Anisotropic cosmological model with a massive scalar field in $f(R,T)$ gravity

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Abstract. The goal of this paper is to investigate the cosmic evolution of a cosmological model with a massive scalar field in $f(R,T)$ gravity (R and T denote the Ricci scalar and the energy-momentum tensor trace, respectively). We reconstruct the $f(R,T)$ model using a correspondence scheme that includes (i) a power-law relationship between the scalar field and the average scale factor, and (ii) the expansion scalar of the space-time is proportional to the shear scalar, resulting in a metric potential relationship. With the assistance of evolutionary trajectories of the equation of state and deceleration, as well as statefinder diagnostic parameters, the qualitative analysis of the obtained model is studied. The equation of state parameter is found to describe the Universe's phantom epoch, whereas the deceleration parameter depicts smooth transition from the early decelerated phase to the present accelerated phase, while the statefinder plane corresponds to the Chaplygin gas model and the model finally approaches ΛCDM model.
Key words: Bianchi type-I model; dark energy model; anisotropic model; massive scalar field; $f(R,T)$ gravity.

Introduction

Observational data has confirmed the recent concept of cosmic acceleration in our universe, and the cause for this is believed to be the presence of a mysterious force known as dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999; Peebles and Ratra 2003; Komatsu et al. 2009). Two concepts have been proposed to explain the universe's late-time acceleration. One of them is to investigate the dynamical DE models such as Chaplygin gas (Bento et al. 2002; Zhang et al. 2006), holographic models (Hsu 2004; Li 2004), etc. Because the simple DE model, specifically the cosmological constant, is plagued by coincidence and other major difficulties in general relativity, the above models have been investigated. Modifying Einstein's theory of gravity is another way to explain the cosmic acceleration. A nice review of dark energy and modified theories of gravitation are presented in the literature (Nojiri and Odintsov 2011; Harko and Lobo 2012; Nojiri and Odintsov 2007; Bamba et al. 2012). The $f(R)$, $f(R,T)$ (Capozziello et al. 2003; Nojiri and Odintsov 2003; Harko et al. 2011) theories of gravity are among the most important adaptations. We are mostly interested in $f(R,T)$ gravity here. The gravitational Lagrangian has been assumed to be an arbitrary function of the Ricci scalar R and the trace T of the matter-energy tensor in this theory.

The field equations of the $f(R,T)$ theory of gravity are produced by combining gravity, matter, and a scalar field in the following action (Harko et al. 2011; Sharif and Nawazish 2017):

$$S=\frac{1}{16\pi}\int f(R,T)\ \sqrt{-g}\ d^4x+\int (L_m+L_\varphi)\ \sqrt{-g}\ d^4x. \tag{1}$$

L_m and L_φ are the matter and scalar field Lagrangian densities, respectively. Only L_m and L_φ allow minimum coupling in the gravity Lagrangian $f(R,T)$

(Harko et al. 2011). R is the Ricci scalar, and T is the trace of the energy-momentum tensor of matter. The combined energy-momentum tensor of matter and scalar field is defined as

$$\Im_{ij}=-\frac{2\delta(\sqrt{-g}(L_m+L_\varphi))}{\sqrt{-g}\delta g^{ij}}, \qquad i,j=1,2,3,4. \tag{2}$$

We got the field equations of $f(R,T)$ gravity by assuming that the matter Lagrangian L_m and scalar field Lagrangian L_φ depend only on the metric tensor components g_{ij}

$$\begin{aligned} f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}- &\ (\nabla_i\nabla_j-g_{ij}\ \Box)f_R(R,T) \\ &=8\pi\Im_{ij}-f_T(R,T)(\Im_{ij}+\Theta_{ij}) \end{aligned} \tag{3}$$

and

$$\Theta_{ij}=-2\Im_{ij}+g_{ij}(L_m+L_\varphi)-2g^{lk}\frac{\partial^2(L_m+L_\varphi)}{\partial g^{ij}\partial g^{lm}}. \tag{4}$$

Here

$$\Box=\nabla^i\nabla_i,\ \ f_R(R,T)=\frac{\partial f(R,T)}{\partial R},\ \ f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$$

and ∇_i denotes the covariant derivative. For a perfect fluid, the energy- momentum tensor is

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij} \tag{5}$$

where ρ and p denote energy density and pressure, respectively, and u_i denotes the fluid's four-velocity vector. We also assume an attracting massive scalar field with a tensor of energy-momentum equal to

$$T^{\varphi}_{ij}=\varphi_{;i}\varphi_{;j}-\frac{1}{2}g_{ij}\left(\varphi_{;k}\varphi^{,k}-M^2\varphi^2\right). \tag{6}$$

where M is the mass of the scalar field φ which satisfies the Klein-Gordon equation

$$g^{ij}\varphi_{;ij}+M^2\varphi=0. \tag{7}$$

Ordinary and covariant differentiation is indicated by a comma and a semicolon, respectively, and φ is a function of time t. The Lagrangian density of perfect fluid and scalar fields for action (1) is defined as (Harko et al. 2011; Sharif and Nawazish 2017)

$$L_m=-p, \quad L_\varphi=\frac{1}{2}\left(M^2\varphi^2-\dot{\varphi}^2\right). \tag{8}$$

Now with the use of Eqs. (4) and (8) we obtain the tensor Θ_{ij} as

$$\Theta_{ij}=-2\Im_{ij}-\frac{g_{ij}}{2}\left(2p+\dot{\varphi}^2-M^2\varphi^2\right). \tag{9}$$

The tensor Θ_{ij}, which describes the physical constitution of the matter field, and affects the field equations in general. As a result, in the case of $f(R,T)$ gravity, we have many theoretical models corresponding to each choice of $f(R,T)$ provided by each choice of $f(R,T)$ (Harko et al. 2011)

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) . \end{cases} \tag{10}$$

Assuming the function $f(R,T)$ as

$$f(R,T) = R + 2f(T) \tag{11}$$

where $f(T)$ is an arbitrary function of the trace of stress-energy tensor of matter (perfect fluid and massive scalar field), we get the gravitational field equations of $f(R,T)$ gravity from Eq. (3) as

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi \mathfrak{T}_{ij} - 2\left(\mathfrak{T}_{ij} + \Theta_{ij}\right) f'(T) + f(T) g_{ij} \tag{12}$$

where the prime denotes differentiation with respect to the argument.

The study of cosmological models in the $f(R)$ and $f(R,T)$ theories of gravity in the presence of various source terms that represent stress-energy tensors has generated a lot of interest. This is because $f(R,T)$ gravity models represent the universe's early inflation and late-time acceleration. Capozziello and Laurentis (2011), Nojiri and Odintsov (2011), and Nojiri et al. (2017) provide a review of modified gravity models to explain dark energy. Sharif and Shamir (2009) and Shamir (2010) studied Bianchi type-I, III, V, and Kantowski-Sachs models in $f(R)$ theory. Several researchers have investigated numerous anisotropic Bianchi-type cosmological models in $f(R)$ theory under various physical conditions (Katore 2015; Santhi et al. 2018; Aditya and Reddy 2018c). In $f(R,T)$ gravity, Rao et al. (2016) studied Bianchi type-VI_h model, Mishra et al. (2015) examined non-static cosmological models, Sahoo et al. (2016) examined Bianchi-type string cosmological models and Aditya et al. (2016) investigated numerous Bianchi-type models. Sahoo et al. (2020) explored the bouncing scenario, whereas Maurya and Ortiz (2020) studied anisotropic fluid spheres in $f(R,T)$ theory of gravity. Singh and Beesham (2020) investigated LRS Bianchi type-I model with constant Hubble parameter in $f(R,T)$ gravity. Sharma (2021) constructed the Tsallis holographic dark energy model in $f(R,T)$ gravity. Nishant et al. (2020) investigated a flat accelerating universe of the model with a linearly varying deceleration parameter.

The study of scalar fields (SFs) in general relativity has stimulated the interest of numerous researchers, because of their physical significance in cosmology. Now particle physics theories confirm the presence of SFs. For example the Higgs mechanism explaining the mass of the particles is a massive SF. Also, the recent scenario of accelerated expansion of the universe is explained by quintessence SF. It, also, helps to produce inflation at early stages of evolution of the universe. Scalar fields are thought to accelerate the expansion of the universe and aid in the solution of the horizon problem. Scalar fields in cosmology are matter fields with spin-less quanta that characterize gravitational fields. Massive scalar fields and zero mass scalar fields are the two types of scalar fields. Long-range interactions are defined by zero-mass scalar fields, while short-range interactions are described by massive scalar fields. In general relativity, massless and massive scalar meson fields have been widely explored. Singh (2009) obtained Bianchi type-V model in Lyra's geometry in the presence of massive scalar field. Singh and Rani (2015) presented Bianchi type-III cosmological model in Lyra's geometry in the presence of a massive scalar field. The following studies are relevant to our research. Several researchers have investigated DE models with SFs in the context of the anisotropic background in the literature (Prasanthi and Aditya 2020, 2021; Aditya et al. 2019; Aditya and Reddy 2018a, 2018b, 2019; Santhi et al. 2016a, 2016b, 2017a, 2017b, 2017c). Naidu et al. (2015) investigated Bianchi type-V DE models in the presence of a scalar-meson field in general relativity. Reddy et al. (2019) investigated the Kantowski-Sachs DE model in the presence of the scalar-meson field. Aditya et al. (2019) studied the Kaluza-Klein DE model in the Lyra manifold in the presence of massive SF. Aditya and Reddy (2019) discussed the Bianchi type-III model in the presence of a massive SF in $f(R,T)$ gravity. Raju et al. (2020a, 2020b, 2020c) studied several aspects of anisotropic DE models with a massive SF. Aditya et al. (2021a) looked into the Bianchi type-VI_0 DE model with SF, whereas Naidu et al. (2020, 2021) explored Bianchi type-I and Kaluza-Klien cosmological models in the presence of a perfect fluid and an attracting massive SF. Observational constraints on dark energy and SF models were examined by Naidu et al. (2021), Aditya et al. (2021b), and Bhaskara Rao (2021, 2022). In the Lyra manifold, Aditya et al. (2022) investigated the Bianchi type-IX dark energy model in the presence of a massive scalar meson field.

The theoretical argument and recent experimental data support the existence of an anisotropic phase that approaches an isotropic phase (in the future). Hence, this motivates one to consider universe models with an anisotropic background. Bianchi type metrics are the best for Bianchi type universes which are the class of cosmological models that are homogeneous but not necessarily isotropic. The significance of Bianchi type cosmological models resides in their homogeneity and anisotropy, which allows for the study of the Universe's isotropization through time. Due to the simplicity of the field equations and relative ease of solution, Bianchi space-times proved useful in developing models of spatially homogeneous and anisotropic cosmologies. An increasing interest in anisotropic cosmological models of the Universe has been generated by anomalies observed in the cosmic microwave background (CMB) and research into big scale structure.

The primary objective of this study is to investigate the dynamics of the Locally-Rotationally-Symmetric (LRS) Bianchi type-I cosmological model in the presence of a perfect fluid and a massive scalar meson field. Because they are among the simplest models with an anisotropic backdrop, Bianchi type-I models are essential in cosmology. The following is how we organised our work in this paper: We derive the $f(R,T)$ gravity field equations using LRS Bianchi type-I space-time in the presence of perfect fluid and attracting massive scalar field in Sect. 2. Also, we solve the field equations and provide the corresponding cosmological model in Sect. 2. Sect. 3 contains the cosmic parameters that correspond to our model and discusses their physical importance in modern cosmology. A summary and conclusions are included in the final section.

1 Field equations and the model

We consider the LRS Bianchi type-I metric of the form, which is spatially homogeneous and anisotropic

$$ds^2=dt^2-A^2(t)dx^2-B^2(t)(dy^2+dz^2), \tag{13}$$

where $A(t)$ and $B(t)$ represent the scale factors of the universe. The volume (V) and average scale factor $(a(t))$ of LRS Bianchi type-I space-time are specified as

$$V=\sqrt{-g}=AB^2, \quad a(t)=(AB^2)^{\frac{1}{3}} \tag{14}$$

The anisotropic parameter A_h is given by

$$A_h=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{15}$$

where $H_1=\frac{\dot{A}}{A}, H_2=H_3=\frac{\dot{B}}{B}$ are the directional Hubble's parameters and $H=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)$ is the mean Hubble's parameter. The expansion scalar (θ) and shear scalar (σ^2) are defined as

$$\theta=\frac{\dot{A}}{A}+2\frac{\dot{B}}{B} \tag{16}$$

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2 \tag{17}$$

The deceleration parameter is given by

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1. \tag{18}$$

To derive the field equations of our model, we assume the particular choice of the function $f(R,T)$ as (Harko et al. 2011)

$$f(T)=\lambda T, \quad \lambda=\text{constant}. \tag{19}$$

Now, for the metric (13), using comoving coordinates and Eqs. (5)-(6), the explicit form of the $f(R,T)$ gravity field equations (12) and the Klein-Gordon equation, take the form:

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-p\left(8\pi+3\lambda\right)+\lambda\rho=4\pi\left(\dot{\varphi}^2-M^2\varphi^2\right) \tag{20}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-p\left(8\pi+3\lambda\right)+\lambda\rho=4\pi\left(\dot{\varphi}^2-M^2\varphi^2\right) \tag{21}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\rho\left(8\pi+3\lambda\right)-\lambda p=-(4\pi+2\lambda)\dot{\varphi}^2-4\pi M^2\varphi^2 \tag{22}$$

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)+M^2\varphi=0 \tag{23}$$

where an overhead dot denotes differentiation with respect to cosmic time t. The geometrical and physical parameters to utilize in solving the $f(R,T)$ field equations for the Bianchi type-I space-time provided by Eq. (13) are given in Eqs. (14)-(18).

Field Eqs. (20)–(23) are a set of four independent equations with five unknowns A, B, φ, p and ρ. Hence, we employ the following physically plausible conditions to obtain a determinate solution:

- We may use that the shear scalar σ^2 is proportional to the scalar expansion θ (Collins et al. 1980) which leads to

$$A=B^k \tag{24}$$

  where $k \neq 1$ is a constant that maintains the spatial anisotropic nature. Given that the Hubble expansion of the current universe is isotropic within 30% (Thorne 1967; Kantowski and Sachs 1966; Kristian and Sachs 1966), velocity-redshift measurements for extragalactic sources are possible. In addition, the red-shift study determined that the limit in our current Galaxy is $\frac{\sigma}{H} \leq 0.3$. Collins et al. (1980) have also shown that the normal congruence to the homogeneous expansion meets the requirement that $\frac{\sigma}{H}$ is constant.
- A power-law relation between the scalar field $\varphi(t)$ and the average scale factor $a(t)$ of the model is given by (Johri and Sudharsan 1989; Johri and Desikan 1994)

$$\varphi \propto [a(t)]^n \tag{25}$$

  where n is a power index. To reduce the mathematical complexity of the system, here, we consider the following relation between the scalar field and the metric potentials

$$\varphi=\varphi_0\ [a(t)]^n \tag{26}$$

  which is, clearly, a consequence of Eq. (25). Several authors, in the literature, have considered the above relation to study anisotropic cosmological models with massive scalar fields (Singh and Rani 2015; Raju et al. 2020a, 2020b; Aditya et al. 2021a; Naidu et al. 2020, 2021; Bhaskara Rao et al. 2021, 2022). Here the question of over-determinacy is settled by satisfying the field equations.

Now from Eqs. (23), (24) and (26), we get the metric potentials

$$A=(k_1 sinh\left(k_2 t+c_2\right)\ )^{\frac{3k}{(n+3)(k+2)}}$$

$$B = \left(k_1 sinh\left(k_2 t + c_2\right)\right)^{\frac{3}{(n+3)(k+2)}} \tag{27}$$

and scalar field $\varphi(t)$ as

$$\varphi\left(t\right) = \varphi_0 \left(k_1 sinh\left(k_2 t + c_2\right)\right)^{\frac{n}{n+3}} \tag{28}$$

where $k_1 = \sqrt{-\frac{c_1 n\left(\frac{n}{3}+1\right)(k+2)^2}{3M^2}}$, $k_2 = \sqrt{-\frac{3M^2\left(\frac{n}{3}+1\right)}{n}}$, c_1 and c_2 are integrating constants. Since Pradhan et al. (2012) and Mishra et al. (2013) presented an average scale factor $a(t) = [sinh(t)]^{\frac{1}{n}}$ in conjunction with the investigation of dark energy models in anisotropic backdrop, it is worth noting that the solution achieved here is quite interesting and physically acceptable. They discovered that it produces some feasible results that match modern cosmic data. Many researchers have used this type of average scale factor to study various elements of dark energy models in the literature (Amirhashchi et al. 2011; Mishra et al. 2016; Rao and Prasanthi 2017). Based on these results, studying the scalar field model with this hyperbolic solution for scale factors (Eq. (27)) is interesting.

Now the metric (13) with the help of Eq. (27) can be written as

$$ds^2 = dt^2 - \left(\left(k_1 sinh\left(k_2 t + c_2\right)\right)^{\frac{6k}{(n+3)(k+2)}}\right)\left(dx^2\right) - \left(\left(k_1 sinh\left(k_2 t + c_2\right)\right)^{\frac{3}{(n+3)(k+2)}}\right)\left(dy^2 + dz^2\right). \tag{29}$$

Eq. (29) along with Eq. (28) shows LRS Bianchi type-I universe with massive scalar fields in $f(R,T)$ theory of gravity along with the following physical and cosmological parameters which are very crucial in the discussion of cosmology.

2 Cosmological parameters and discussion

Spatial volume

$$V(t) = \left(k_1 sinh\left(k_2 t + c_2\right)\right)^{\frac{3}{n+3}}. \tag{30}$$

The average scale factor is given by

$$a\left(t\right) = \left(k_1 sinh\left(k_2 t + c_2\right)\right)^{\frac{1}{n+3}}. \tag{31}$$

The mean Hubble parameter is

$$H\left(t\right) = \frac{k_2 coth\left(k_2 t + c_2\right)}{n+3}. \tag{32}$$

The scalar expansion is

$$\theta = \frac{3k_2 coth\left(k_2 t + c_2\right)}{n+3}. \tag{33}$$

The shear scalar is
$$\sigma^2=\frac{(k-1)k_2coth\,(k_2t+c_2)}{(n+3)(k+2)}. \tag{34}$$
The average anisotropic parameter is
$$A_h=2\left(\frac{k-1}{k+2}\right)^2 \tag{35}$$
Now using Eqs. (27) and (28) in the field equations (20)-(22) we get the energy density ρ and the pressure p as
$$\begin{aligned}\rho=&\frac{1}{(8\pi+3\lambda)^2-\lambda^2}\left\{\frac{3\lambda}{2}\left\{\frac{\left(9\ k_2{}^2coth(k_2t+c_2)^2\right)}{(n+3)^2(k+2)^2}+\frac{3\ k_2{}^2}{(n+3)\,(k+2)}\right.\right.\\&\left.-\frac{3\ k_2{}^2cosh(k_2t+c_2)^2}{(n+3)\,(k+2)\,sinh(k_2t+c_2)^2}\right\}+\left\{\frac{\left(9k^2k_2{}^2cosh(k_2t+c_2)^2\right)}{(n+3)^2(k+2)^2sinh(k_2t+c_2)^2}\right.\\&+\frac{kk_2{}^2}{(n+3)\,(k+2)}-\frac{3kk_2{}^2cosh(k_2t+c_2)^2}{(n+3)\,(k+2)\,sinh(k_2t+c_2)^2}\\&\left.+\frac{9k_2{}^2cosh(k_2t+c_2)^2}{(n+3)^2(k+2)^2sinh(k_2t+c_2)^2}\right\}-\frac{(8\pi+3\lambda)}{(coth(k_2t+c_2))^{-2}}\left\{\frac{9k_2{}^2}{(n+3)^2(k+2)^2}\right.\\&\left.+\frac{18kk_2{}^2}{(n+3)^2(k+2)^2}\right\}-\frac{4\lambda\pi}{(k_1sinh\,(k_2t+c_2)\,)^{\frac{-2n}{(n+3)}}}\left\{\frac{\varphi_0{}^2n^2k_2{}^2}{(n+3)^2tanh(k_2t+c_2)^2}\right.\\&\left.\left.\left.-M^2\varphi_0{}^2\right\}-(8\pi+3\lambda)\left\{4\pi M^2\varphi_0{}^2+\frac{(4\pi+2\lambda)\,\varphi_0{}^2n^2k_2{}^2}{(n+3)^2tanh(k_2t+c_2)^2}\right\}\right\}\right\}\end{aligned} \tag{36}$$
$$\begin{aligned}p=&\frac{1}{(8\pi+3\lambda)^2-\lambda^2}\left\{\frac{3(8\pi+3\lambda)}{2}\left\{\frac{\left(9k_2{}^2coth(k_2t+c_2)^2\right)}{(n+3)^2(k+2)^2}+\frac{3k_2{}^2}{(n+3)\,(k+2)}\right.\right.\\&\left.-\frac{3k_2{}^2cosh(k_2t+c_2)^2}{(n+3)\,(k+2)\,sinh(k_2t+c_2)^2}\right\}+\left\{\frac{\left(9\ k^2k_2{}^2cosh(k_2t+c_2)^2\right)}{(n+3)^2(k+2)^2sinh(k_2t+c_2)^2}\right.\\&\left.+\frac{3kk_2{}^2}{(n+3)\,(k+2)}-\frac{3kk_2{}^2coth(k_2t+c_2)^2}{(n+3)\,(k+2)}+\frac{9k_2{}^2coth(k_2t+c_2)^2}{(n+3)^2(k+2)^2}\right\}\\&-\frac{\lambda}{coth(k_2t+c_2)^2}\left\{\frac{9k_2{}^2}{(n+3)^2(k+2)^2}+\frac{18kk_2{}^2}{(n+3)^2(k+2)^2}\right\}\\&-\frac{4(8\pi+3\lambda)\pi}{(k_1sinh\,(k_2t+c_2)\,)^{\frac{-2n}{(n+3)}}}\left\{-M^2\varphi_0{}^2+\frac{\varphi_0{}^2n^2k_2{}^2}{(n+3)^2tanh(k_2t+c_2)^2}\right\}\\&\left.-\frac{\lambda}{(k_1sinh\,(k_2t+c_2)\,)^{\frac{-2n}{(n+3)}}}\left\{4\pi M^2\varphi_0{}^2+\frac{(4\pi+2\lambda)\,\varphi_0{}^2n^2k_2{}^2}{(n+3)^2tanh(k_2t+c_2)^2}\right\}\right\}\end{aligned} \tag{37}$$

Scalar Field:

The behavior of a scalar field over cosmic time for various values of a parameter n that plays an essential role in its evolution is seen in Fig. 1. With cosmic time, we see that the scalar field diminishes. We can observe that the SF is decreasing and hence the kinetic energy is increasing. This result is quite similar to that of the scalar field models in literature (Jawad et al. 2015; Singh and Rani 2015; Naidu et al. 2015; Aditya et al. 2019). It can also be shown that when the parameter n is increased, the SF $\varphi(t)$ decreases initially whereas it increases at present epoch. Hence, we plotted the other cosmological parameters for various values of n to analyze the effects of the SF on the evolution of other dynamical parameters.

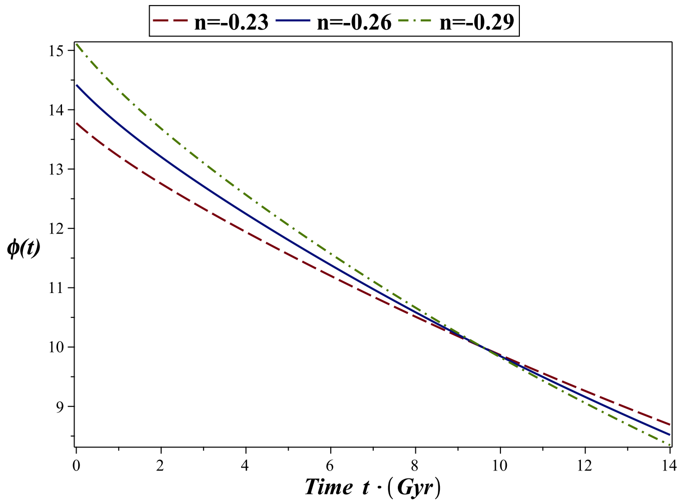

Fig. 1. Plot of scalar field versus cosmic time t for $c_2 = 0.84$, $M = 0.115$, $c_1 = 0.00001$, $\varphi_0 = 10$ and $k = 0.95$.

Energy conditions:

We investigate the well-known energy conditions for our model. The energy conditions are also used to demonstrate a number of general theorems about the behavior of large gravitational fields. The standard energy conditions are as follows:
– Null energy conditions (NEC): $\rho_{DE} + p_{DE} \geq 0$,
– Weak energy conditions (WEC): $\rho_{DE} \geq 0$, $\rho_{DE} + p_{DE} \geq 0$,
– Strong energy conditions (SEC): $\rho_{DE} + p_{DE} \geq 0$, $\rho_{DE} + 3p_{DE} \geq 0$,
– Dominant energy condition (DEC): $\rho_{DE} \geq 0$, $\rho_{DE} \pm p_{DE} \geq 0$.
Fig. 2 depicts the model's energy conditions. As cosmic time t increases, the energy density appears to decrease. Energy density increases when the scalar field increases, as illustrated in Fig. 2. Based on these results we can conclude that, when the scalar field is modified, the behavior of energy density changes significantly. The NEC is violated, causing the model to have a Big Rip. As predicted, our model fails the other energy conditions. This is due to the uni-

verse's late-time acceleration, which current observational evidence supports. Hence, the model suggests early inflation.

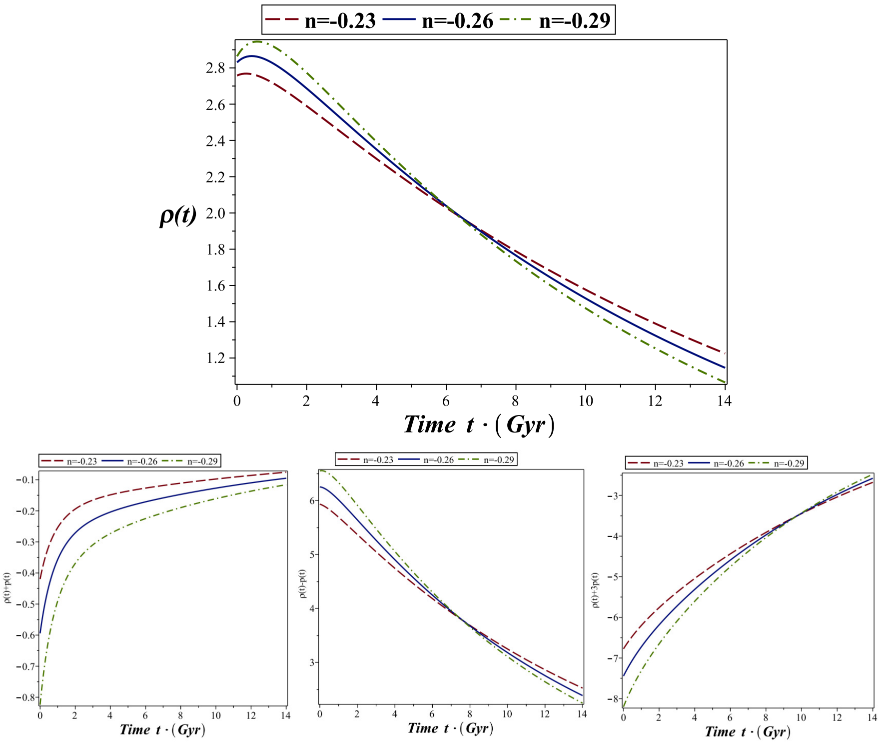

Fig. 2. Plot of energy conditions versus cosmic time t for $c_2 = 0.84$, $M = 0.115$, $c_1 = 0.00001$, $\varphi_0 = 10$, $k = 0.95$ and $\lambda = -8.65$.

EoS parameter:

The equation of state (EoS) parameter describes the behavior of our model. For standard cosmological theories, we have the $\omega = -1$ vacuum model, the $\omega = 0$ dust model, the $\omega = \frac{1}{3}$ radiation model, and the $\omega = +1$ stiff fluid model. It is defined as

$$\omega = \frac{p}{\rho} \tag{38}$$

The behavior of our model's EoS parameter in terms of cosmic time is seen in Fig. 3. It can be observed that the EoS value totally varies in the phantom region ($\omega_{de} < -1$) and it differs in the aggressive phantom region as the scalar field grows. The present values of the EoS parameter of our model are (n, ω)=

$(-0.23, -1.025)$, $(-0.26, -1.09)$, $(-0.29, -1.12)$. It is also worth mentioning that the EoS value in our model matches recent Planck observations very well (Aghanim et al. 2020).

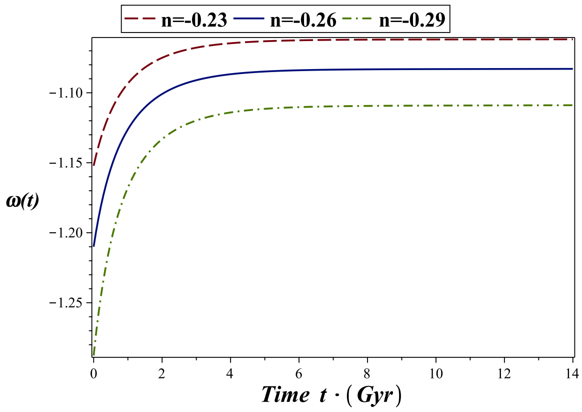

Fig. 3. Plot of equation of state parameter versus cosmic time t for $c_2 = 0.84$, $M = 0.115$, $c_1 = 0.00001$, $\varphi_0 = 10$, $k = 0.95$ and $\lambda = -8.65$.

Deceleration parameter:

The deceleration parameter ($q(t)$) is critical in describing the model's behavior. When $q > 0$, the cosmological model decelerates as expected, however when $q = 0$, the model expands at a constant rate. If $-1 \leq q < 0$, we have a cosmological model with accelerated expansion, and when $q < -1$, we have a model with exponential growth. For our model, it is obtained as

$$q(t) = -1 + (n+3)(sech(k_2 t + c_2))^2 \tag{39}$$

The deceleration parameter of our model varies in the area $q > 0$ and eventually reaches the value $q = -1$, as shown in Fig. 4. This proves that our model exhibits a transition from an early decelerated phase to the present accelerated epoch of the universe. This is compatible with the recent scenario of the Universe.

Statefinder parameters:

These parameters help us identify between the many DE models that have been developed over time. Because the parameters r and s are directly derived from the metric, they have a geometrical character, making them model dependent. Sahni et al. (2003) proposed two new dimensionless parameters known as statefinders, which have the following definitions:

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}.$$

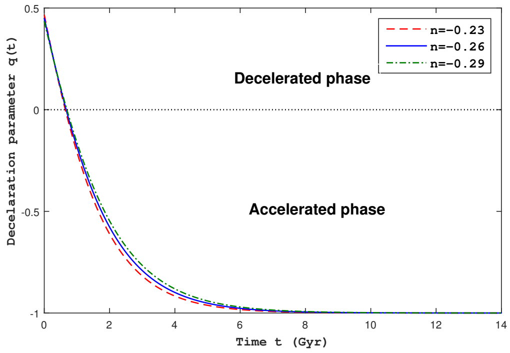

Fig. 4. Plot of deceleration parameter versus cosmic time t for $c_2 = 0.84$, $M = 0.115$, $c_1 = 0.00001$, $\varphi_0 = 10$ and $k = 0.95$.

For our model, the above parameters are obtained as

$$r = 1 + \frac{3(n+3)}{(tanh\,(k_2t+c_2))^2} - \frac{(n+3)(2n+3)}{(sinh\,(k_2t+c_2))^2} - \frac{2(n+3)^2}{(coth\,(k_2t+c_2))^2} \quad (40)$$

$$s = \left\{ \frac{3(n+3)}{(tanh\,(k_2t+c_2))^2} - \frac{(n+3)(2n+3)}{(sinh\,(k_2t+c_2))^2} - \frac{2(n+3)^2}{(coth\,(k_2t+c_2))^2} \right\} \times \left\{ -\frac{9}{2} + 3(n+3)(sech(k_2t+c_2))^2 \right\}^{-1} \quad (41)$$

There is a limit for $(r, s) = (1, 1)$ (CDM limit) and for $(r, s) = (1, 0)$ (ΛCDM limit). The model corresponds to the Chaplygin gas model for $r > 1$ and $s < 0$. When $r < 1$ and $s > 0$, the DE model's quintessence and phantom regions are also obtained. It is very clear from Fig. 5 that our model becomes a ΛCDM model at late times. Also, in terms of evolution, our model is similar to the Chaplygin gas model.

r-q plane:

The fixed points $(r, q) = (1, 0.5)$ and $(r, q) = (1, -1)$, respectively, exhibit the standard cold dark matter (SCDM) and steady state (SS) models. Starting at the fixed point of the SCDM model $(r, q) = (1, 0.5)$, the ΛCDM model evolves along the vertical dotted line ($r = 1$) untill it reaches the fixed point of the SS model $(r, q) = (1, -1)$. The evolution of our model in the $r - q$ plane is seen in Fig. 6. The $r - q$ trajectory shows that the deceleration parameter (q) has shifted sign, going from positive to negative. At a later stage, our model approaches the SS model. The $r - q$ trajectory also shows that our massive scalar field model in the $f(R, T)$ theory of gravity behaves very similarly to the scalar field models in literature (ref. Singh and Kumar 2016; Aditya et al. 2022).

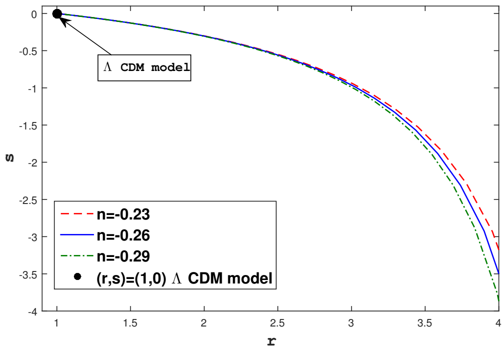

Fig. 5. Plot of statefinder parameters for $c_2 = 0.84$, $M = 0.115$, $c_1 = 0.00001$, $\varphi_0 = 10$ and $k = 0.95$.

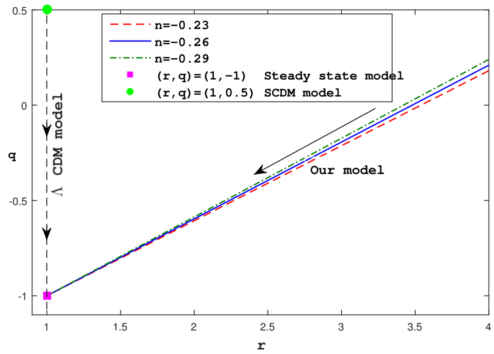

Fig. 6. Plot of $r - q$ for $c_2 = 0.84$, $M = 0.115$, $c_1 = 0.00001$, $\varphi_0 = 10$ and $k = 0.95$.

3 Conclusions

Several researchers are looking at cosmological models based on the $f(R,T)$ theory of gravity in order to explain the interesting question of accelerated expansion of the universe. Hence, we solved the $f(R,T)$ field equations in the presence of matter and a massive SF in this work and constructed a Bianchi type-I universe. We used a relationship between the metric potentials and a power law between the SF and the average scale factor to construct a deterministic model. In light of new cosmological scenarios and observations, we analyzed several dynamical cosmological parameters and gave their physical discussion. Some conclusions are as follows:

– The spatial volume of our model increases with cosmic time, hence the universe is expanding spatially. The physical quantities H, θ, σ^2 of the model are finite initially (at $t = 0$) and tend to a constant value for sufficiently large values of cosmic time. The average anisotropy parameter of our model is constant and hence the model is anisotropic. However it may be noted that it becomes isotropic and shear free for $n = 1$. The scalar field in the model decreases with cosmic time and hence the corresponding kinetic energy increases. This result is quite similar to that of the scalar field models in literature (Jawad et al. 2015; Singh and Rani 2015; Naidu et al. 2015; Aditya et al. 2019).

- The energy density of our model appears to decrease with the passage of time whereas the energy density increases when the scalar field increases (Fig. 2). When the scalar field is modified, the behavior of energy density changes significantly. The NEC is violated, causing the model to have a Big Rip. Based on these results, the model suggests an early inflation.
- The EoS parameter of our model totally varies in the phantom region (Fig. 3). Also, its behavior differs in the aggressive phantom region as the scalar field grows (Fig. 3). The present values of the EoS parameter of our model are (n, ω)= $(-0.23,\ -1.025)$, $(-0.26, -1.09)$, $(-0.29, -1.12)$. It is important to mention that the EoS value in our model matches recent Planck observations very well (Aghanim et al. 2020)
  $\omega_{de} = -1.56^{+0.60}_{-0.48}$(Planck + TT + lowE);
  $\omega_{de} = -1.58^{+0.52}_{-0.41}$(Planck + TT,TE,EE+lowE);
  $\omega_{de} = -1.57^{+0.50}_{-0.40}$(Planck + TT,TE,EE+lowE+lensing);
  $\omega_{de} = -1.04^{+0.10}_{-0.10}$(Planck + TT,TE,EE+lowE+lensing+BAO).
- We observe that, initially, the deceleration parameter $(q(t))$ of our model is positive and finally approaches -1. Hence, the model starts in a decelerated phase and finally approaches an accelerated expansion (Fig. 4). The present value of $q(t)$ for our model is $q \approx -0.98$ which is in accordance with the observational data (Capozziello et al. 2019; Amirhashchi and Amirhashchi 2019) given as $q = -0.930 \pm 0.218$ $(BAO + Masers + TDSL + Pantheon + H_z)$ and $q = -1.2037 \pm 0.175$ $(BAO + Masers + TDSL + Pantheon + H_0 + H_z)$. Statefinders analysis confirms that our model behaves as the Chaplygin gas model and finally approaches to ΛCDM (Fig. 5). It can be seen from the $r - q$ trajectory that our model approaches the steady state model at late times (Fig. 6). This behavior is quite similar to that of the scalar field models in literature (Singh and Kumar 2016; Aditya et al. 2022).

All of the dynamical parameters are observed to behave in a way that is consistent with current experimental observations in modern cosmology. Hence, our model will help to understand the significance of massive scalar fields in modern cosmology.

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