On reconstruction of astronomical images in observations through turbulent atmosphere

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Abstract. We demonstrate the technique for reconstruction of astronomical images, obtained during observations through a turbulent atmosphere. We use combined method, which includes: averaging of centered images; the filtration of their Fourier transforms by the inverse optical transfer function for the Kolmogorov model of atmospheric turbulence, and subsequent inverse Fourier transform. We use observational data from the high-speed CMOS camera on the Zeiss-600 telescope of Andrushevska Observatory with an exposure time of 0.01 sec and a shooting frequency of 30 frames per second. We show that the angular resolution can be improved up to the diffraction limit for observations made on small telescopes.
Key words: atmospheric turbulence - image reconstruction - stars: individual: 61 Cyg A

Introduction

Atmospheric turbulence distorts the wavefront of the light wave. This results in degradation of the image, loss of angular resolution, which can be from one to several arc seconds.

Fried (1967) established some important properties of atmospheric turbulence: the coherence radius r_0 , the coherence time, and the isoplanatic angle. The coherence radius is defined as the length of a wavefront area over which the rms phase variations are equal to 1 rad. At best sites r_0 ranges from 10-30 cm, the coherence time is 10-50 milliseconds and the isoplanatic angle is 2-10 arc seconds. Another important property related to turbulence is seeing. It is defined as the full width of the star image at half maximum (FWHM). At best sites seeing disk is 1/2 to 1/3 arc second.

Image degradation has the main two types: image motion and image blurring. The motion of the image is due to the distortion of the slopes of the wavefront. The blurring looks like a spotted pattern. The way to reduce the turbulence effect is to reduce the exposure time, so that it freezes the atmospheric turbulence. The exposure time is usually chosen to be equal to the coherence time.

The degradation of the image is usually eliminated by methods of adaptive optics. At the same time, it is possible to achieve a diffraction quality of the image in a small area of the focal plane (area of isoplanatism). It requires, however, a complex and expensive technique (Labeyrie, 2013). We can specify one of the image recovery methods. This is the so-called "lucky imaging". The randomness that prevails in a turbulent field rarely produces "lucky" images of high quality. A special technique (stacking, shift-and-add) allows to combine these images together, achieving nearly diffraction quality.

A number of large telescopes in the world are equipped with systems for observing "lucky imaging". Here should be mentioned the 5 m telescope at

Mount Palomar, the 3.5 m New Technology Telescope (NTT) in La Cilla in Chile (Hippler et al., 2009; Hormuth et al., 2007, 2008), the 2.56 m Nordic Optical Telescope (Baldwin et al., 2001), and others.

We use data from photometric observations of stars made with a high-speed CMOS camera. Photometric observations were performed with the Cassegrain focus (F = 7.2 m) on the Zeiss-600 telescope of the Andrushevka Observatory on July 30, 2016 in white light. The camera used was ZWO ASI174MM, 1936x1216, pixel size 5.86 microns. The observed object was the star 61 Cyg A, V = 5.20, with an exposure time of 0.01 sec, and shooting frequency of 30 frames per second. Note that during observations the average quality of images FWHM was about 3 arc seconds.

In this paper, we show that using this technique the angular resolution can be improved up to the diffraction limit for observations with small telescopes.

1 Algorithm of reconstruction of images

The goal is to restore a degraded image to its original form. The point scatter function (PSF) and the optical transfer function (OTF) are the main elements of the theory of image reconstruction in a turbulent atmosphere. Below we will operate with two kinds of images - with short and long exposure.

The scatter function PSF $h(x, y)$ describes the brightness distribution of the point source in the image plane (Rodionov, 2000).

Let's make some definitions:

- $I(x, y)$ - the image before degradation, 'true image'
- $\acute{I}(\acute{x}, \acute{y})$ - the image after degradation, 'observed image'
- $h(x, y)$ - the degradation filter, 'the scatter function PSF'
- $H(f_x, f_y)$ - the Fourier transform of h, ' the optical transfer function'
- $G(f_x, f_y)$ - the Fourier transform of I
- $I_R(x, y)$ - the estimate of $I(x, y)$ computed from $\acute{I}(\acute{x}, \acute{y})$

An observed image can be defined as:

$$\acute{I}(x, y) = \int\int h(x - \acute{x}, y - \acute{y})\, I(\acute{x}, \acute{y})\, d\acute{x}, d\acute{y}$$

where the integral is a convolution, h is the point scatter function of the atmosphere. In the symbolic form we have:

$$\acute{I}(x, y) = h(x, y) * I(x, y)$$

$$\acute{G}(u, v) = H(u, v)\, G(u, v)$$

We restore the degraded image $I(x, y)$ using *many realizations* of the turbulence-distorted images with short-exposure $\acute{I}(r, t)$. According to Kandidov (1998), distortions of short-exposure images $\acute{I}(r, t)$ include three main components:

- diffraction distortions associated with the finiteness of the receiving aperture,
- blurring caused by the averaged contribution of small-scale atmospheric inhomogeneities,
- large-scale image shifts associated with random changes the inclination of the wavefront within the limits of the aperture.

In one of the methods (Kandidov, 1998), the procedure of averaging the short-range images $\tilde{I}(r,t)$ is used to eliminate large-scale distortions: $< I(r) >= \frac{1}{M}\sum_{\mu=1}^{M}\tilde{I}(r)$. We determine the image spectra for a long exposure $G_L(\Omega)$, where $M \gg 1$, and for the averaged $G_{<>}(\Omega)$, where M is of the order of several units: $G_L(\Omega) = F[I_L(r)]$ and $G_{<>}(\Omega) = F[< I > (r)]$. Here $\Omega(f_x, f_y)$ is the angular spatial frequency. According to Kandidov (1998), the restored image will have the form:

$$I_R(r) = F^{-1}\left[G_{<>}(\Omega)/H_L(\Omega)\right],$$

or

$$I_R(r) = F^{-1}\left[\frac{1}{M}\sum_{\mu=1}^{M}\tilde{I}(r)/H_L(\Omega)\right],$$

where F^{-1} is the inverse Fourier transform operator.

The expression for $H_L(\Omega)$ in the case of the Kolmogorov spectrum of fluctuations in the refractive index has the form (Kandidov, 1998):

$$H_L(\Omega) = \exp(-3.44(\lambda\Omega/r_0)^{5/3}), \qquad (1)$$

where r_0 is the Fried radius, λ is the wavelength.

Another method (Averin, 1210; 2014) uses the expression for the averaged OTF $H(\Omega)$ calculated for the case of short exposures after elimination of large-scale image shifts caused by random changes in the wavefront inclination angle within the aperture range:

$$H(\Omega) = \exp(-3.44\{(\lambda\Omega/r_0)^{5/3}[1 - \alpha(\frac{\Omega}{\Omega_0})^{1/3}]\}) \qquad (2)$$

Here $\Omega_0 = D/\lambda$ is the spatial truncating frequency of the optical system spectrum; D is the diameter of the aperture; r_0 is the Fried parameter. The parameter $\alpha = 1$ for "near field" (when only phase effects are significant) and 0.5 for "far field" (when amplitude and phase distortions are equally significant); $\alpha = 0$ corresponds to the case of prolonged exposure. In this case, formula (2) coincides with formula (1).

The near-field condition, when only phase aberrations are important in a turbulent atmosphere, has the form: $D \gg (L * \lambda)^{1/2}$, where L is the path length of the beam through a turbulent atmosphere. It is satisfied for a telescope with an aperture greater than 6 cm.

The essence of the method of image reconstruction consists in using OTF as the reverse filter by the formula (2). This will correct the residual small-scale blurring of the image.

Thus, to restore the image, the following operations are performed: the random shifts $\Delta x, \Delta y$ of the images are measured by calculating the cross-correlation function, these shifts are eliminated by centering the images, then the centered images are added together. The summation is recursively performed with the recursion coefficient R of the order of 0.01-0.05 (Averin et al., 2011):

$$I_{0n+1}(x, y) = (1 - R) \cdot I_{0n}(x, y) + R \cdot I_n(x + \Delta x, y + \Delta y) \tag{3}$$

Next, the Fried parameter is estimated and the inverse OTF $H^{-1}(\Omega)$ is calculated according to the formula (2). And, finally, performing the deconvolution operation, we get the reconstructed image:

$$I_R(r) = F^{-1}[F(I_{0n+1})/H(\Omega)] \tag{4}$$

As a result of these operations, a stabilized image of diffraction quality is obtained if the aperture of the receiving system is not too large in comparison with the Fried parameter ($D \leq (5 \div 7)\, r_0$) (Averin et al., 2011).

To estimate the Fried parameter, we use the expression for atmospheric OTF (Tokovinin, 2005). The atmospheric OTF $H_A(\Omega)$ is related to the statistics of phase aberrations in a turbulent atmosphere, the so-called phase structure function $D_\varphi(\lambda\Omega)$:

$$H_A(\Omega) = exp(-0.5\, D_\varphi(\lambda\Omega)).$$

The Kolmogorov model of atmospheric distortions gives the following form of the phase structure function:

$$D_\varphi(r) = 6.88 \left(\frac{r}{r_0}\right)^{5/3}$$

This formula contains only the Fried parameter. Using this model, we obtain an expression for atmospheric OTF in the form:

$$H_A(\Omega) = \exp(-3.44(\lambda\Omega/r_0)^{5/3})$$

It is easy to see that this formula coincides with formula (1) for OTF under long exposures. Applying the inverse Fourier transform to this equation, we obtain the atmospheric PSF. The numerical value of the half-width of atmospheric PSF (FWHM) is usually called the image quality β. The calculations give the following relationship between the image quality for long exposures and the Fried parameter (Tokovinin, 2005):

$$\beta = 0.98\, \frac{\lambda}{r_0}$$

In particular, it is easy to estimate from this that at a wavelength of 0.5 μm and the image quality of 1 angular second, the Fried parameter corresponds to $r_0 = 10.1$ cm.

Note, as practice shows, when we observe an extended object on the surface routes, the final image is improved with the accumulation of frames. When processing approximately 100 consecutive frames, a stabilized image is practically obtained of diffraction quality (Averin et al., 2011).

2 Results

A quantitative measure of image quality is the Strehl ratio, which is equal to the ratio of the intensity at the maximum of the distorted image to the intensity at the maximum of the Airy disk. As a typical example, we give data for the 2.56 m Nordic Optical Telescope with an average image quality FWHM $\sim$ 0.5 arc second (Baldwin, et al, 2001). The histogram of the Strehl ratio from the data of about 6000 measurements of the ϵ Aquilae within 30 sec gives the maximum of the ratio in the range of values 0.1-0.15.

"Good" images have frames in which the ratio of Strehl is greater than a certain preset level, say 0.25-0.30. The frames corresponding to this criterion are subject to summation after the corresponding "shift-and-add". Fried (1967) calculated the probability P to obtain a "good" image on an aperture of diameter D with an image quality determined by the Fried parameter r_0:

$$P = 5.6 \exp(-0.1557(D/r_0)^2)$$

This means that for the aperture $D = (5, 6, 7)\ r_0$, the probability of a "lucky" exposure is 11, 2 and 0.3 %, respectively. And for $D = 10\ r_0$, only 10^{-6}. Apparently, this limits the use of the proposed image reconstruction technique with large telescopes.

In practice, the calculation of the Strehl ratio is a difficult task. For the Zeiss-600 telescopes, the diameter of the Airy circle for the wavelength λ = 0.55 μm is about 0.23 arc second. However, the residual aberrations of the optics give the scattering function of the point FWHM $\sim$ 0.6-0.7 arc second. We approximate the scattering circle by a Gaussian curve when calculating the modified Strehl ratio. We must proceed from a scattering circle determined by the residual aberrations of the telescope. In subsequent calculations, we used frames with a Strehl ratio greater than 0.25-0.30.

With a developed turbulence, variations of the Fried parameter are observed. As noted in the work (Averin et al., 2010) it is possible to change it by an iterative way, proceeding from the best quality of the image correction, and also by changing the recursion coefficient in the formula (3).

The images of the 61 Cyg A star reconstructed in 100 consecutive frames (FWHM = 0.65 arc second) and with an exposure of 1 second (FWHM = 2.28 arc second) are shown in Fig. 1, and their photometric sections in Fig. 2. The images are normalized to unit intensity at the maximum.

Fig. 3 shows the resultant point scatter function, and the optical transfer function (frequency-contrast characteristic) of the reconstructed image of the 61 Cyg A star. The resultant point scatter function has FWHM $\sim$ 0.65 arc second. As noted, for the Zeiss-600 telescope, residual aberrations are

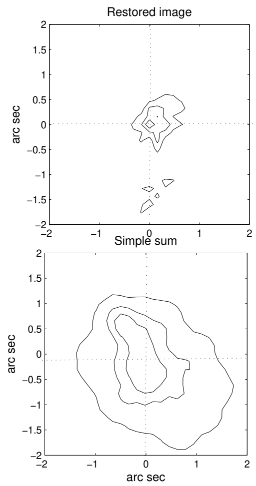

Fig. 1. Contour images of the star 61 Cyg A, reconstructed (top picture) and with an exposure of 1 second (lower figure).

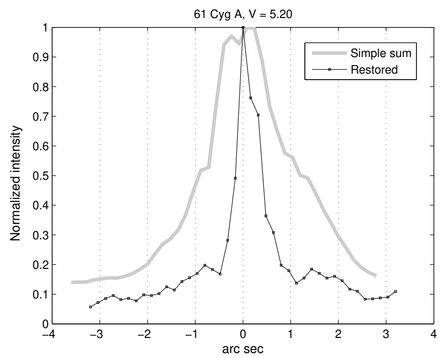

Fig. 2. Photometric sections of the star images 61 Cyg A, FWHM = 2.28 arc second (exposure 1 sec), FWHM = 0.65 arc second (reconstructed image).

about 0.6 arc second. Thus, we can conclude that the reconstructed image is close to the theoretical limit.

3 Conclusion

We obtain a diffraction quality corrected images by applying a sequence of steps. We use a high speed camera in order to accumulate a large number of images with frozen atmospheric turbulence. The random shifts are eliminated through the centering of the images, which are then averaged

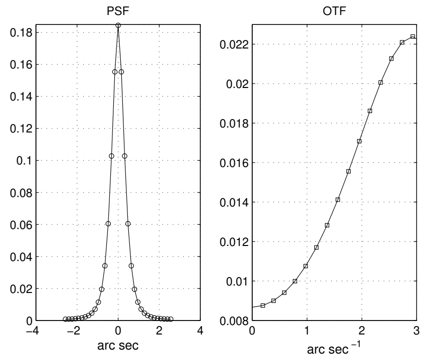

Fig. 3. The resulting normalized point scatter function (left image) and module of the optical transfer function (right image) of the reconstructed image of the star 61 Cyg A, in the scale of angular seconds and reverse angular seconds, respectively. Two-dimensional PSF and OTF are normalized to a unit volume.

in order to eliminate large scale distortions. Finally a deconvolution step is applied to account for the atmospheric PSF.

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References

Averin A.P., Pryanichnikov V.S., Tyapin V.V., 2010, *Quantum Electronics, 40, No. 5, 418*

Averin A.L., Morozov Yu.B., Pryanichnikov V.S., et al., 2011, *Quantum Electronics, 41, No. 5, 457*

Averin A.P., Morozov Yu.B., Pryanichnikov V.S. et al., Tyapin V.V., 2013, *A method for determining the parameters of a turbulent atmosphere, (RF patent 2488095, July 20, 2013)*

Baldwin J. E., Tubbs R. N., Cox G. C. et al., 2001, *Astron. Astrophys., 368, L1*

Fried D. L., 1967, *Proc. IEEE 55, 57*

Labeyrie A., 2013, *EAS Publications Series, 59, 5*

Kandidov V.P., Chesnokov S.S., Shlenov S.A., 1998, *Optics of Atmosphere and Ocean, 11, No. 4, 522*

Hippler S., Bergfors C., Brandner W. et al., 2009, *The ESO Messenger, 137*

Hormuth F., Brandner W., Hippler S. et al., 2007, *Astron. Astrophys., 463, 707*

Hormuth F., Hippler S., Brandner W. et al., 2008, *SPIE, 7014, 48*

Tokovinin A., 2005, *Adaptive optics tutorial at CTIO. Imaging through the turbulence, URL: http://www.ctio.noao.edu/?atokovin/tutorial/part1/turb.html*

Rodionov S.A., 2000, *Foundations of Optics, SPb, GITMO*