A stochastic sampling method for the analysis of eclipsed pulsations

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(Submitted on 30.09.2019. Accepted on 25.11.2019)

Abstract. Studies of stellar oscillations have a history of several decades. Their importance lies in their information content about the internal stellar structure. Recent space missions have hugely extended our knowledge about them. However, in order to go beyond the general characterization of stellar interiors and perform asteroseismic inversions, the identification of many modes is still required. In this respect pulsating stars in eclipsing binaries offer a unique opportunity, if the effective surface sampling of the eclipse events can be properly analyzed. Here we present an investigation of new possibilities to perform mode identification in such systems, using parameter fitting driven by stochastic methods.

Key words: pulsating stars, ecipsing binaries, MCMC, Gibbs sampling

Introduction

Pulsating stars are one of the most important type of variable stars in stellar astrophysics. The periodic variation of their brightness can be easily observed from ground observatories or space telescopes. This light variation is caused by the internal waves generated by self-sustaining processes in a wide variety of stellar evolutionary stages. Thanks to this, we can determine in principle the properties of the stellar interior and the absolute physical parameters (e.g. stellar radius, mass) with a proper analysis, which is the main task of asteroseismology (Aerts, Christensen-Dalsgaar & Kurtz, 2010). This field of astronomy has experienced significant advances in recent years thanks to the high precision photometry missions (e.g. *MOST*, *Kepler*).

The mode identification of the stellar pulsations is a crucial step in the analysis. It is an especially hard task because the pulsation patterns cannot be directly seen on the surface of distant stars. There are several methods for solving this challenge of determining the degree ℓ and azimuthal order m of the modes for each frequency. For single stars this involves a sophisticated analysis of either the brightness, or, better, high-resolution spectral feature variations. Another interesting possibility is the so-called *eclipse tomography*, which requires an eclipsing stellar companion for the pulsating star. In this case the other component is periodically mapping the surface of the pulsating star. In a good geometric configuration, the modulation of amplitude and phase variations can be distinguished between different (ℓ,m) mode numbers (Fig. 1). This method, termed *Dynamic Eclipse Mapping*, was presented by Bíró, I. B. 2011, 2000, and successfully applied to an eclipsing binary system by Bíró I. B., Bókon A. 2017.

Thanks to the high precision measurement of *Kepler*, a new era has begun in the observational astronomy. The properties of stars can be inferred in unprecedented details, and new challenges and questions have been found in different territories of observational astrophysics. From our viewpoint, one of the biggest challenges is the hundreds of frequencies found in the Fourier spectra of pulsating stars. The modulation of these frequency components can make the task of eclipse tomography very difficult.

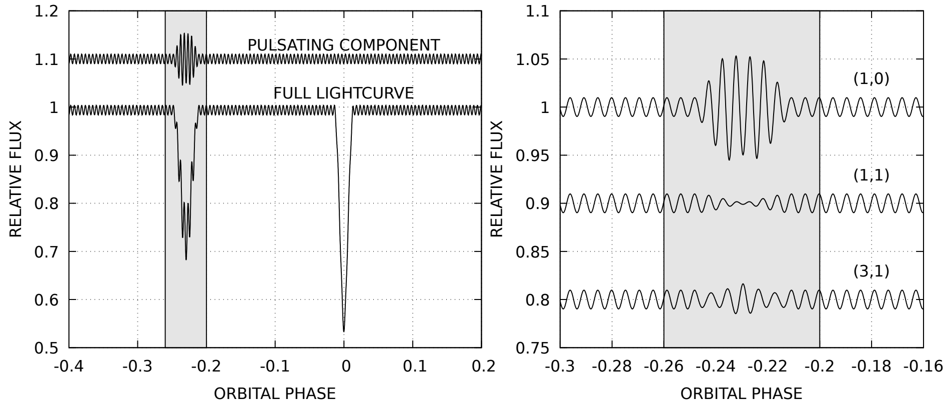

Fig. 1. An example of the modulation effect of the eclipses on the pulsations. Left panel: a full lightcurve of an eclipsing binary system (e=0.485; ω=0.29) with a pulsating secondary component. The light contribution of the secondary component is also shown vertically shifted upwards. Right panel: the amplitude modulations due to the eclipse of various (ℓ,m) nonradial oscillation patterns. The secondary eclipse region is highlighted with gray in both panels.

A second key issue is that the developed fitting methods search for one solution only. Specifically, the aim of Dynamic Eclipse Mapping is to find the simplest pulsating pattern of the stellar disc consistent with the observations. Another method, the Direct Fitting, obtains the best fit of spherical harmonics for each frequency. None of them provides any information about the relevance and the probabilities of the (ℓ,m) solutions specifically.

In this paper we present a novel method to solve the challenges outlined above. To our best knowledge, there is no other attempt to date addressing this problem. In Sec. 1 we describe the methods and their implementation. In Sec. 2 we present the result and discussion of tests carried out so far. In Sec. 3 we summarize our conclusions.

1. Methods

One of the common choices for this type of problem is Markov-chain Monte Carlo (*MCMC*). It has a large number of implementations, with the common aim to achieve ergodic sampling of the parameter space in terms of the posterior probability distribution. The ergodic property ensures that the joint *aposteriori* distribution of the parameters is proportional to the sample density in the parameter space. In addition, we can also incorporate *apriori* information about the parameters via the well-known Bayes' theorem.

We choose the Metropolis-Hastings algorithm because it is easily available in almost any programming language. This algorithm carries out random steps in the parameter space, accepting or rejecting the new parameter set depending on conditions crafted to ensure an ergodic sampling of the posterior distribution. Although it is implemented in various `Python` packages, we wrote our own implementation, because the nonradial mode numbers are restricted to integer numbers, and they have a special constraint ($|m| \leq \ell$).

We have also considered another commonly used stochastic method, the Gibbs sampling, as an alternative for performing this task. The Gibbs sampling is used for restricted parameter spaces, it has a simpler algorithm, but requires more steps and thus more execution time, because it tries every possible value for each parameter while keeping the others fixed. The chosen parameter is randomly weighted according to the aposteriori probability of the possible values. It is also implemented by us, in order to have the same output as MCMC.

Each implementation assumes that the pulsational pattern can be described by spherical harmonics. This is a strong restriction, because there should not be any distortion in the equilibrium state of the star (e.g. fast rotation, tidal distortion). This restriction could be, however, easily relaxed in the future, by allowing more general amplitude profiles – at the expense of introducing additional parameters, of course.

According to Bayes' theorem the *aposteriori* probability density distribution (pdf) is proportional to the likelihood of the parameters, multiplied by their *apriori* pdf. For normally distributed measurement errors the likelihood is expressed by the the chi-squared function as

$$\mathcal{L}(\boldsymbol{p}) = a \cdot \exp\left(-\frac{1}{2}\,\chi^2(\boldsymbol{p})\right), \tag{1}$$

where χ^2 is the sum of squared residuals, and a is a normalization constant. In the absence of any *apriori* information we have a uniform prior, and the posterior pdf is in essence the likelihood $\mathcal{L}$.

Both methods have a common key issue, that is the treatment of the discrete variables ℓ and m. Besides being discrete, there is a specific constraint between them ($|m| < \ell$). We tried several ways to find the one that best suits our purpose:

- The first one was the use of ℓ and m mode numbers as normal variables independently, which we called *classical variables* in the following part of our paper.
- The second one was treating the pair of (ℓ, m) numbers, fully describing the mode, as a set of *category variables*. For the criterion of their ordering we used the deviation of their amplitude modulation during eclipse, compared to that of the radial case $(0, 0)$. (The ordering is required in order to have *distances* defined between two variables, crucial for the sampling methods in computing the next step of the chain.)

Beside the programs of MCMC & Gibbs, we have also created some auxiliary routines to do the analysis of the Markov chains. Calculating and inspecting the autocorrelation of parameters is essential in order to assess the reliability of the chains. For the visualization of the joint as well as the marginalized posterior distributions it is customary to use *corner plots*. They contain information about the most probable parameter values and correlations between them. We created a modified version of this plot for several reasons. The biggest change is using a colorbar to encode the sampling number of the individual modes. An example is given in Fig. 2.

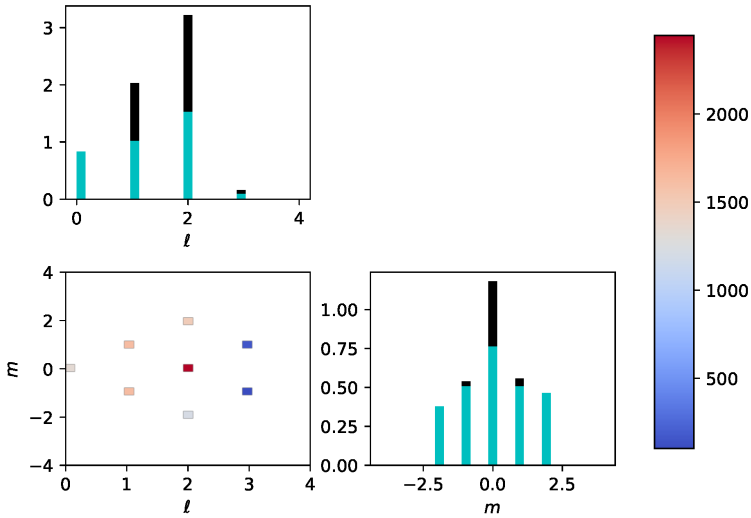

Fig. 2. An example for the modified corner-type plots. The joint posterior distribution inferred from the Markov-chain is in the left bottom figure. The colors encode how many times each mode appears in the chain, which is proportional to their posterior probability. It can be easily seen that $\ell = 2$ and $m = 0$ is the most probable oscillation mode in this particular example. Diagonally running are the marginal distributions of the mode degree and azimuthal order, shown with black-colored bars. The cyan color is used for showing the highest number of sampling of the parameter.

Several synthetic lightcurves were modeled in order to check the reliability of the algorithm. The modeled systems were detached eclipsing binary systems, with the secondary components nonradially pulsating simultaneously in 1, 2, 3 and 4 frequencies. The nonradial pulsations were selected with ℓ between 1 and 4. The total number of lightcurves was about 100, which were inspected with the created programs described above.

In order to further explore the limits of our programs, after the initial tests we modeled a lightcurve of an eclipsing binary system with the secondary component pulsating in 16 simultaneous modes with various frequencies (Fig. 3). There were two kinds of accomplished tests. The first was a full simultaneous MCMC, which means that all the parameters were sampled. The second one was a manually conducted whitening process similar to the methods used in the harmonic analysis of multiperiodic signals, where the individual peaks are fitted and subtracted in decreasing order of amplitude from the signal. The difference here was that we grouped the frequencies in packs, so that frequencies within a pack had the same order

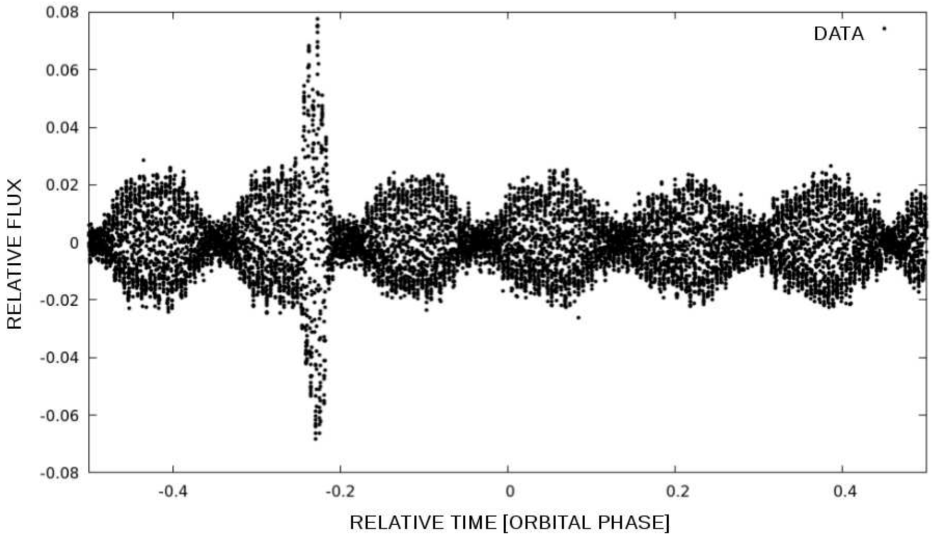

Fig. 3. The lightcurve used for the comprehensive test of our method using MCMC and Gibbs algorithm. The number of frequencies are 8, and there was another number of sinusoids which is considered as a noise.

of contribution to the lightcurve; then the pack with the highest contribution was sampled, after which its result was subtracted from the lightcurve before turning to the next pack. At any stage, the unprocessed frequencies were assigned the mode numbers (0,0) as a default assumption.

2. Results and conclusions

The results of the selected runs is summarized in Tab. 1 and 2. The tables compare the selected runs for the given (ℓ,m) nonradial mode numbers, the variable types which were adapted (cl = classical, ca = ordered categorical variable), and the top 3 candidates (ℓ,m) with the percentages compared to the total elements of the Markov chain.

The first key point derived from the results was that the selectivity is preserved. The selectivity means that specific (ℓ,m) nonradial pulsations have characteristic modulations from which the mode numbers can be the most easily determined by the inverse methods. This property remained for the MCMC and Gibbs, with high probability (~80-90%) of the considered (ℓ,m).

The second remark is that similar modulations of typical (ℓ,m) nonradial modes (e.g. (1,0) and (2,1)) have similar probabilities. This property is actually the aim which we wanted to achieve, because the possibility of other nonradial oscillations for a given frequency could be assigned a probability. In general the original input mode numbers were among the top three candidates.

(ℓ,m)	Var. type	N.o. 1 (%)	N.o. 2 (%)	N.o. 3 (%)
(1,1)	cl	(2,0) (29.96)	**(1,1) (18.3)**	(2,0) (17.0)
(1,1)	ca	(2,2) (21.53)	(2,0) (19.95)	**(1,1) (18.73)**
(2,2)	cl	**(2,2) (41.9)**	(2,0) (21.3)	(1,1) (14.8)
(2,2)	ca	**(2,2) (37.53)**	(1,1) (18.82)	(2,0) (16.2)
(3,2)	cl	**(3,2) (90.5)**	(2,1) (6.1)	(2,-1) (2.3)
(3,2)	ca	**(3,2) (99.88)**	(2,1) (0.05)	(2,-2) (0.04)

Table 1. Selected results from the run of MCMC, tested on the dataset with 1 frequency. In each column can be found in order from left to right: the given (ℓ,m) nonradial mode number, the variable type which was adapted (cl = classical, ca = ordered categorical variable), and the top 3 candidate (ℓ,m) with the percentages of Markov chain.

(ℓ,m)	Var. type	N.o. 1 (%)	N.o. 2 (%)	N.o. 3 (%)
(1,1)	cl	(0,0) (51.83)	(2,2) (15.5)	**(1,1) (11.2)**
(1,1)	ca	(2,2) (20.61)	(2,0) (18.97)	**(1,1) (17.5)**
(2,2)	cl	(0,0) (46.8)	**(2,2) (28.9)**	(1,1) (14.3)
(2,2)	ca	**(2,2) (39.4)**	(1,1) (17.31)	(2,0) (16.1)
(3,2)	cl	**(3,2) (100)**	-	-
(3,2)	ca	**(3,2) (99.70)**	(2,1) (0.06)	(2,-2) (0.03)

Table 2. Selected results from the run of Gibbs sampling, tested on the dataset with 1 frequency. In each column can be found in order from left to right: the given (ℓ,m) nonradial mode number, the variable type which was adapted (cl = classical, ca = ordered categorical variable), and the top 3 candidate (ℓ,m) with the percentages of Markov chain.

The third conclusion was that the use of ordered category variables is more suited to our problem than the classical ones, especially in the case of Gibbs sampling. A plausible explanation could be that the Gibbs algorithm cannot sample sufficiently the parameter space with constraints specified between parameter sets.

The ability of assigning probabilities to possible nonradial mode numbers for each frequency can be very useful for later work, as well. In direct modeling, the oscillation modes are given just in a probabilistic way at specified frequency ranges. This information can be compared in order to check the validity of the known pulsating models. Naturally, we should study many eclipsing binaries with pulsating components in order to get reliable results.

Shortcomings were also found during the analysis of the Markov chains. The biggest issue can be that specific mode numbers, e.g. (1,0) cannot be found by MCMC with category variables. The cause for this key problem could be due to the ordering, because some mode numbers are in the second half of the list. This issue can be solved by using improved alternative orderings of the categories, for example sorting by the amplitudes of the oscillations *on the stellar surface*. Obviously, more investigation is required in this direction.

Another challenge is that as the number of the pulsation modes included in the analysis increases, the sampling for the modes with smaller amplitudes become less reliable, when compared to the larger amplitude modes. The reason for this is yet unclear, but we could tackle this shortcoming by the whitening procedure mentioned in Sec. 1, involving fitting of frequencies grouped in sets of similar amplitudes.

3. Summary

We have successfully applied MCMC and Gibbs sampling algorithms for obtaining probability information on the modes of nonradial oscillations of components of eclipsing binaries. Although the parameter sets were well restricted because of the integer values of (ℓ,m), we managed to solve them with the proper handling of the variables. The new programs were tested with lightcurves modeled with different nonradial modes.

After executing the required tests, the following key points were found:

- the selectivity of the method remained (oscillation modes with well distinguishable modulations have clearly the highest probability, giving back the true, original modes),
- in the case of ambiguous mode numbers, probability can be assigned to different modes.

These conclusions show that the main goal of our work was accomplished in large part. Some refinement is necessary (fine tuning in MCMC or other ordering method of categoric variable), but the results are promising.

We plan to further develop the code by including an *automatic whitening process*. This means that all the modeling and subtraction mechanisms, which were conducted manually as written in Sec. 1, will be done by a frame code that drives the algorithms used so far. This core program is needed for the larger problem, which attempts to solve the issue of the large number of parameters for frequencies with considerably differing amplitudes.

We plan to apply the existing and/or future programs to analyze eclipsing binary systems observed by the *Kepler* space mission. So far we have investigated only one system (KIC 3858884), but in the near future we would like to study other systems as well. Presumably the TESS survey mission will also provide further systems of interest.

Acknowledgments: We thank Barna Imre Bíró for providing us the `pulzem` suite containing the modeling and Direct Fitting codes. We thank the anonymous referee for the suggestions and the comments that improved our paper. This work was supported by the New National Excellence Program (UNKP-18-318-3-I-SZTE-13) of the Ministry of Human Capacities of Hungary.

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