Application of Kolmogorov-Smirnov Test in the Distribution of Saturn's Regular Satellites

Fabao Gao, Huan Cheng
School of Mathematical Science, Yangzhou University, Yangzhou 225002, China
gaofabao@sina.com
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Abstract. This article focuses on the application of nonparametric statistical inference methods in astronomy, aiming to reveal the potential distribution of physical characteristics of Saturn's regular satellites. Based on the Kolmogorov-Smirnov test, we fit the eight physical features (equatorial radius, equatorial circumference, volume, density, mass, surface area, surface gravity, and escape velocity) of Saturn's regular satellite data one by one. It is found that except for the two characteristics of density and surface gravity, which obey the Stable distribution and the generalized Pareto distribution, respectively, the best-fitting distributions of the other six physical characteristics are all Lognormal distributions. The rationality of this result is verified by comparison with the cumulative distribution function derived from an analytical perspective. In addition, an example of predicting the physical properties of Saturn's satellite Pandora is added to illustrate the effectiveness of distribution inference to further illustrate in practical applications.
Key words: Saturn's regular satellites; physical characteristics; Kolmogorov-Smirnov test; Lognormal distribution

1 Introduction

In October 2019, the team of Sheppard at the *Carnegie Institute for Science* in the United States announced the newly discovered 20 Saturn satellites (Sheppard 2019). This discovery made Saturn become the planet with the most satellites in the solar system by having 82 natural satellites. Saturn's satellites can be divided into regular and irregular satellites, which can be mathematically represented as a function of orbital eccentricity, orbital inclination, and distance from Saturn (Denk et al. 2018). More detailed information can be found in . Saturn and its satellites are usually called "the small solar system". Researchers are keen to study the Saturn system with multiple satellites (Canup 2010, Dones et.al 2009, Hirata 2016, Castillo-Rogez et.al 2019, Dorofeeva 2016, Dubinski 2019, Mitri et al. 2021, Hand et.al 2020, Neveu & Rhoden 2019). The research results will be helpful to predict the unknown nature of natural satellites, understand the relationship between satellites and reveal their origins. In the future, when natural resources are increasingly depleted, Saturn and its satellites may even become an important place to obtain resources.

With the help of space telescopes and space probes, such as the Cassini spacecraft, scientists can obtain relevant data on most planets. However, in the case of satellites, data is sometimes difficult to obtain or inaccurate. It is a significant effort to apply statistical knowledge to the study of planets and establish mathematical models based on statistical results to help us better understand some of the laws behind the data (Gao et al. 2018). Gao et al. (2018 & 2021) determined the best-fitting distribution of each physical feature based on the p-value of the Kolmogorov-Smirnov (K-S) test. When they studied the physical distribution of the irregular moons of Jupiter by fitting, they found that in most cases, the Loglogistic distribution is the best. According to the K-S test, the best-fitting distribution of physical properties may be affected by the number of satellites and the fitted distribution function. Theoretically, more satellites and fitting distribution functions will make the best-fitting distribution result more accurate. With the discovery of more Saturn satellites, the question whether there are specific laws in the physical properties of Saturn satellites has attracted many researchers. For example, Tattersa (2013) proposed that the physical properties of planets exhibit Lognormal distribution, and Müller (2010 & 2015) proposed that the distribution of mass and orbital period in the Saturn's satellite system can be described according to the scaling law: $M = \mu T^D$, where M is the mass of the satellite, T is the orbital period, D and μ are constants. When studying the particles in the inner magnetosphere of Saturn, Martínez-Gómez et al. (2017) selected six probability distribution functions: Normal, Exponential, Logistic, Lognormal, Weibull, and Extreme values to fit the physical system. The logistic distribution is regarded as the best fitting distribution of the system according to the Anderson-Darling statistics. In addition, Demer's law and Titius-Bode's law can also approximate the distribution of planets in the solar system to a certain extent (Ballesteros et al. 2019, Huang 2014, Bovaird & Lineweaver 2013), although there is no strict mathematical explanation yet. On this basis, Bovaird & Lineweaver (2013) analyzed the planet's orbital period and successfully predicted the existence of five exoplanets; by determining the diameters of Himalia and Phoebe, Grav et al. (2015) predicted the diameters and albedo of 12 satellites that were not accurately measured. It is speculated from similar albedo that they may have a common origin.

This paper aims to use the K-S non-parametric test method to study the physical characteristics of Saturn's regular satellites and reveal the possible distribution laws hidden behind the data. See Appendix A for the relevant data on the physical characteristics (equatorial radius, equatorial circumference, volume, density, mass, surface area, surface gravity, and escape velocity). The structure of this paper is as follows: Section 2 introduces the theoretical method (K-S test) used in this paper. In Section 3, the MATLAB mathematical software is used to fit the distribution of each physical characteristic based on the K-S test. Among 22 common distribution functions, it is found that the main best-fitting distribution of the physical characteristics of regular satellites is the Lognormal distribution. Section 4 verifies the feasibility of the results of statistical inference based on the relationship between the physical characteristics of the satellites. In Section 5, a specific example will be used to demonstrate the effectiveness of the K-S test in predicting the physical characteristics of the satellite Pandora. This paper ends with Section 6, which summarizes the results of applying statistical inference to the physical characteristics of Saturn's satellites.

2 Theoretical method

Both natural sciences and social sciences are faced with the problem of dealing with data obtained through various observations or experiments. Regardless whether it is based on actual needs or curiosity, we all hope to reveal the statistical laws hidden behind these data, and infer which types of regular distributions fit a bunch of seemingly disorderly data. Of course, they may indeed belong to chaotic distribution in nature. The K-S test is a commonly used statistical test method to solve such problems.

The K-S test is a non-parametric test method of goodness-of-fit. It judges whether the empirical distribution of sample observations is consistent with an existing theoretical distribution based on the cumulative distribution function. By analyzing the difference between the two distributions, we can determine whether there are sufficient reasons to believe that the observations of the sample come from the population of the given theoretical distribution. Let $S_n(x)$ be the empirical distribution function of sample observations, and $S(x)$ be a specific theoretical distribution function, then we define the difference between the two distributions as $D = |S_n(x) - S(x)|$. When x is fixed, if the difference between $S_n(x)$ and $S(x)$ is very small, that is, the value of D is very small, it indicates that the fitting degree between the empirical distribution function and this specific theoretical distribution function is very high, so we have sufficient reason to believe that the sample data comes from the given theoretical distribution function.

The K-S test mainly examines the largest deviation in the value of D, which is represented by $D_{max} = \max |S_n(x) - S(x)|$. The steps of K-S test are as follows:

(1). Propose the null hypothesis $\mathrm{H}_0 : S_n(x) = S(x)$ and the alternative hypothesis $\mathrm{H}_1 : S_n(x) \neq S(x)$, which is to assume whether this set of data obeys a specific theoretical distribution function;

(2). Calculate the D value corresponding to each x value and find the largest statistic D_{max};

(3). According to the number of sample data n and the significance level α, compare D_{max} with the critical value $D(n, \alpha)$, then one can determine whether to reject the null hypothesis. If the $D_{max} \geq D(n, \alpha)$, then the null hypothesis H_0 is rejected; if the $D_{max} < D(n, \alpha)$, one cannot reject the null hypothesis H_0, but it does not mean that the null hypothesis is acceptable.

In addition to the D value, the p-value can better reflect the evidence that the sample data support the null hypothesis. In general, the minimum significance level of rejecting the null hypothesis H_0 is called p-value in a certain hypothesis testing problem. It is the probability of sample observation or more extreme results occurring when the null hypothesis is true. The larger the p-value, the more likely that the null hypothesis will be satisfied. Therefore, we do not have sufficient evidence to reject the null hypothesis; the smaller the p-value, the less likely the null hypothesis will be. According to a given significance level α (for example, we take $\alpha = 0.05$ in this paper), when $p \leq 0.05$, the null hypothesis is considered invalid, that is, the sample data does not obey the given theoretical distribution; the null hypothesis H_0 is retained under the significance level α when $p > 0.05$.

In the actual hypothesis testing process, we often encounter the same sample data that may not reject multiple theoretical distribution functions. How should we choose the best theoretical distribution function? We could choose the distribution with the largest p-value as the best-fitting distribution and consider the confidence interval of the parameter if the p-value is similar.

3 Inference Results of Physical Characteristic Distribution

This section will explore the possible distribution laws of the physical characteristics of Saturn's regular satellites (see Appendix A), so that researchers can better understand the relevant characteristics of these satellites and can use these potential laws to study further the satellites of Saturn for which there is no sufficient data available yet and even help discover new ones.

According to the method introduced in the previous section, we performed one-sample K-S test on the eight physical characteristics (i.e. equatorial radius, equatorial circumference, volume, density, mass, surface area, surface gravity, and escape velocity) of Saturn's regular satellites one by one (see Appendix B), and obtained the corresponding best-fitting distribution. The specific types of best-fitting distributions are listed in Table 1. Except for the density, which obeys the Stable distribution, and the surface gravity, which obeys the Generalized Pareto distribution, the best-fitting distributions of the other six physical characteristics are the Lognormal distribution. The p-values in Table 1 indicate that the p-values of these six physical characteristics are all greater than 0.988, which gives us reason to believe that the Lognormal distribution has an excellent performance in fitting the physical properties of these satellites. However, although the best-fitting of the surface gravity is the Generalized Pareto distribution, it is noticed that the p-values corresponding to this distribution and the Lognormal distribution differ by only about 0.07, which is almost negligible. If the range of the confidence interval is also considered (See Table 2), it seems that the Lognormal distribution can be used as a best-fitting alternative for the surface gravity. In short, the results of the inference of the distribution of Saturn's regular satellites support the Lognormal distribution. This distribution can better describe the physical characteristics of Saturn's regular satellites compared with other known distributions.

The Lognormal distribution sometimes referred to as the Galton distribution, is obtained after logarithmic transformation of the Gaussian distribution (also known as the Normal distribution), just like the Logistic distribution after logarithmic transformation is the Loglogistic distribution. The Lognormal distribution applies when the variable must be positive, since $\ln(x)$ is only available when x is positive. If x follows the Lognormal distribution with location parameter μ and scale parameter σ, then $\ln(x)$ obeys the Normal distribution with the mean value of μ and the standard deviation of σ. Similarly, if x follows the Normal distribution with the parameters μ and σ, then $\exp(x)$ obeys the Lognormal distribution with two parameters μ and σ.

The probability density function of the Lognormal distribution is as follows:

$$f(x|\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp \frac{-(\ln x - \mu)^2}{2\sigma^2}, \tag{1}$$

and the cumulative distribution function of Lognormal distribution can be expressed as:

$$F(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^x \frac{1}{t} \exp \frac{-(\ln t - \mu)^2}{2\sigma^2} dt, \tag{2}$$

where $-\infty < \mu < +\infty$ is the mean of the logarithmic values and non-negative σ is the standard deviation of the logarithmic values.

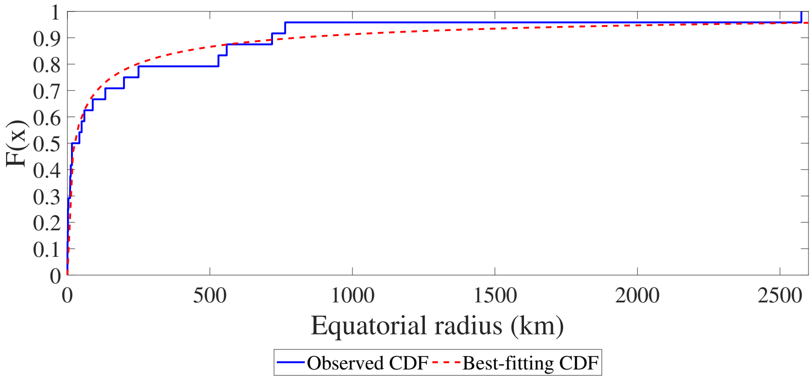
Table 1: The best-fitting distribution of the physical characteristics of Saturn's regular satellites.

Characteristic	**Best-fitting Distribution**	**Parameters**	**Confidence Intervals**	**p-value**
Equatorial radius (km)	Lognormal	$\mu = 3.21852$ $\sigma = 2.70957$	$\mu \in [2.07437, 4.36267]$ $\sigma \in [2.10592, 3.80088]$	0.9957
Equatorial circumference (km)	Lognormal	$\mu = 5.32783$ $\sigma = 2.47549$	$\mu \in [4.25735, 6.39831]$ $\sigma \in [1.91453, 3.50369]$	0.9880
Volume (km^3)	Lognormal	$\mu = 11.8913$ $\sigma = 7.44553$	$\mu \in [8.67159, 15.111]$ $\sigma \in [5.75834, 10.538]$	0.9885
Density (kg/m^3)	Stable	$\alpha = 0.4$ $\beta = 0.244005$ $c = 27.0693$ $\mu = 504.349$	$\alpha \in [0, 2]$ $\beta \in [-1, 1]$ $c \in [0, Inf]$ $\mu \in [-Inf, Inf]$	0.3440
Mass (kg)	Lognormal	$\mu = 39.1272$ $\sigma = 7.82406$	$\mu \in [35.7438, 42.5106]$ $\sigma \in [6.05109, 11.0738]$	0.9889
Surface area (km^2)	Lognormal	$\mu = 9.50876$ $\sigma = 4.95446$	$\mu \in [7.36629, 11.6512]$ $\sigma \in [3.83175, 7.0123]$	0.9881
Surface gravity (km/h^2)	Generalized Pareto	$k = 2.84814$ $\sigma = 20.5499$ $\theta = 0$	$k \in [1.19334, 4.50295]$ $\sigma \in [5.91591, 71.3836]$ $\theta = 0$	0.8822
Escape velocity (km/h)	Lognormal	$\mu = 3.23069$ $\sigma = 2.96181$	$\mu \in [1.9499, 4.51147]$ $\sigma \in [2.29065, 4.19201]$	0.9971

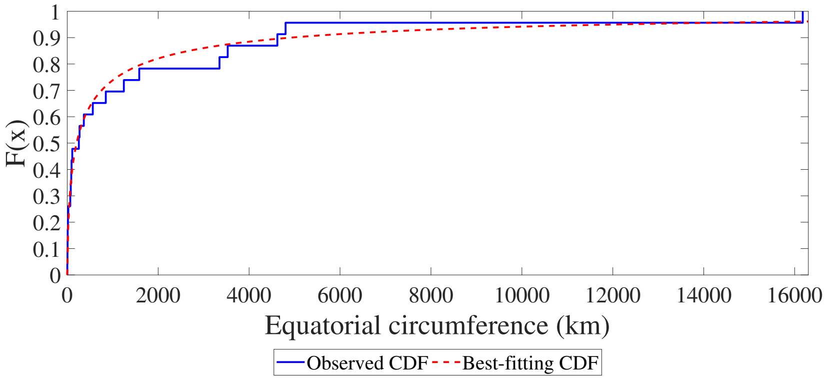
Table 2: Comparison of two distributions with close p-values when fitting surface gravity.

Characteristic	**Parameters**	**Confidence Intervals**	**p-value**
Generalized Pareto	$k = 2.84814$ $\sigma = 20.5499$ $\theta = 0$	$k \in [1.19334, 4.50295]$ $\sigma \in [5.91591, 71.3836]$ $\theta = 0$	0.8822
Lognormal	$\mu = 4.35907$ $\sigma = 2.89612$	$\mu \in [3.10669, 5.61144]$ $\sigma \in [2.23984, 4.09902]$	0.8095

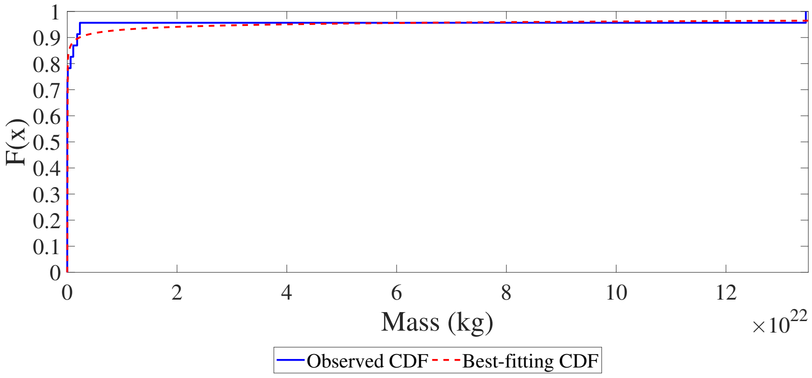
The p-value may be a good reflection of the adjustment effect between the sample observation data and the distribution function, and we draw the real observed CDF and the best-fitting CDF of physical characteristics to understand the relationship between the size of the p-value and the fitting effect more intuitively. It can be seen from Figures 1 and 2 that there is a close relationship between the size of the p-value and the fitting effect of the function. For example, the best-fitting distribution of density is the Stable distribution, and its p-value is only 0.3440, while the best-fitting distribution of the equatorial radius is the Lognormal distribution, and its p-value reaches 0.9957. The fitting effect of the Stable distribution of density is not as good as the fitting effect of the equatorial radius.



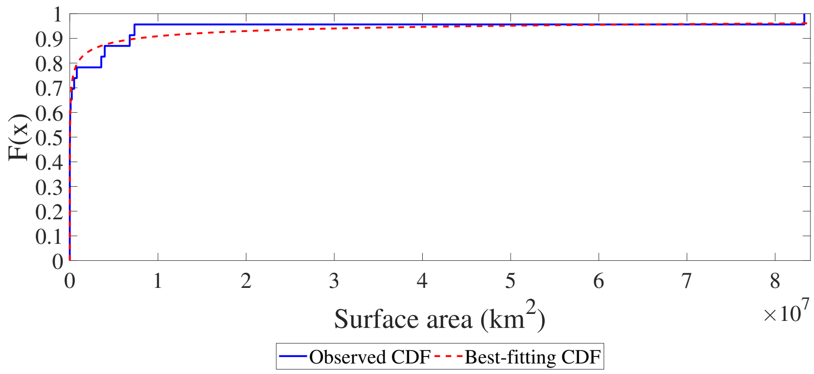
(a) Comparison of the real Observed CDF and the Best-fitting CDF of Equatorial radius.



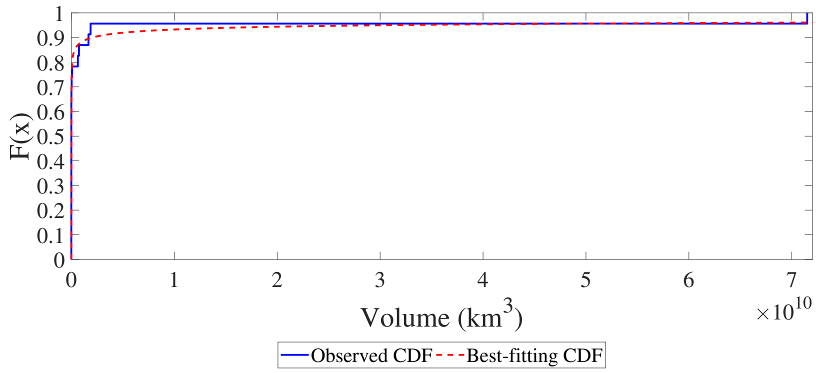
(b) Comparison of the real Observed CDF and the Best-fitting CDF of Equatorial circumference.



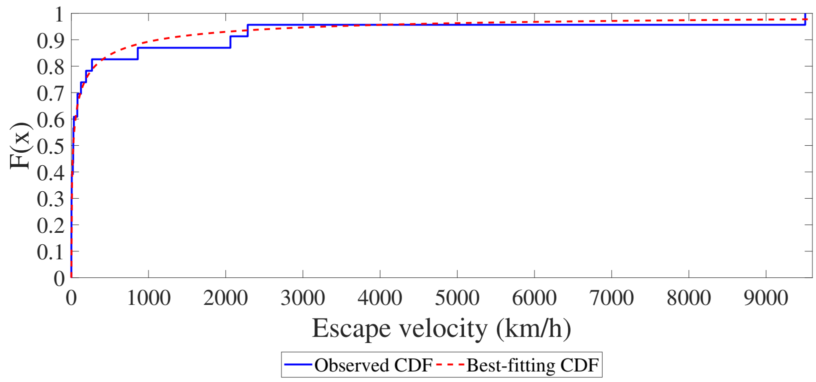
(c) Comparison of the real Observed CDF and the Best-fitting CDF of Mass.



(d) Comparison of the real Observed CDF and the Best-fitting CDF of Surface area.

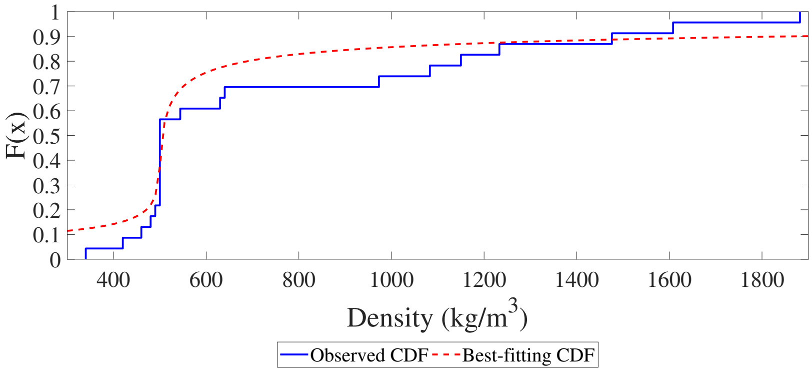


(e) Comparison of the real Observed CDF and the Best-fitting CDF of Volume.

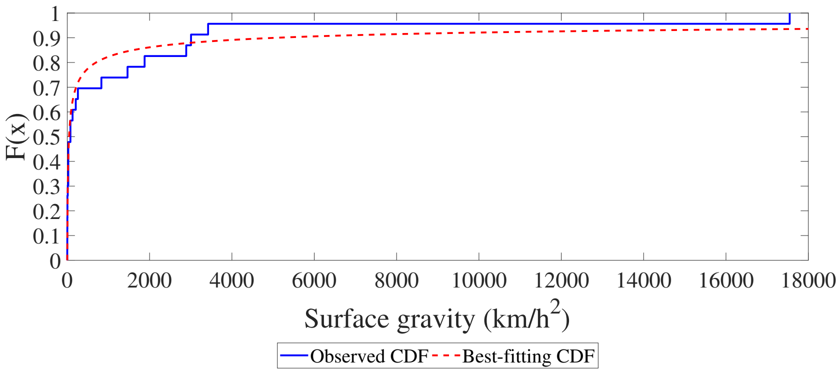


(f) Comparison of the real Observed CDF and the Best-fitting CDF of Escape velocity.

Fig. 1: The real Observed CDF and the Best-fitting CDF of six physical characteristics with Lognormal distribution.



(a) Comparison of the real Observed CDF and the Best-fitting CDF of Density.



(b) Comparison of the real Observed CDF and the Best-fitting CDF of Surface gravity.

Fig. 2: The real Observed CDF and the Best-fitting CDF of two physical characteristics without Lognormal distribution.

4 Validation of Statistical Inference

In the previous section, we obtained that the Lognormal distribution is the best-fitting distribution of the physical characteristics of Saturn's regular satellites. At the same time, it should be noted that there may be a certain relationship between different physical characteristics, such as the nonlinear relationship between equatorial radius (R) and surface area (S), $S = g(R) = 4\pi R^2$, $R(S) = \sqrt{S/4\pi}$. It is easy to find that both $g(R)$ and $R(S)$ are strictly monotonically increasing with respect to R and S, respectively. Denote the probability density function obtained by statistical prediction of R as $f_{pre,R}(R|\mu,\sigma)$, note that the best distributions of R and S are Lognormal distributions, and the corresponding distribution parameters are ($\mu = 3.21852$, $\sigma = 2.70957$) and ($\mu = 9.50876$, $\sigma = 4.95446$), so the probability density function $f_{ana,S}(S|\mu,\sigma)$ of S which is analytically derived from R can be written as

$$\begin{aligned} f_{ana,S}(S|\mu,\sigma) &= f_{pre,R}(R(S)|\mu,\sigma)|R'(S)| \\ &= \frac{1}{4\sqrt{\pi S}} f_{pre,R}(\sqrt{S/4\pi}|\mu,\sigma) \\ &= \frac{1}{2\sigma\sqrt{2\pi}S} \exp\left[\frac{-(\ln\sqrt{S/4\pi}-\mu)^2}{2\sigma^2}\right] \\ &= \frac{0.0736}{S} \exp\left[-0.0681\left(\ln(0.2821\sqrt{S}) - 3.21852\right)^2\right] \end{aligned} \tag{3}$$

and the probability density function $f_{pre,S}(S|\mu,\sigma)$ of S obtained through statistical prediction can be expressed as

$$\begin{aligned} f_{pre,S}(S|\mu,\sigma) &= \frac{1}{S\sigma\sqrt{2\pi}} \exp\frac{-(\ln S-\mu)^2}{2\sigma^2} \\ &= \frac{0.0805}{S} \exp\left[-0.0204\,(\ln S - 9.50876)^2\right] \end{aligned} \tag{4}$$

The graphs of $f_{pre,S}$ and $f_{ana,S}$ are drawn with MATLAB, these two graphs agree very well obviously (see Figure 3).

Similarly, the volume V obeys the Lognormal distribution with parameters $\mu = 11.8913$ and $\sigma = 7.44553$, it is easy to get that $V = 4\pi R^3/3$ is strictly monotonically increasing with respect to R, also the derivative are $V'(R) = 4\pi R^2$ and $R'(V) = \frac{(3/4\pi)^{\frac{1}{3}}}{3} V^{-\frac{2}{3}}$. So there are

$$\begin{aligned} f_{ana,V}(V|\mu,\sigma) &= f_{pre,V}(R(V)|\mu,\sigma)|R'(V)| \\ &= \frac{(3/4\pi)^{\frac{1}{3}}}{3} V^{-\frac{2}{3}} f_{pre,R}\left((\frac{3V}{4\pi})^{\frac{1}{3}}|\mu,\sigma\right) \\ &= \frac{0.0491}{V} \exp\left[-0.0681\left(\ln(0.6204V^{\frac{1}{3}}) - 3.21852\right)^2\right]. \end{aligned} \tag{5}$$

and

$$\begin{aligned} f_{pre,V}(V|\mu,\sigma) &= \frac{1}{V\sigma\sqrt{2\pi}} \exp\frac{-(\ln(V)-\mu)^2}{2\sigma^2} \\ &= \frac{0.0536}{V} \exp\left[-0.0090\,(\ln V - 11.8913)^2\right]. \end{aligned} \tag{6}$$

We use the MATLAB again to draw the graphs of $f_{pre,V}$ and $f_{ana,V}$ as shown in the Figure (4), and the two graphs are in good agreement.

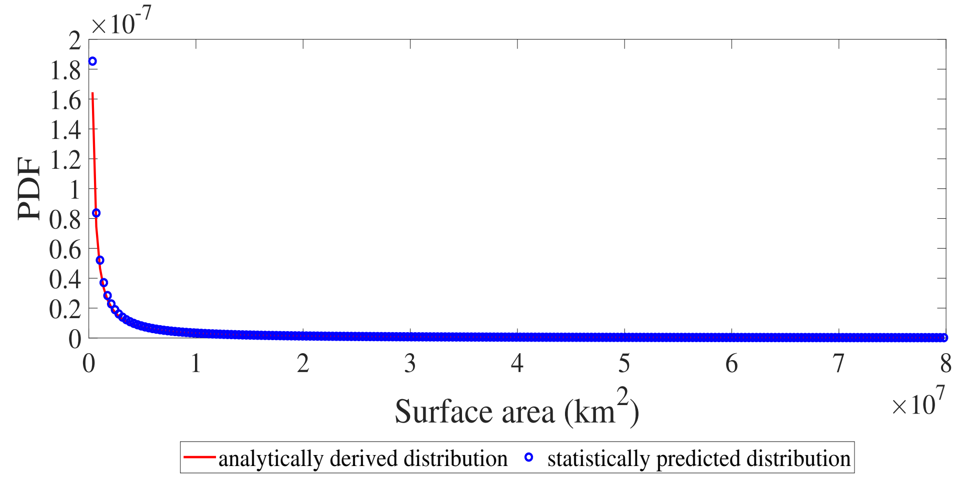


Fig. 3: Comparison of analytical derivation distribution and statistical prediction distribution of Surface area.

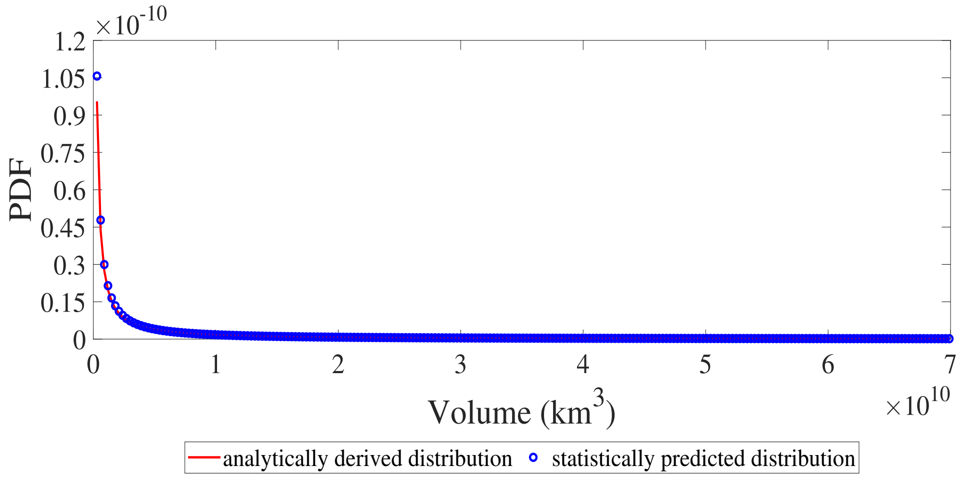


Fig. 4: Comparison of analytically derived distribution and statistically predicted distribution of Volume.

5 Application of Statistical Inference Results

We inferred that the Lognormal distribution is the best-fitting distribution of regular satellites through the K-S test in Section 3, and verified the rationality of this statistical inference in Section 4. In this section, we will show the application of this inference method. Without loss of generality, now we assume that some data of the physical characteristics such as equatorial radius, equatorial circumference, volume, mass, surface area, and escape velocity of Saturn's satellite Pandora are missing or at least not well studied, and then based on the data of the rest of Saturn's regular satellites, use the K-S test to determine the best-fitting distribution of each physical feature. The specific results are listed in Table 3. The results show that the Lognormal distribution is still the best-fitting distribution for other regular satellites.

Note that $\ln(x)$ will follow Normal distribution with mean μ and standard deviation σ when x follows Lognormal distribution with parameters μ and σ. Therefore, according to Table 3, we list the predicted data of the six physical characteristics of the Pandora satellite mentioned above and the actual data (see Appendix A) after taking the logarithm in Table 4, where R is the equatorial radius, C is the equatorial circumference, V is the volume, D is the density, M is the mass, A is the surface area, G is the surface gravity and EV is the escape velocity. It is easy to find that the predicted values are very close to the observed values.

In addition, although we also know that the best-fitting distribution of Pandora's density follows a Stable distribution, the parameter α is less than 1, making the corresponding mean value tend to infinity, which has no actual physical meaning. For surface gravity, although the logarithmic mean value of the best-fitting Birnbaum Saunders distribution is 7.3393, the range of its confidence interval is too wide. If the width of the confidence interval and the log-normal distribution of the other six features are considered to predict the density and surface gravity of Pandora, their mean values are 6.5159 and 4.3593, and the actual density and surface gravity values are 6.1940 and 4.3536 respectively. The comparison between the predicted and actual values of Pandora's various physical properties is shown in Figure 5. The slight error between them illustrates the substantial effect of the Lognormal distribution in predicting the physical properties of satellites.

Table 3: Distribution inference results of physical characteristics after removing Pandora.

Characteristic	**Best-fitting Distribution**	**Parameters**	**Confidence Intervals**	**p-value**
Equatorial radius (km)	Lognormal	$\mu = 3.19595$ $\sigma = 2.76816$	$\mu \in [1.99891, 4.39299]$ $\sigma \in [2.14088, 3.91792]$	0.9921
Equatorial circumference (km)	Lognormal	$\mu = 5.318$ $\sigma = 2.53329$	$\mu \in [4.19481, 6.4412]$ $\sigma \in [1.94899, 3.62023]$	0.9766
Volume (km^3)	Lognormal	$\mu = 11.8613$ $\sigma = 7.61932$	$\mu \in [8.48307, 15.2395]$ $\sigma \in [5.86194, 10.8885]$	0.9784
Density (kg/m^3)	Stable	$\alpha = 0.960913$ $\beta = 1$ $c = 108.081$ $\mu = 516.6$	$\alpha \in [0, 2]$ $\beta \in [-1, 1]$ $c \in [0, Inf]$ $\mu \in [-Inf, Inf]$	0.1830
Mass (kg)	Lognormal	$\mu = 39.1116$ $\sigma = 8.00782$	$\mu \in [35.5612, 42.6621]$ $\sigma \in [6.16082, 11.4437]$	0.9463
Surface area (km^2)	Lognormal	$\mu = 9.489$ $\sigma = 5.07012$	$\mu \in [7.24103, 11.737]$ $\sigma \in [3.90071, 7.24553]$	0.9771
Surface gravity (km/h^2)	Birnbaum Saunders	$\beta = 94.3206$ $\gamma = 5.53598$	$\beta \in [34.4674, 154.174]$ $\gamma \in [3.89641, 7.17556]$	0.9427
Escape velocity (km/h)	Lognormal	$\mu = 3.18009$ $\sigma = 3.02132$	$\mu \in [1.84051, 4.51967]$ $\sigma \in [2.32446, 4.31766]$	0.9784

Table 4: Prediction of physical characteristics based on Lognormal distribution.

Characteristic	Predictive value	Actual value
ln(R)	3.19595	3.73767
ln(C)	5.318	5.544
ln(V)	11.8613	12.5511
ln(M)	39.1116	39.4695
ln(A)	9.489	9.943
ln(EV)	3.18009	4.34381

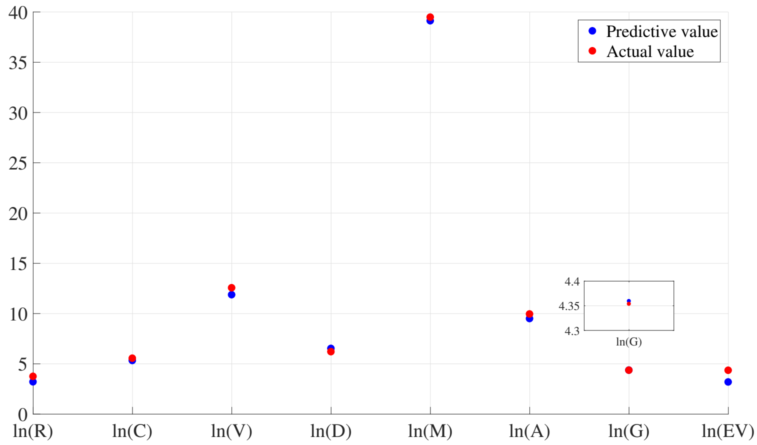


Figure 5: Comparison of Pandora's actual and predicted values.

6 Conclusions

This paper uses the K-S non-parametric test method to carry out statistical research on the equatorial radius, equatorial circumference, volume, density, mass, surface area, surface gravity, and escape velocity of Saturn's regular satellites. We use the MATLAB software to assist in finding that six of the eight physical features (except for density and surface gravity) can be characterized by the lognormal distribution as the best-fitting distribution function. It is noted that the particle size distribution produced by random impact crushing is one of the broad applications of the lognormal distribution. This fact inspires us to suspect that these satellites may admit a common origin. They are likely to be fragments of the parent body that are shattered by impact or torn apart by the gravity of giant planets and then form these natural satellites. Although there is a lack of rigorous evidence, there is certain rationality. We verified the rationality of the best-fitting distribution by comparing the distribution obtained through mathematical deduction with the distribution obtained based on statistical prediction.

In addition, we also demonstrate the application of distributed reasoning through a practical example. That is to verify that this method can predict the corresponding physical feature value when data on the Pandora satellite is missing. We found that the deviation between the predicted value of this method and the actual value is tiny, so we have reason to believe that the solution can even be used to help researchers discover new satellites of Saturn or study giant planetary systems with multiple satellites. Based on the statistical law obtained by the K-S scheme, we can also establish a corresponding dynamic model from a statistical perspective. For example, the dynamic behavior analysis of the Saturn satellite under the variable mass law will soon appear in our next manuscript.

Appendix

A. Physical Characteristics Data of Saturn's Regular Satellites

see Table A1

B. Inference Results of Physical Characteristics of Saturn's Regular Satellites

see Tables B1-B4

Table A1: Physical characteristics data of Saturn's regular satellites[1]

Name	Equatorial radius (km)	Equatorial circumference (km)	Volume (km^3)	Density (kg/m^3)	Mass (kg)	Surface area (km^2)	Surface gravity (km/h^2)	Escape velocity (km/h)
Aegaeon	0.25	1.9	0.1	500	59,946,737,324	1.13	0.5184	1
Anthe	0.5	5.7	3	500	1,498,668,433,097	10.18	1.5552	2
Atlas	16	94.9	14,422	460	6,594,141,105,627,650	2,865.26	25.92	27
Calypso	9.5	67.2	5,131	500	2,547,736,336,265,230	1,438.72	12.96	20
Daphnis	3.5	23.9	230	340	77,930,758,521,054	181.46	4.6656	6
Dione	559	3,529.3	742,338,322	1476	1,095,745,430,185,280,000,000	3,964,776.51	3006.72	0.510228649
Enceladus	249.5	1,584.0	67,113,076	1608	107,944,591,230,692,000,000	798,648.27	1464.48	861
Epimetheus	59.5	365.1	821,518	640	526,032,620,017,115,000	42,419.17	129.6	125
Helene	16	110.6	22,836	500	11,389,880,091,538,700	3,892.56	25.92	33
Hyperion	133	848.2	10,305,995	544	5,585,537,250,153,240,000	229,022.10	259.2	268
Iapetus	718	4,621.9	1,667,300,080	1083	1,805,952,411,282,580,000,000	6,799,755.63	2890.08	2061
Janus	89	562.3	3,003,016	630	1,892,818,231,001,750,000	100,659.77	207.36	191
Methone	1.5	10.1	17	500	8,992,010,598,583	32.17	2.9808	3
Mimas	198.5	1,245.3	32,613,662	1150	37,505,676,206,690,400,000	493,647.75	829.44	0.158912805
Pallene	2	15.7	65	500	32,970,705,528,138	78.54	4.536	5
Pan	10	88.6	11,742	420	4,945,605,829,220,740	2,498.32	25.92	25
Pandora	42	255.7	282,405	490	138,476,963,218,181,000	20,816.07	77.76	77
Polydeuces	2	8.2	9	500	4,496,005,299,292	21.24	2.3328	2
Prometheus	50	270.8	335,367	480	160,956,989,714,639,000	23,343.42	77.76	80
Rhea	764	4,802.2	1,870,166,133	1233	2,307,089,151,289,080,000,000	7,340,701.82	3421.44	2285
S/2009 S1	0.15	——	——	——	——	——	——	——
Telesto	12	77.9	7,986	500	4,046,404,769,362,420	1,932.21	25.92	24
Tethys	530	3,348.9	634,264,255	973	617,551,805,221,061,000,000	3,569,967.66	1879.2	0.39321974
Titan	2575	16,177.5	71,496,320,086	1882	134,552,523,083,241,000,000,000	83,305,418.53	17547.84	9507

[1] See https://sites.google.com/carnegiescience.edu/sheppard/moons/saturnmoons and https://solarsystem.nasa.gov/moons/saturn-moons/overview.

Table B1: Inference results of physical characteristics of Saturn's regular satellites

		Beta	**Birnbaum-Saunders**	**Burr**	**Exponential**	**Extreme Value**	**Gamma**	**Generalized Extreme Value**	**Generalized Pareto**	**Half Normal**	**Inverse Gaussian**	**Logistic**
Equatorial radius(km)	h	null	0	0	1	1	0	0	0	1	1	1
	p		0.5735	0.8360	0.0001	0.0005	0.5145	0.7112	0.7395	0.0000	0.0007	0.0080
	parameter		β=18.3165 γ=4.82569	α=89437.5 c=0.439212 k=21.352	μ=251.704	μ=587.697 σ=889.617	a=0.298751 b=842.522	k=2.66518 σ=16.2368 μ=6.13581	k=2.42117 σ=9.83729 θ=0	μ=0 σ=593.688	μ=251.704 λ=1.59654	μ=141.38 σ=199.011
	confidence interval		$\beta\in$[7.08163, 29.5513] $\gamma\in$[3.4534, 6.19798]	$\alpha\in$[9.25613e-21, 8.64191e+29] $c\in$[0.233204, 0.827205] $k\in$[2.78022e-09, 1.63982e+11]	$\mu\in$[175.041, 392.847]	$\mu\in$[207.257, 968.137] $\sigma\in$[697.884, 1134.03]	$a\in$[0.191147, 0.46693] $b\in$[357.44, 1985.91]	$k\in$[1.45147, 3.8789] $\sigma\in$[4.47083, 58.9674] $\mu\in$[-1.5294, 13.801]	$k\in$[0.851826, 3.99052] $\sigma\in$[2.68653, 36.0213] θ=0	μ=0 $\sigma\in$[463.568, 825.91]	$\mu\in$[-1012.69, 1516.1] $\lambda\in$[0.69323, 2.49986]	$\mu\in$[11.9494, 270.81] $\sigma\in$[138.749, 285.445]

		Loglogistic	**Lognormal**	**Nakagami**	**Negative Binomial**	**Normal**	**Poisson**	**Rayleigh**	**Rician**	**t Location-Scale**	**Weibull**	**Stable**
Equatorial radius(km)	h	0	0	0	null	1	1	1	1	1	0	0
	p	0.9842	0.9957	0.4651		0.0098	0.0000	0.0000	0.0000	0.0002	0.8229	0.6205
	parameter	μ=3.29035 σ=1.58039	μ=3.21852 σ=2.70957	μ=0.124538 ω=352465		μ=251.704 σ=549.254	λ=251.704	B=419.801	s=17.2122 σ=419.665	μ=2.00668 σ=3.81758 ν=0.309943	A=91.2806 B= 0.42845	α=0.400002 β= 0.999998 c=6.58557 μ=4.50262
	confidence interval	$\mu\in$[2.16896, 4.41174] $\sigma\in$[1.14048, 2.18998]	$\mu\in$[2.07437, 4.36267] $\sigma\in$[2.10592, 3.80088]	$\mu\in$[0.0816295, 0.190001] $\omega\in$ [113439, 1.09514e+06]		$\mu\in$[19.7745, 483.634] $\sigma\in$[426.888, 770.472]	$\lambda\in$[245.357, 258.051]	$B\in$[350.081, 524.456]	$s\in$[0,1335.52] $\sigma\in$[340.203, 517.688]	$\mu\in$[-0.842635, 4.856] $\sigma\in$[0.601205, 24.2412] $\nu\in$[0.160105, 0.60001]	$A\in$[33.9982, 245.076] $B\in$[0.31433, 0.584003]	$\alpha\in$[0,2] $\beta\in$[-1,1] c$\in$[0,Inf] $\mu\in$[-Inf,Inf]

		Beta	**Birnbaum-Saunders**	**Burr**	**Exponential**	**Extreme Value**	**Gamma**	**Generalized Extreme Value**	**Generalized Pareto**	**Half Normal**	**Inverse Gaussian**	**Logistic**
Equatorial circumference(km)	h	null	0	0	1	1	0	0	0	1	1	1
	p		0.7210	0.8611	0.0004	0.0008	0.5533	0.8368	0.9313	0.0000	0.0129	0.0118
	parameter		β=178.709 γ=3.98004	α=11955.2 c=0.50836 k=5.02731	μ=1657.21	μ=3796.36 σ=5620.55	a=0.326223 b=5080	k=2.29441 σ=139.609 μ=60.8598	k=2.079 σ=101.794 θ=0	μ=0 σ=3817.66	μ=1657.21 λ=21.125	μ=952.023 σ=1294.11
	confidence interval		$\beta\in$[68.5966, 288.821] $\gamma\in$[2.82859, 5.13149]	$\alpha\in$[0.000311388, 4.59001e+11] $c\in$[0.247136, 1.0457] $k\in$[0.00540183, 4678.76]	$\mu\in$[1144.34, 2614.25]	$\mu\in$[1341.04, 6251.68] $\sigma\in$[4381.04, 7210.76]	$a\in$[0.206054, 0.516475] $b\in$[2170.54, 11889.4]	$k\in$[1.3144, 3.27442] $\sigma\in$[44.2552, 440.418] $\mu\in$[-4.26974, 125.989]	$k\in$[0.661246, 3.49676] $\sigma\in$[30.4673, 340.103] θ=0	μ=0 $\sigma\in$[2967.14, 5355.26]	$\mu\in$[-4341.44, 7655.87] $\lambda\in$[8.91555, 33.3344]	$\mu\in$[90.0625, 1813.98] $\sigma\in$[896.549, 1867.96]

		Loglogistic	**Lognormal**	**Nakagami**	**Negative Binomial**	**Normal**	**Poisson**	**Rayleigh**	**Rician**	**t Location-Scale**	**Weibull**	**Stable**
Equatorial circumference(km)	h	0	0	0	null	1	1	1	1	1	0	0
	p	0.9852	0.9880	0.4370		0.0140	0.0000	0.0000	0.0000	0.0054	0.8173	0.7402
	parameter	μ=5.35923 σ=1.44764	μ=5.32783 σ=2.47549	μ=0.133793 ω=1.45745e+07		μ=1657.21 σ=3516.5	λ=1657.21	B=2699.49	s=112.426 σ=2698.59	μ=55.3638 σ=74.1115 ν=0.432087	A=680.14 B=0.458562	α=0.4 β=0.998418 c=57.4477 μ=40.2808
	confidence interval	$\mu\in$[4.30881, 6.40965] $\sigma\in$[1.03792, 2.01909]	$\mu\in$ [4.25735, 6.39831] $\sigma\in$[1.91453, 3.50369]	$\mu\in$[0.0867719, 0.206294] $\omega\in$[4.76826e+06, 4.4548e+07]		$\mu\in$[136.562, 3177.86] $\sigma\in$[2719.65, 4977.09]	$\lambda\in$[1640.58, 1673.85]	$B\in$[2243.21, 3390.52]	$s\in$[0,8929.8] $\sigma\in$[2176.27, 3346.29]	$\mu\in$[-2.28409, 113.012] $\sigma\in$[33.7878, 162.559] $\nu\in$[0.258181, 0.723136]	$A\in$[264.769, 1747.15] $B\in$[0.335045, 0.627614]	$\alpha\in$[0.399897, 0.400103] $\beta\in$[0.971185,1] c$\in$[57.0098, 57.8856] $\mu\in$[36.1953, 44.3662]

Table B2: Inference results of physical characteristics of Saturn's regular satellites (Cont'd).

		Beta	**Birnbaum-Saunders**	**Burr**	**Exponential**	**Extreme Value**	**Gamma**	**Generalized Extreme Value**	**Generalized Pareto**	**Half Normal**	**Inverse Gaussian**	**Logistic**
Volume(km^3)	h		1	0	1	1	0	1	0	1	1	1
	p		0.0432	0.8622	0.0000	0.0000	0.3846	0.0000	0.1449	0.0000	0.0000	0.0000
	parameter	null	β=80885.7 γ=279.537	α=3.99372e+10 c=0.16858 k=5.23994	μ=3.32717e+09	μ=1.2861e+10 σ=2.613e+10	a=0.082804 b=4.01813e+10	k=0.856997 σ=4857.74 μ=2724.27	k=11.0101 σ=32.4636 θ=0	μ=0 σ=1.49186e+10	μ=3.32717e+09 λ=2.18558	μ=3.32713e+09 σ=3.92127e+09
	confidence interval		$\beta\in$[31884.5, 129887] $\gamma\in$[198.633, 360.442]	$\alpha\in$[1.59737e-13, 9.98502e+33] $c\in$[0.0823605, 0.345058] $k\in$[0.00462084, 5941.97]	$\mu\in$[2.29748e+09, 5.24861e+09]	$\mu\in$[1.43859e+09, 2.42835e+10] $\sigma\in$[2.04671e+10, 3.33599e+10]	$a\in$[0.0541667, 0.126582] $b\in$[9.12587e+09, 1.76918e+11]	$k\in$[0.733295, 0.980699] $\sigma\in$[2725.08, 8659.44] $\mu\in$[610.076, 4838.47]	$k\in$[5.89825, 16.122] $\sigma\in$[2.77344, 379.993] θ=0	μ=0 $\sigma\in$[1.15949e+10, 2.09272e+10]	$\mu\in$[-1.92434e+12, 1.931e+12] $\lambda\in$[0.922398, 3.44876]	$\mu\in$[-1.48986e+09, 8.14412e+09] $\sigma\in$[1.99513e+09, 7.70697e+09]

		Loglogistic	**Lognormal**	**Nakagami**	**Negative Binomial**	**Normal**	**Poisson**	**Rayleigh**	**Rician**	**t Location-Scale**	**Weibull**	**Stable**
Volume(km^3)	h	0	0	0		1	1	1		1	0	1
	p	0.9859	0.9885	0.3326		0.0000	0.0000	0.0000		0.0007	0.8204	0.0002
	parameter	μ=11.9924 σ=4.35148	μ=11.8913 σ=7.44553	μ=0.0387529 ω=2.22563e+20	null	μ=3.32717e+09 σ=1.48697e+10	λ=3.32717e+09	B=1.0549e+10	null	μ=7073.84 σ=16188.1 ν=0.365182	A=5.28955e+06 B=0.152677	α=0.4 β=0.97589 c=2.93768e+06 μ=1.80558e+06
	confidence interval	$\mu\in$[8.83562, 15.1492] $\sigma\in$[3.11962, 6.06976]	$\mu\in$[8.67159, 15.111] $\sigma\in$[5.75834, 10.538]	$\mu\in$[0.025556, 0.0587644] $\omega\in$[2.79154e+19, 1.77445e+21]		$\mu\in$[-3.10295e+09, 9.75729e+09] $\sigma\in$[1.15001e+10, 2.10458e+10]	$\lambda\in$[3.32715e+09, 3.32719e+09]	$B\in$[8.76597e+09, 1.32494e+10]		$\mu\in$[-17804.3, 31952] $\sigma\in$[558.828, 468937] $\nu\in$[0.148708, 0.896776]	$A\in$[311077, 8.99436e+07] $B\in$[0.111541, 0.208986]	$\alpha\in$[0.399897, 0.400103] $\beta\in$[0.894072,1] $c\in$[0,2.49642e+09] $\mu\in$[-1.15075e+09, 1.15436e+09]

		Beta	**Birnbaum-Saunders**	**Burr**	**Exponential**	**Extreme Value**	**Gamma**	**Generalized Extreme Value**	**Generalized Pareto**	**Half Normal**	**Inverse Gaussian**	**Logistic**
Density(kg/m^3)	h		1	0	1	1	1	0	1	1	1	0
	p		0.0259	0.0502	0.0016	0.0226	0.0284	0.1030	0.0407	0.0105	0.0277	0.0941
	parameter	null	β=677.022 γ=0.48851	α=431.381 c=14.408 k=0.153217	μ=756.913	μ=992.544 σ=506.3	a=4.08607 b=185.242	k=0.534244 σ=168.23 μ=521.639	k=-0.599796 σ=1195.57 θ=0	μ=0 σ=868.41	μ=756.913 λ=2993.19	μ=677.508 σ=229.758
	confidence interval		$\beta\in$[545.912, 808.132] $\gamma\in$[0.347339, 0.629681]	$\alpha\in$[383.709, 484.976] $c\in$[7.08481, 29.301] $k\in$[0.065378, 0.359072]	$\mu\in$[522.663, 1194.03]	$\mu\in$[771.878, 1213.21] $\sigma\in$[381.396, 672.108]	$a\in$[2.3434, 7.12467] $b\in$[102.521, 334.709]	$k\in$[0.169496, 0.898993] $\sigma\in$[107.074, 264.317] $\mu\in$[445.403, 597.875]	$k\in$[-0.990706, -0.208886] $\sigma\in$[718.185, 1990.28] θ=0	μ=0 $\sigma\in$[674.94, 1218.17]	$\mu\in$[601.357, 912.469] $\lambda\in$[1263.24, 4723.15]	$\mu\in$[514.014, 841.002] $\sigma\in$[161.375, 327.116]

		Loglogistic	**Lognormal**	**Nakagami**	**Negative Binomial**	**Normal**	**Poisson**	**Rayleigh**	**Rician**	**t Location-Scale**	**Weibull**	**Stable**
Density(kg/m^3)	h	0	1	1	0	1	1	1	1	1	0	0
	p	0.0876	0.0360	0.0247	0.0290	0.0237	0.0000	0.0389	0.0388	0.0004	0.0542	0.3440
	parameter	μ=6.43181 σ=0.275089	μ=6.50192 σ=0.488656	μ=1.0805 ω=754135	R=4.10275 P=0.00539115	μ=756.913 σ=435.264	λ=756.913	B=614.058	s=55.6024 σ=612.972	μ=500 σ=1.00362e-06 ν=0.172015	A=861.139 B=1.9371	α=0.4 β=0.244005 c=27.0693 μ=504.349
	confidence interval	$\mu\in$[6.23299, 6.63062] $\sigma\in$[0.194866, 0.388339]	$\mu\in$[6.29061, 6.71323] $\sigma\in$[0.377924, 0.691621]	$\mu\in$[0.64731, 1.80359] $\omega\in$[508980, 1.11737e+06]	$R\in$[1.80955, 6.39595] $P\in$[0.00220376, 0.00857853]	$\mu\in$[568.691, 945.135] $\sigma\in$[336.631, 616.052]	$\lambda\in$[745.669, 768.157]	$B\in$[510.267, 771.248]	$s\in$[0,4626.29] $\sigma\in$[413.124, 909.495]	$\mu\in$[-Inf,Inf] $\sigma\in$[-Inf,Inf] $\nu\in$[-Inf,Inf]	$A\in$[687.877, 1078.04] $B\in$[1.43946, 2.60678]	$\alpha\in$[0,2] $\beta\in$[-1,1] $c\in$[0,Inf] $\mu\in$[-Inf,Inf]

Table B3: Inference results of physical characteristics of Saturn's regular satellites (Cont'd).

		Beta	Birnbaum-Saunders	Burr	Exponential	Extreme Value	Gamma	Generalized Extreme Value	Generalized Pareto	Half Normal	Inverse Gaussian	Logistic
Mass(kg)	h	null	1	0	1	1	0	1	0	1	1	1
	p		0.0345	0.8643	0.0000	0.0000	0.3145	0.0000	0.1723	0.0000	0.0000	0.0000
	parameter		β=8.9038e+16 γ=370.466	α=3.76388e+20 c=0.170255 k=2.83657	μ=6.11012e+21	μ=2.40706e+22 σ=4.92323e+22	a=0.0760157 b=8.03796e+22	k=0.569397 σ=1.32889e+16 μ=7.07519e+15	k=11.3686 σ=1.55618e+13 θ=0	μ=0 σ=2.8064e+22	μ=6.11012e+21 λ=1.29748e+12	μ=6.11012e+21 σ=1.54407e+22
	confidence interval		β∈[3.5716e+16, 1.4236e+17] γ∈[263.409, 477.524]	α∈[664.59, 2.13166e+38] c∈[0.0739575, 0.39194] k∈[0.0204874, 392.734]	μ∈[4.21915e+21, 9.63871e+21]	μ∈[2.54927e+21, 4.5592e+22] σ∈[3.85631e+22, 6.28534e+22]	a∈[0.049786, 0.116065] b∈[1.72057e+22, 3.75508e+23]	k∈[0.494556, 0.644238] σ∈[7.47124e+15, 2.36367e+16] μ∈[1.14168e+15, 1.30087e+16]	k∈[6.12161, 16.6156] σ∈[1.31349e+12, 1.84372e+14] θ=0	μ=0 σ∈[2.18117e+22, 3.93671e+22]	μ∈[-Inf,Inf] λ∈[-Inf,Inf]	μ∈[-Inf,Inf] σ∈[-Inf,Inf]

		Loglogistic	Lognormal	Nakagami	Negative Binomial	Normal	Poisson	Rayleigh	Rician	t Location-Scale	Weibull	Stable
Mass(kg)	h	0	0	0	null	1	1	1	1	1	0	1
	p	0.9759	0.9889	0.2574		0.0000	0.0000	0.0000	0.0000	0.0000	0.7818	0.0000
	parameter	μ=39.1312 σ=4.58867	μ=39.1272 σ=7.82406	μ=0.0357998 ω=7.8759e+44		μ=6.11012e+21 σ=2.80064e+22	λ=6.11012e+21	B=1.98443e+22	s=1 σ=2.71315e+08	μ=-1.54662e+15 σ=1.81001e+16 ν=0.177628	A=4.42076e+18 B=0.142598	α=0.4 β=0.987279 c=3.62044e+18 μ=2.24553e+18
	confidence interval	μ∈[35.7976, 42.4648] σ∈[3.29169, 6.39669	μ∈[35.7438, 42.5106] σ∈[6.05109, 11.0738]	μ∈[0.0236219, 0.0542557] ω∈[9.08296e+43, 6.82924e+45]		μ∈[-6.00076e+21, 1.8221e+22] σ∈[2.166e+22, 3.96389e+22]	λ∈[6.11012e+21, 6.11012e+21]	B∈[1.64901e+22, 2.49241e+22]	s∈[1,1] σ∈[2.71315e+08, 2.71315e+08]	μ∈[-Inf,Inf] σ∈[-Inf,Inf] ν∈[-Inf,Inf]	A∈[2.12356e+17, 9.20299e+19] B∈[0.104361, 0.194845]	α∈[0,2] β∈[-1,1] c∈[0,Inf] μ∈[-Inf,Inf]

		Beta	Birnbaum-Saunders	Burr	Exponential	Extreme Value	Gamma	Generalized Extreme Value	Generalized Pareto	Half Normal	Inverse Gaussian	Logistic
Surface area(km^3)	h	null	0	0	1	1	0	1	0	1	1	1
1	p		0.2240	0.8614	0.0000	0.0001	0.4377	0.0000	0.2640	0.0000	0.0000	0.0004
	parameter		β=9401.08 γ=30.5568	α=4.83652e+07 c=0.2538 k=5.0864	μ=4.63922e+06	μ=1.56767e+07 σ=3.01688e+07	a=0.133749 b=3.4686e+07	k=2.01614 σ=276.879 μ=127.213	k=6.58682 σ=159.537	μ=0 σ=1.75317e+07	μ=4.63922e+06 λ=21.258	μ=1.36406e+06 σ=4.0781e+06
	confidence interval		β∈[3552.88, 15249.3] γ∈[21.7111, 39.4025]	α∈[2.486e-08, 9.40946e+22] c∈[0.123558, 0.52133] k∈[0.00517215, 5002.06]	μ∈[3.20347e+06, 7.31838e+06]	μ∈[2.48947e+06, 2.8864e+07] σ∈[2.36237e+07, 3.85272e+07]	a∈[0.0867442, 0.206226] b∈[1.04636e+07, 1.14981e+08]	k∈[1.58, 2.45228] σ∈[123.868, 618.902] μ∈[24.308, 230.118]	k∈[3.32277, 9.85088] σ∈[24.0848, 1056.76]	μ=0 σ∈[1.36259e+07, 2.45928e+07]	μ∈[-9.04015e+08, 9.13293e+08] λ∈[8.9717, 33.5444]	μ∈[-1.13043e+06, 3.85856e+06] σ∈[2.76187e+06, 6.0216e+06]

		Loglogistic	Lognormal	Nakagami	Negative Binomial	Normal	Poisson	Rayleigh	Rician	t Location-Scale	Weibull	Stable
Surface area(km^3)	h	0	0	0	null	1	1	1	1	1	0	1
	p	0.9854	0.9881	0.3627		0.0010	0.0000	0.0000	0.0000	0.0000	0.8181	0.0024
	parameter	μ=9.57279 σ=2.89699	μ=9.50876 σ=4.95446	μ=0.0599785 ω=3.07362e+14		μ=4.63922e+06 σ=1.72868e+07	λ=4.63922e+06	B=1.23968e+07	s=2479.18 σ=25733.6	μ=19.8191 σ=39.5262 ν=0.132679	A=147113 B=0.229201	α=0.449075 β=0.243243 c=130818 μ=24232.8
	confidence interval	μ∈[7.4708, 11.6748] σ∈[2.07702, 4.04066]	μ∈[7.36629, 11.6512] σ∈[3.83175, 7.0123]	μ∈[0.0393967, 0.0913126] ω∈[5.79333e+13, 1.63069e+15]		μ∈[-2.83614e+06, 1.21146e+07] σ∈[1.33695e+07, 2.44669e+07]	λ∈[4.63834e+06, 4.6401e+06]	B∈[1.03014e+07, 1.55702e+07]	s∈[0,Inf] σ∈[2.22507e-308, Inf]	μ∈[-5.08037, 44.7185] σ∈[8.05647, 193.921] ν∈[0.08406, 0.20942]	A∈[22280, 971376] B∈[0.167459, 0.313708]	α∈[0,2] β∈[-1,1] c∈[0,Inf] μ∈[-Inf,Inf]

Table B4: Inference results of physical characteristics of Saturn's regular satellites (Cont'd).

		Beta	**Birnbaum-Saunders**	**Burr**	**Exponential**	**Extreme Value**	**Gamma**	**Generalized Extreme Value**	**Generalized Pareto**	**Half Normal**	**Inverse Gaussian**	**Logistic**
Surface gravity(km/h^2)	h	null	0	0	1	1	0	0	0	1	1	1
	p		0.8659	0.8113	0.0000	0.0003	0.3528	0.8809	0.8822	0.0000	0.0078	0.0041
	parameter		β=94.4132 γ=5.41462	α=70.6412 c=0.590039 k=0.984127	μ=1388	μ=3695.71 σ=6216.05	a=0.247349 b=5611.53	k=2.68888 σ=46.939 μ=17.7068	k=2.84814 σ=20.5499 θ=0	μ=0 σ=3864.73	μ=1388 λ=5.70516	μ=674.359 σ=1137.36
	confidence interval		$\beta\in$[35.4658, 153.361] $\gamma\in$[3.84601, 6.98324]	$\alpha\in$[0.00380188, 1.31256e+06] c$\in$[0.15481, 2.24886] $k\in$[0.0273833, 35.3685]	$\mu\in$[958.444, 2189.58]	$\mu\in$[979.239, 6412.18] $\sigma\in$[4859.63, 7951.07]	$a\in$[0.157774, 0.387779] $b\in$[2199.26, 14318.2]	$k\in$[1.59853, 3.77924] $\sigma\in$[12.6784, 173.781] $\mu\in$[-3.66088, 39.0745]	$k\in$[1.19334, 4.50295] $\sigma\in$[5.91591, 71.3836] θ=0	μ=0 $\sigma\in$[3003.72, 5421.3]	$\mu\in$[-7459.74, 10235.7] $\lambda\in$[2.4078, 9.00253]	$\mu\in$[-61.5403, 1410.26] $\sigma\in$[785.221, 1647.42]

		Loglogistic	**Lognormal**	**Nakagami**	**Negative Binomial**	**Normal**	**Poisson**	**Rayleigh**	**Rician**	**t Location-Scale**	**Weibull**	**Stable**
Surface gravity(km/h^2)	h	0	0	0	null	1	1	1	1	1	0	0
	p	0.8072	0.8095	0.1845		0.0045	0.0000	0.0000	0.0000	0.0001	0.5662	0.7438
	parameter	μ=4.30128 σ=1.70459	μ=4.35907 σ=2.89612	μ=0.103632 ω=1.49362e+07		μ=1388 σ=3687.95	λ=1388	B=2732.78	s=98.0776 σ=2732.11	μ=3.47145 σ=4.93339 ν=0.248852]	A=325.585 B=0.375115	α=0.400289 β=0.962349 c=28.4608 μ=19.0402
	confidence interval	$\mu\in$[3.06048, 5.54207] $\sigma\in$[1.22355, 2.37476]	$\mu\in$[3.10669, 5.61144] $\sigma\in$[2.23984, 4.09902]	$\mu\in$[0.0675475, 0.158994] $\omega\in$[4.19659e+06, 5.31596e+07]		$\mu\in$[-206.784, 2982.79] $\sigma\in$[2852.24, 5219.74]	$\lambda\in$[1372.78, 1403.23]	$B\in$[2270.87, 3432.33]	$s\in$[0,8912.76] $\sigma\in$[2209.73, 3377.97]	$\mu\in$[0.492099, 6.4508] $\sigma\in$[1.12045, 21.722] $\nu\in$[0.145826, 0.424666]	$A\in$[102.564, 1033.56] $B\in$[0.27519, 0.511322]	$\alpha\in$[0.375664, 0.424914] $\beta\in$[0.837029,1] $c\in$28.4608[0,57.8612] $\mu\in$[-0.266788, 38.3472]

		Beta	**Birnbaum-Saunders**	**Burr**	**Exponential**	**Extreme Value**	**Gamma**	**Generalized Extreme Value**	**Generalized Pareto**	**Half Normal**	**Inverse Gaussian**	**Logistic**
Escape velocity(km/h)	h	null	0	0	1	1	0	0	0	1	1	1
	p		0.3914	0.9761	0.0000	0.0002	0.2377	0.6709	0.5903	0.0000	0.0015	0.0010
	parameter		β=37.288 γ=6.20235	α=13.9436 c=0.633499 k=0.815327	μ=678.437	μ=1943.4 σ=3399.33	a=0.220358 b=3078.79	k=2.78144 σ=14.9256 μ=5.45411	k=2.85733 σ=6.70243 θ=0	μ=0 σ=2092.77	μ=678.437 λ=1.67816	μ=258.528 σ=601.271
	confidence interval		$\beta\in$[13.4422, 61.1338] $\gamma\in$[4.39769, 8.00701]	$\alpha\in$[0.0985432, 1972.98] c$\in$[0.274447, 1.46229] $k\in$[0.118939, 5.58905]	$\mu\in$[468.474, 1070.24]	$\mu\in$[457.869, 3428.93] $\sigma\in$[2656.66, 4349.6]	$a\in$[0.141074, 0.344201] $b\in$[1157.61, 8188.41]	$k\in$[1.63254, 3.93034] $\sigma\in$[3.86914, 57.5769] $\mu\in$[-1.39049, 12.2987]	$k\in$[1.16695, 4.54772] $\sigma\in$[1.84734, 24.3175] θ=0	μ=0 $\sigma\in$[1626.53, 2935.66]	$\mu\in$[-4896.43, 6253.3] $\lambda\in$[0.708246, 2.64807]	$\mu\in$[-123.416, 640.472] $\sigma\in$1[408.286, 885.476]

		Loglogistic	**Lognormal**	**Nakagami**	**Negative Binomial**	**Normal**	**Poisson**	**Rayleigh**	**Rician**	**t Location-Scale**	**Weibull**	**Stable**
Escape velocity(km/h)	h	0	0	0	null	1	1	1	1	1	0	0
	p	0.9920	0.9971	0.0890		0.0006	0.0000	0.0000	0.0000	0.0007	0.7954	0.5496
	parameter	μ=3.16235 σ=1.70579	μ=3.23069 σ=2.96181	μ=0.0928146 ω=4.3797e+06		μ=678.437 σ=2024.25	λ=678.437	B=1479.81	s=58.1696 σ=1479.37	μ=1.71088 σ=2.58995 ν=0.278257	A=110.299 B=0.35192	α=0.400005 β=0.994997 c=14.9179 μ=9.96514
	confidence interval	$\mu\in$[1.93303, 4.39168] $\sigma\in$[1.21936, 2.38626]	$\mu\in$[1.9499, 4.51147] $\sigma\in$[2.29065, 4.19201]	$\mu\in$[0.060609, 0.142133] $\omega\in$[1.14514e+06, 1.67506e+07]		$\mu\in$[-196.913, 1553.79] $\sigma\in$[1565.54, 2865.02]	$\lambda\in$[667.793, 689.082]	$B\in$[1229.69, 1858.63]	$s\in$[0,4427.27] $\sigma\in$[1196.52, 1829.1]	$\mu\in$[-0.0361466, 3.45791] $\sigma\in$[0.81286, 8.25215] $\nu\in$[0.167458, 0.462368]	$A\in$[32.155, 378.352] $B\in$[0.260225, 0.475925]	$\alpha\in$[0.399902, 0.400108] $\beta\in$[0.947878,1] $c\in$[0.329883, 29.506] $\mu\in$[-0.11517, 20.0455]

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