\mathrm{Zw}} - \lg \bar{D}''$ and $\lg \bar{d}''$.

Table 12

$\bar{D}, \bar{d}$	Ellipticals		Spirals		All galaxies	
	3''.355	6''.71	3''.355	6''.71	3''.355	6''.71
$\bar{D}''_{\text{V-V}}$					$20^m.72$	$19^m.21$
$\bar{d}''_{\text{V-V}}$					18.55	17.13
$\bar{D}''_{\mathrm{Kal}}$	$20^m.2$	$18^m.60$	$21^m.6$	$19^m.8$	21.09	19.40
$\bar{d}_{\mathrm{Kal}}$	19.8	17.8	18.8	17.1	18.94	17.23
$\bar{D}''_{\mathrm{M}}$	20.1	18.6	21.7	20.0	21.11	19.50
$\bar{d}''_{\mathrm{M}}$	20.0	17.8	18.5	17.0	18.66	17.03
$\bar{D}''_{\mathrm{St}}$	21.2	19.4	21.0	19.3	20.60	18.95
$\bar{d}''_{\mathrm{St}}$	20.4	17.9	18.7	17.0	19.06	17.18

Relations (131)—(136), as it is seen from confidence limits, are suitable for prediction (or extrapolation) over quite a large range of m_{Zw}.

Control measurements of diameters and counting over several Palomar prints show that counting of galaxies can be carried out certainly down to 0.03 mm (1 division of the eyepiece scale) or $3^m.355$, while for measurements of galaxy diameters—down to 0.06 mm (2 divisions), at then the probability for an adoption of stars instead of galaxies being insignificant.

Limiting magnitudes for 1 and 2 divisions for Palomar print 64 O are given in Table 12. Above regressions for ellipticals and spiral, as well as for all galaxies have been used.

As it is seen from Table 12, counting of galaxies can be carried out up to 21^m and measurements of galaxy diameters — up to $19^m.0$—$19^m.5$.

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ИЗМЕРВАНЕ НА ДИАМЕТРИ НА ГАЛАКТИКИ ВЪРХУ ПАЛОМАРСКИЯ АТЛАС

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(Резюме)

Дават се резултатите от измерванията на външните и вътрешните диаметри на галактиките до $15^m,7$ върху копия 1398 и 64 от Паломарския атлас. Всички измервания са извършени независимо от трима автори (табл. 1 и 7; включени са и измерванията на Воронцов-Вельяминов).

Определени са регресиите между m_{Zw} и $B(0)$. Определени са и връзките между измерените средни диаметри и $B(0)$.

Установена е необходимостта от логаритмична трансформация на диаметрите при регресиите със звездните величини.

За всички наблюдатели са определени както регресиите, така и доверителните интервали между измерените от тях средни диаметри и звездните величини по Цвики. С помощта на методи на многомерния статистически анализ е установено, че външните видими диаметри са репрезентативни. Аналогичен извод е получен и за сферичностите на галактиките.

Показано е, че ефективно преброяване на галактиките върху Паломарския атлас може да бъде проведено до $20^m,7-21^m,0$, а ефективно измерване на техните диаметри — до $19^m,0-19^m,5$, като при това за всяко копие от регресиите и доверителните интервали могат да бъдат направени уверени оценки на зездните величини въз основа на видимите средни диаметри на галактиките.

ИЗМЕРЕНИЕ ДЕАМЕТРОВ ГАЛАКТИК НА ПАЛОМАРСКОМ АТЛАСЕ

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(Резюме)

Приведены результаты измерения внешних внутренних диаметров галактик до $15^m,7$ на копиях 1398 и 64 Паломарского атласа. Все измерения проведены тремя авторами независимо друг от друга (табл. 1 и 7, включающие и измерения Воронцова-Вельяминова).

Определены регрессии между m_{Zw} и $B(0)$. Определены также корреляции между средними замеренными диаметрами и $B(0)$.

Установлена необходимость логарифмической трансформации звездных величин.

Для всех наблюдателей определены как регрессии, так и доверительные интервалы между измеренными средними диаметрами и звездными величинами по Цвики. Методами многомерного статистического анализа установлена репрезентативность внешних видимых диаметров. Аналогичный вывод получен и для сферичности галактик.

Показано, что эффективный подсчет галактик на Паломарском атласе может проводится до $20^m,7$—$21^m,0$, а эффективное измерение их диаметров — до $19^m,0$—$19^m,5$ причем для каждой копии регрессии и доверительных интервалов можно сделать оценку звездных величин на основании видимых средних диаметров галактик.