Extended Theories of Gravity and Their Applications to Neutron Stars

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Neutron stars are the last stage of the evolution of massive stars. Their small sizes ($R_{NS} \approx 10 - 15$km) and masses that can exceed 2 solar masses (Antoniadis et al. 2013), lead to densities higher than the density of the atomic nuclei and their magnetic fields are some of the strongest known to humanity ($10^{14} - 10^{15}$G). Those features make them great stellar laboratories for studying physics at conditions unobtainable on Earth. The gravitational field generated by neutron stars is also extreme as it is $\sim 10^{11}$ times stronger than Earth's. This makes the relativistic effects substantial and one needs relativistic theory to describe them.

The observable evidence of the existence of dark matter and dark energy comes from astrophysics and cosmology, like gravitational lensing measurements, galaxy rotation curves of spiral galaxies, etc. for dark matter, and the accelerated expansion of the Universe, etc. for dark energy. One of the possible explanation is that general relativity is not the theory of gravity, but it is part of some broader extended theory of gravity (Sotiriou & Faraoni 2010), which effects become significant at large energies. One of the places where those theories can be tested are the neutron stars, where deviations from general relativity and new effects predicted by the extended theories could be found (Staykov et al. 2014).

The main purpose of the dissertation is to study the effects on neutron stars of a model of extended theory of gravity. The model used in the thesis is called minimal dilatonic gravity (MDG). It is a model alternative to general relativity with Brans-Dicke parameter $w = 0$ and a potential (Fiziev, 2000). MDG uses one gravitational-dilaton field Φ and offers simultaneous explanation of effects connected with dark matter and dark energy. A special class of potentials are introduced in the model. They confine dynamically the values of the dilaton Φ in the physical domain. There is a correspondence between MDG and f(R) theories. They are related via Legendre transformations, but the two models are equivalent only under the additional assumption that the potential has the withholding property.

The system of equations describing neutron stars in MDG is composed of differential equations for the mass, pressure and two new variables introduced in the model, the dilaton Φ and the dilaton pressure p_Φ. The procedure for the derivation of the equations is similar to the one in general relativity, though it is more complicated. The equations are solved from the center to the surface of the star. Since on the surface of the star the dilaton pressure $p^\star_\Phi \neq 0$, we continue integrating shortened version of the system describing neutron stars outwards, using the results on the surface as left boundary conditions and the cosmological horizon as right boundary condition.

The problems in MDG require, like problems in general relativity, the solving of nonlinear differential equations. In the case of neutron stars, where the matter part of the equation is essential, the problem is practically unsolvable analytically, but can be treated numerically. The physical situation leads to differential equations with boundary conditions at two points, the starting and ending points of the integration. This is called two-points boundary value problem. In the thesis the numerical method used to perform the calculations is the shooting method, which reduces the boundary value problem to system of initial value problems. In general, due to the free parameters, the boundary conditions at the end point are not satisfied and the correct initial conditions have to be found.

The main results and conclusions of our study can be summarized as follows (Fiziev & Marinov 2015; 2017; Marinov & Fiziev 2018):

1. We obtained results for stable static spherically-symmetric neutron stars in the model of minimal dilatonic gravity with polytropic equation of state and with four realistic equations of state (SLy, BSk19, BSk20, BSk21) (Potekhin et al., 2013). The models of neutron stars in MDG do exist for certain domain of parameters and the proper domain is not universal, but it depends on the equation of state.

2. Different values of the free parameter d (the dimensionless Compton length of the dilaton) were considered in the model, which gave us insight how different physical quantities depend on the dilaton field. Parallel to that, the research included the initial conditions for the equations describing neutron stars, further broadening the picture. A new effect in MDG was observed, showing shrinkage of the domain of the initial conditions due to the approaching of a bifurcation point.

3. Very important feature of the model is the existence of dilaton sphere around neutron stars. It is like an atmosphere composed of dilaton, called dilasphere. It carries about $15\% - 20\%$ of the mass of the object and plays the role of dark matter halo. The dilasphere is the main reason behind the lager masses of neutron stars in MDG compared to General Relativity. The total maximum masses (pure neutron star and dilasphere) of neutron stars depend on the mass of the dilaton. The obtained values for maximum masses are in accordance with the observational data of neutron stars with $M_{obs} > 2M_{\odot}$.

4. A detailed study concerning quantities interpreted as dark matter and dark energy was performed. The values of the pressure interpreted as dark matter and the pressure interpreted as dark energy are comparable with the matter pressure, thus they play significant role in the structure of a neutron star in MDG.

5. A research on the structure of a neutron star in the MDG model was performed, showing how different quantities change from the center to the surface of the star. This includes mass, pressure, dilaton and quantities concerning dark matter and dark energy.

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