The origin of impact craters: some ideas

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Abstract. The surfaces of solid objects in the planetary system are dotted by impact craters. From the viewpoint of condensed matter physics, the problem of the origin of these craters can be expressed in the following way: in a material of given chemical composition, there exists a hole of given dimensions. Assuming that this hole is a consequence of an impact, what can be concluded on the impactor? This problem can be analyzed in two ways: by using scaling laws and by the standard laws of condensed matter physics. The aim of this paper is to present basic notions of the two approaches, and give some of the results.

Key words: impacts craters, condensed matter, scaling laws

1.Introduction

Observation, ranging from those performed by the earliest telescopes to current work by space probes, shows that surfaces of many solid bodies in the planetary system are filled with craters. A part of them are of volcanic origin and they are outside the scope of this paper. The subject of this paper are impact craters; that is craters which are due to impacts of smaller solid bodies (often called impactors) in the surfaces of larger objects in the Solar system. Some impact craters have halos (Bart et al.,2019). Physically, the impactors are remnants from the epoch of formation of the Solar system (for example Perryman, 2011,Carlson, 2019).

According to the database kept at the University of New Brunswick in Canada at http://www.passc.net/, at present there are 190 known impact craters on the Earth. Due to their number, their purely scientific interest, but also the possible dramatic consequences which an impact could have, the study of impact craters has become a well developed sub-field of planetary science.

The most widely known impact crater is certainly the Barringer crater in the Arizona desert,which is mentioned in publications ranging from elementary school textbooks to scientific papers. A much less studied impact structure seems to exist in the Falkland Plateau, north-west of the West Falkland island. Seismic reflection profiles indicate the presence of a large roughly circular basin with a diameter of approximately 250 km. The best explanation of this structure is the presence of a large burried impactor (Rocca et al.,2016).A much more recent impact event, which did not produce a crater, is the one of 15 February 2013, when a small asteroid entered the atmosphere over the city of Chelyabinsk in Russia. There were no victims, but a large number of people have been injured by shattered glass.

Finally, at the time of this writing (July 2019) there appeared an analysis concerning the possibility of Earth impact by asteroid $2009FD$. The result, obtained using recent astrometry and radar data, is that the main impact possibility in 2185 is ruled out, but there remains a very small possibility in 2190 (DelVigna et al.,2019).

The aim of this paper is to present an introduction to two different theoretical approaches to the problem of impact craters. The second section is devoted to the approach using scaling theory, while the third part uses standard condensed matter physics. Both approaches treat the same problem: having data on the target and the crater, what can be concluded on the impactor and the impact?

2. The scaling theory

Predicting the outcome of one event from the result of another is often called *scaling*. The relation used in this predicting is called the *scaling law*, and the parameters which change between the two events are named *scaled* variables. The need for the application of scaling in the problem of impact craters is obvious: impact craters in reality have diameters of the order of meters to kilometers, while craters in terrestrial experiments are smaller (Holsapple,1993).

There exist three ways in which the form of scaling laws can be predicted: these are impact experiments, analytical calculations and approximate theoretical calculations. The principle of impact experiments is simple, so it may be imagined that they are simple to perform. There is, however, a problem in impact experiments: currently accessible impactor velocities in laboratories are smaller than theoretically expected collision velocities (for example, Hibbert et al., 2017). In principle, projectiles of various mass and composition are fired with different speeds into targets of different composition. The aim is to measure diameters of the resulting craters as a function of the impact velocities and the chemical compositions of the target and the impactor. A detailed presentation of one of the actual installations for impact experiments, at the University of California in Davis, can be found at http://geology.ucdavis.edu/read/stewart$_s hockwave$.

Analytical calculations on the impacts are founded on the laws of balance of mass, momentum and energy, with the addition of the equation of state (EOS) of the material of the target and of the impactor. The problem with this approach are the details in the knowledge of the (EOS) and thermodynamic potentials of the target and of the impactor. Work in this direction will be somewhat easier in the future, owing to results of big calculational projects in material science. Examples of such projects are the Materials Genome Initiative http://www.mgi.gov in the US and the mainly European project Novel Materials Discovery http://www.nomad-coe.eu.

These projects give material characteristics as a function of various parameters. However, the choice of a material for any particular astronomically interesting case is beyond their scope. In terrestrial applications it is (in principle) possible to collect pieces of the impactor and of the target. However, in the case of asteroid(s) moving towards the Earth, the only way of determining their composition is spectroscopy. Problems can arise if an asteroid is made up of a material with small albedo,which implies that the composition will be determined with a large relative error. Assuming that the impactor is sufficiently massive, and that the speed of impact is sufficiently high, in the moment of impact there occurs a phase transition $solid \rightarrow gas$ or even $solid \rightarrow plasma$. After a rapid cooling, the result of the impact can be analyzed within solid state physics.

The basic idea of the approximate theoretical solutions is the so called "point source model". Simply stated, the phase immediately after impact is described as a "point source" of shock waves propagating through the target. It was developed near the middle of the last century for analysis of nuclear explosions.

Attempting to predict the volume V of a crater formed in an impact is the simplest case of application of scaling laws in these problems. Denote the radius of the impactor by r, its velocity by v and density by ρ_1. The target has density ρ, gravitational acceleration at the surface g, and it is made up of material with material strength X. Material strength is defined as the ability of a material to withstand load without failure. In that case:

$$V = f[\{r, v, \rho_1\}, \{\rho, X\}, g]. \tag{1}$$

The first three variables describe the impactor, the following two - the material making up the planet and the last one - the surface gravity of the planet. Scaling laws can be derived from this expression by dimensional analysis.

It can be shown that equation (1) leads to

$$\frac{\rho V}{m} = f_1[\frac{gr}{v^2}, \frac{X}{\rho v^2}, \frac{\rho}{\rho_1}], \tag{2}$$

where $m = \frac{4\pi}{3}\rho_1 r^3$ is the mass of the impactor. The quantity on the left side is the ratio of the mass of the material within the crater to the mass of the impactor. It is usually called cratering efficiency and denoted by π_V. The first term in the function is the ratio of the lithostatic pressure $\rho g r$ to the initial dynamic pressure ρv^2, generated by the impactor. The lithostatic pressure at a certain depth is defined as the pressure exerted by the material above it. This ratio is denoted by π_2; the second term is the ratio of the material strength to the dynamical pressure, denoted by π_3. The final term is the ratio of the mass densities. If all the parameters of eq.(2) were known, or could be determined, it would not be too difficult to determine the volume of an impact crater. Finding the general solution of this equation is an open problem. As a consequence, solutions of this equation are usually studied in two limiting cases: the "strength" regime and the "gravity" regime.

The main difference of these two situations is in the value of the ratio of strength of the surface material and the lithostatic pressure.The "strength" regime is the situation in which the strength of the surface material is larger than the lithostatic pressure, which implies impactors with diameters of approximately one meter (Holsapple,1993). This means that

$$\frac{\rho V}{m} = f_1[\frac{X}{\rho v^2}], \tag{3}$$

where it was assumed that the ratio of the densities is approximately one. In this regime, the volume of the impact crater increases linearly with the volume of the impactor, its mass and its energy. Any dimension of the crater increases with the radius of the impactor. In the opposite case, when the diameter of the impactor is of the order of a kilometer or more, the lithostatic pressure is bigger than the material strength, meaning that

$$\frac{\rho V}{m} = f_1[\frac{gr}{v^2}]. \quad (4)$$

This is the definition of the "gravity" regime. Various experiments (discussed in Holsapple, 1993) have been performed on the dependence of π_V on π_2, the result being an exact power law. This can be explained, as discussed in (Holsapple, 1993), by the assumption that whenever there is a dependence on the impactor size and speed, it is actually the dependence on its kinetic energy. This idea was used in the early sixties, in scaling from a nuclear event called "Teapot ESS" to the creation of the Meteor Crater in Arizona. For some more details on this aspect of the problem see,for example Celebonovic (2017).

3.Condensed matter physics

Surfaces of objects in the solar system which contain impact craters are solid. It is known that impactors are solid objects, so the ensuing question is what can be concluded about the impacts by using laws of condensed matter physics and all kinds of measurable parameters of the target. Possibilities exist, and they will be discussed using previous results of the present author (Celebonovic and Souchay,2010 ; Celebonovic,2013,2017).

The first condition for the creation of an impact crater is the velocity of the impactor, which must exceed some minimal value. This was determined in Celebonovic and Souchay (2010). It was postulated in that paper that a crater will be formed if the kinetic energy of a unit volume of the impactor is equal to the internal energy of a unit volume of the material of the target. As discussed in that paper, the minimal speed of the impactor is given by

$$v^2 = \frac{\pi^2}{5\rho_1}\frac{(k_B T)^4}{\hbar^3}(\frac{\partial P}{\partial \rho})^{-3/2}, \quad (5)$$

where ρ_1 is the mass density of the impactor, T the temperature of the target, and P, ρ are the pressure and mass density of the material of the surface of the target respectively. From the point of view of condensed matter physics, this equation is physically correct. In order to test its applicability in a real astronomical situation, it was applied to the case of an impactor made of olivine $(Mg, Fe)_2SiO_4$. The minimal speed of this object would be 16.3 km/s. For comparison, note that the impact velocity of a real object, asteroid 99942 Apophis, is estimated to be between 13 and 20 km/s. This means that two completely different methods: celestial mechanics and condensed matter physics give mutually close results on the same problem, which is extremely encouraging.

It is stated sometimes that condensed matter physics can not be applied to impact cratering because in hyper velocity impacts the material of the target melts and can even evaporate. The final result of an impact is a crater. If the kinetic energy of the impactor is high enough, and if the target has a suitable value of the heat capacity, a consequence of the impact will be heating of the target. Heating in impacts has been studied in Celebonovic and Nikolic (2015). Depending on the kinetic energy of the impactor, the target may heat enough so as to melt, and possibly even evaporate at the point of impact. In this regime, condensed matter physics cannot be applied. Regardless of the amount of heating in the impact, the outcome is always the same: a certain quantity of material of the target gets "pushed aside" at the point of impact, implying the creation of a crater of certain dimensions. The aim of the calculations outlined here is to draw conclusions about the impactor using measurable dimensions of the crater and various parameters of the target.

Using the vocabulary of condensed matter physics, the problem of formation of impact craters can be expressed as the following equivalent problem: how big must be the kinetic energy of the impactor in order to produce a hole of given dimensions in a target material with known parameters (Celebonovic,2013). It was assumed that the target is a crystal, that one of the usual types of bonding exists in it, and that the target does not melt as a consequence of the impact.

The calculation is based on a simple and acceptable physical idea: the kinetic energy of the impactor must be greater than, or equal to the internal energy of some volume V_2 of the target. The kinetic energy of the impactor of mass m_1 and speed v_1 is obviously

$$E_k = \frac{1}{2} m_1 v_1^2, \tag{6}$$

and the internal energy E_I consists of three components: the cohesion energy E_C, the thermal energy E_T and $E_H(T)$ - the energy required for increasing the temperature of the material at the point of impact by an amount ΔT. Therefore,

$$E_I = E_C + E_T + E_H(T), \tag{7}$$

and the condition for the formation of an impact crater as a consequence of an impact is

$$E_I = E_k. \tag{8}$$

Expressions for various terms in E_I exist in the literature, and are given in Celebonovic (2013). Details of the calculation are given in that paper, and the final result is

$$\begin{aligned} & 3k_B T_1 N\nu[1 - \frac{3}{8}\frac{T_D}{T_1} - \frac{1}{20}(\frac{T_D}{T})^2 + \frac{1}{10}(\frac{T_D^2}{TT_1}) \\ & +(\frac{1}{560})(\frac{T_D}{T})^4 - \frac{1}{420}\frac{T^4}{T^3T_1} - \frac{3\bar{u}^2\rho\Omega_m}{np\nu k_B T_1}] = \frac{2\pi\rho_1}{3} r_1^3 v_1^2. \end{aligned} \tag{9}$$

This is the energy condition which has to be satisfied for the formation of an impact crater. Various symbols have the following meanings: the number N is the ratio of the volume of the crater to the volume of the elementary crystal cell,v_e: $N = V/v_e$; k_B is Boltzmann's constant; T is the initial temperature of the target; T_1 is the temperature to which the target heats;

T_D is the Debye temperature of the target; ρ_1,r_1 v_1 - mass density,radius and impact velocity of the projectile respectively; p,n - parameters of the interatomic interaction potential in the material of the target; ν is the number of particles in the elementary crystal cell; $\bar{u}$ is the speed of sound in the material of the target; and Ω_m is the volume per particle pair.

It might seem at first glance, that eq.(9) is just a complicated expression without any physical purpose. Close inspection shows that this equation groups known or measurable parameters on the left hand side, while the right hand side contains parameters of the impactor. It means that this expression gives the possibility of estimating parameters of the impactor from known data on the target.

As a test, this equation was applied to a well known case: the Barringer crater in Arizona. Assuming that the material of the crater is pure Forsterite (Mg_2SiO_4), and making plausible assumptions about the other parameters of eq.(13), it was obtained that $v_1 \cong 41$ km/s, which is far larger than existing estimates obtained by using celestial mechanics.

A possible solution is to assume that the material of the target is a chemical mixture. Suppose that only 10 percent of the material is Forsterite, and keep all the other parameters constant. This will give the value of $v_1 \cong 15$ km/s, for the impact speed, which is much closer to the results obtained by celestial mechanics. Details of this calculation are avaliable in Celebonovic (2013) .

The calculation outlined above was performed using the notion of cohesive energy of solids, which is a very "impractcal"quantity: it is defined as the energy needed to transform a sample of a solid into a gas of widely separated atoms (Marder,2010). As a consequence of this definition, it is difficult to measure experimentally and it is not related to the strength of solids measurable in experiments.

A much more "practical" notion is the stress. It is defined as the ratio of the force applied on a body to the cross section of the surface of a body normal to the direction of the force. After an impact, a crater will form if the stress in the material becomes sufficiently high for the formation of a fracture.

A fracture will occur in a material, if the stress σ in it is greater than the critical value σ_C (Tiley,2004; Celebonovic,2015).

$$\sigma_C = \frac{1}{2}\left(\frac{E\chi\tau}{r_0 w}\right)^{1/2}, \tag{10}$$

where E is Young's modulus of the material, χ is the surface energy, τ is the radius of curvature of the crack, r_0 the interatomic distance at which the stress becomes zero, and w is the length of a crack which preexists in the material. Defined in this way, σ_C has the dimensions of pressure.

Applying the law of conservation of energy to the moment of impact leads to interesting conclusions. The kinetic energy of the impactor, E_k, is used in the moment of impact for fracturing and heating the material of the target. That is

$$E_k = \sigma_C V + C_V V(T_1 - T_0), \tag{11}$$

where V is the volume of the crater formed as a result of the impact, C_V is the heat capacity of the target material, and T_0 is the initial temperature of the target. At this point, one encounters the problem of finding a suitable mathematical approximation of the shape of a crater, in order to be able to make an analytical estimate of the volume V. In accordance with experiments, the volume of the crater is approximated by

$$V = \frac{1}{3}\pi b^2 c, \tag{12}$$

where b is the radius of the "opening" of the crater and c denotes its depth. If one approximates the impactor as a sphere of radius r_1, made up of a material of density ρ_1 and having impact velocity v_1, its kinetic energy is given by $E_k = \frac{2\pi}{3}\rho_1 r_1^3 v_1^2$. It follows from eq.(11) that

$$T_1 = T_0 + \frac{1}{C_V}(\frac{E_k}{V} - \sigma_C), \tag{13}$$

and after some algebra (Celebonovic, 2017):

$$V = \frac{2\pi}{3}\frac{\rho_1 r_1^3 v_1^2}{\alpha C_V T_0 + \sigma_C}, \tag{14}$$

where $T_1 - T_0 = \alpha T_0$, with $\alpha > 0$.Equation (14) can be transformed into the following form

$$V = \frac{E_k}{\alpha C_V T_0 + \sigma_C}, \tag{15}$$

which shows that the volume of a crater is a linear function of the kinetic energy of the impactor.

4.A Granular Target?

It was assumed throughout this paper that the surface of the target is a crystal. This is very often, but not always true. As an example, should an impact occur in a sandy desert like Sahara, the material there certainly could not be approximated as a crystal. In general terms, the question encountered here can be expressed in the following way: how do the consequences of an impact change if the material of the target is granular and not a crystal? From the point of view of condensed matter physics, this "transition" is extremely interesting.

We shall concentrate on two particular aspects: the shape of impact craters when formed in a granular material, as compared to their shape in a crystal, and changes in the quantity of energy needed to heat a granular material compared to the energy needed to heat the same amount of crystalline material for the same temperature difference. In this paper, the volume of a crater was approximated with eq.(12). On the other hand, experiments with normal incidence of a solid spherical impactor into a deep non-cohesive granular bed have shown that the profile of a crater can be fitted by

$$z = z_c + \sqrt{b^2 + c^2 r^2}, \tag{16}$$

where z_c, b and c are fitting parameters (de Vet and de Bruyn,2007). Inserting this expression for the profile of a crater into eq.(12), and using the same notation in both cases, one gets the following result for the difference of crater volumes formed after an impact into a crystal (V_1) and granular material (V_2):

$$V_1 - V_2 = \frac{1}{3}\pi z r^2 - \frac{1}{3}\pi r^2 [z_c + \sqrt{b^2 + c^2 r^2}]. \tag{17}$$

A simple calculation shows that this difference can become equal to zero for

$$z = z_c + \sqrt{b^2 + c^2 r^2} \tag{18}$$

Physically, this means that there exist some conditions of impacts which lead to craters of equal volume in crystal and granular targets, assuming that all the other conditions are the same in the two cases.

Targets similar to terrestrial granular materials have been found in the planetary system. Well known examples are asteroids 253 Mathilde and 25143 Itokawa. It was shown that both of these objects consist of loosely bound pieces, so they later got the name "rubble piles". For a recent review see (Walsh, 2018).

Rubble piles are obviously *porous*,and the the main physical parameter characterizing them is the porosity, defined as the following ratio:

$$\phi = \frac{V_V}{V_T} < 1, \tag{19}$$

where V_V denotes the volume of voids and V_T the total volume of the object. The value of V_T can be decomposed as follows:

$$V_T = V_1 + V_V = V_1 + \phi V_T, \tag{20}$$

which means that

$$V_T = \frac{V_1}{1-\phi}, \tag{21}$$

where V_1 denotes the volume of the "solid component" of V_T.

The quantity of energy needed to heat a volume V_T of a material having the specific heat C_V by a temperature difference of ΔT is given by:

$$Q = C_V V_T \Delta T = (C_{V1} + C_{V2}) \times \frac{V_1}{1-\phi} \times \Delta T, \tag{22}$$

where C_{V1} denotes the specific heat of the "solid component" of V_T and C_{V2} is the specific heat of the voids. Finally,

$$Q = \frac{C_{V1} + C_{V2}}{1-\phi} V_1 \Delta T, \tag{23}$$

which is the expression for the quantity of energy needed to heat up for ΔT a volume V_1 of a solid having the specific heat C_{V1}, the porosity ϕ and the specific heat of voids C_{V2}.

The value of the specific heat C_{V1} depends on the chemical composition of the material. If the composition is known, then this value can either be measured or calculated. More interesting is the problem of the specific heat of the voids,denoted here by C_{V2}. The unknown quantity here is the composition of the voids. If they contain only vacuum, and if there is no source of thermal radiation within the voids, the specific heat C_{V2} will be zero. However, if the voids are filled with some gas, then the value of Q will depend on the ratio of the two specific heats.

How does this expression compare with the result for a pure solid? Using eq.(21), simple reasoning shows that

$$Q = Q_S + C_{V2} V_T \Delta T, \tag{24}$$

where Q_S is the quantity of energy needed to heat a "pure" solid. Clearly, this result strongly depends on the values of C_{V2} and the porosity ϕ. The implication is that some more energy is needed to heat up a porous than a non-porous material, with all the parameters being the same. Obviously, the closer the value of ϕ is to 1, the bigger the value of Q will be.

Conclusion

In this contribution we have presented in some detail two approaches to the problem of impact craters on the surfaces of solid objects in the planetary system. One is the scaling theory, while the other is standard condensed matter physics. Both of these approaches have the same aim: using existing laboratory, field and observational data and relevant laws of physics, draw as much conclusions as possible on the impacts and the impactors.

Scaling theory attempts to link the craters of "celestial " origin with those resulting from man made explosions. The good side of this approach is generality, but the "minus" is the problem of treatment of phase transitions.

The approach founded on standard condensed matter physics is based on well known laws, but also has a weak point (at least one). It can treat either slow impacts or "not very massive" projectiles.

Both approaches give useful contributions to the problem, but they are both in need of improvement in the future.

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