# Characteristics and ellipsoidal motion of $L$ dwarfs within $85^{\circ} \geq b \geq-85^{\circ}$ 

Bushra A. Al-Johani and Amnah S. Al-Johani<br>Mathematics Department, Faculty of Science, University of Tabuk, Tabouk, Saudi Arabia, bushra.a.alrefai@gmail.com and aalgohani@ut.edu.sa

(Submitted on 12.01.2024; Accepted on 21.02.2024)


#### Abstract

Our objective is to construct a model of the velocity ellipsoid and the retrieved Solar velocity for two programs of ultra-cool L dwarfs with 49 points (UCDs 49) and 39 points (UCDs 39) within $85^{\circ} \geq b \geq-85^{\circ}$. Firstly, we calculated the velocity dispersion $\left(\sigma_{1}, \sigma_{2}, \sigma_{3} ; \mathrm{km} \mathrm{s}^{-1}\right)$ ratios $\left(\sigma_{2} / \sigma_{1}=0.78 \pm 0.06 ; ~ \mathrm{UCDs} 49 \& 0.79 \pm 0.05 ;\right.$ UCDs 39) and $\left(\sigma_{3} / \sigma_{1}=0.41 \pm 0.04 ; ~ U C D s ~ 49 \& 0.51 \pm 0.03 ;\right.$ UCDs 39$)$, the vertex longitude $\left(l_{2}\right)$ which has often differed significantly from zero in many analyses (i.e., $l_{2}=-0^{\circ} 8967$; UCDs $49 \&$ -0.4454; UCDs 39). Here, we have computed the Solar velocity $\left(S_{\odot} ; \mathrm{km} \mathrm{s}^{-1}\right)$ as $(32.13 \pm 0.18 ;$ UCDs $49 \& 21.73 \pm$ 0.22 ) and the other Solar elements $\left(l_{A}, b_{A}\right)$. Finally, we have computed Oort's constants $\left(A, B ; k m s^{-1} k p c^{1}\right)$ and the angular velocities for UCDs 49 and UCDs 39 programs where their numerical values are in the same manner.


Key words: Ultra cool dwarfs; L dwarf; Velocity Ellipsoid Parameters (VEPs); Kinematics; Oort constants.

## 1. Introduction

Ultracool dwarfs (UCDs) or brown dwarfs are substellar objects which have masses $\sim 0.1 M_{\odot}$ or $\sim 0.08 M_{\odot}$ (Chabrier \& Baraffe 1997) comprise a significant fraction $20 \%$ of stars in our Galaxy according to Chabrier (2003) and Bochanski et al. (2010). UCDs are Galactic fossils with lifetimes much greater than the Hubble time (Laughlin \& Adams 1997) and they are compelling targets for extrasolar planet surveys (Gaidos et al. 2007, Lunine et al. 2009 and Charbonneau 2009). They are a crucial tool for studying the structure of Galaxies, the evolution of their chemical composition, and the history of star formation (Burgasser 2004, Bochanski et al. 2007, Pirzkal et al. 2009, and Pineda et al. 2013). However, the low luminosities and temperatures of brown dwarfs (Kirkpatrick 2005), and the steady cooling of substellar very low-mass dwarfs over time, have made it difficult to identify and study them in statistically significant numbers.

UCDs evolve on relatively short astronomical periods compared to Hydrogenburning stars, whose position on the main sequence varies relatively little throughout their lifetimes. They have temperatures of about 3000 K as they exit the T Tauri stage, which is comparable to mid-type M dwarfs, but they rapidly cool through mid and late-M before passing through class L to become T dwarfs. UCDs are more luminous at a given temperature (from 1600 to 2300 K) due to their larger radii.

Because the intrinsic velocity dispersion of the stars can be used to infer their age, the kinematics of these stars, particularly those in the Solar neighborhood, are significant (Wielen 1977 and Fuchs et al. 2001). Proper motion measurements have provided the main foundation for kinematic investigations of brown dwarfs (see, for example, Schmidt et al. 2007, Jameson et al. 2008, Casewell et al. 2008, and Faherty et al. 2009). For tiny samples, brown dwarfs'
absolute radial velocities have been measured, for example, by Basri et al. (2000), Bailer-Jones (2004), Blake et al. (2007), or Zapatero Osorio et al. (2007).

Given that they continuously cool over time and never attain a stable state on the main sequence, brown dwarfs may then function in turn as Galactic chronometers (Burgasser 2009).

Large-scale optical and infrared surveys, including the Two Micron AllSky Survey (2MASS; Skrutskie et al. 2006), the Wide-field Infrared Survey Explorer (WISE; Wright et al. 2010), and the Panoramic Survey Telescope And Rapid Response System (Pan-STARRS; Kaiser et al. 2010), have been crucial in the discovery and characterization of the vast majesty of the universe.

The data used in our program UCDs is described in the next portion of the publication. Section 3 discusses computational methods. Section 4 is devoted to the findings discussion and conclusion.

## 1. Observational of UCDs (L - type)

Blake et al. (2010) used the NIRSPEC (McLean et al. 1998) instrument on the Keck II telescope to gather 600 individual observations of a sample of 59 UCDs between 2003 March and 2009 May. A potent tool for high-resolution spectroscopy of cool stars and brown dwarfs, NIRSPEC is a high-resolution, cross-dispersed NIR echelle spectrograph.

Our program of UCDs with L dwarfs (49) listed in Table (1) within Galactic latitudes range $85^{\circ} \geq b \geq-85^{\circ}$ (Blake et al. 2010) ${ }^{1}$, where the first column represents the equatorial coordinates $\left(\alpha_{2000}, \delta_{2000}\right)$, column 2 presents the spectral types, and columns 3,4 , and 5 are devoted to the vector components of the space motion ( $U, V, W ; \mathrm{km} \mathrm{s}^{-1}$ ) as derived by Blake et al. (2010) on the base of their radial velocities supplemented by distances and proper motions taken from Faherty et al. (2009) concerning the local standard of rest (LSR), respectively. Figure (1) shows the radial velocity distribution curve as a function of galactic longitude ( $\imath^{\circ}$ ) with our program UCDs (49 points).


Fig. 1. Distribution of the observed radial velocities ( $V_{r} ; k m s^{-1}$ ) of our program UCDs (L-type)versus their galactic longitudes ( $\imath^{\circ}$ ) within the Galactic latitude range $85^{\circ} \geq b \geq$ $-85^{\circ}$

Table 1. Our program UCDs (49 points) within $85^{\circ} \geq b \geq-85^{\circ}$ adopted by Blake et al. (2010)

| $\left(\alpha_{2000}, \delta_{2000}\right)$ | Spectral type $U\left(\mathrm{~km} \mathrm{~s}^{-1}\right)$ | $V\left(\mathrm{~km} \mathrm{~s}^{-1}\right) W\left(\mathrm{~km} \mathrm{~s}^{-1}\right)$ |  |  |
| :--- | :---: | :--- | :---: | :--- |
| $1.1452,-40.7349$ | L5 | 1.37 | $-109.16^{*}$ | -20.03 |
| $3.9365,35.2674$ | L2 | 13.03 | -42.88 | -6.53 |
| $9.0674,18.3529$ | L3.5 | -42.15 | -5.8 | -11.59 |
| $11.3393,16.5791$ | L2 | -22.96 | -13.41 | -3.98 |
| $25.2634,18.0806$ | L1 | -48.92 | -21.11 | -10.94 |
| $26.1473,-7.2706$ | L5 | -20.82 | -4.41 | 3.88 |
| $33.3700,44.7459$ | L1.5 | 19.54 | -17.52 | -8.81 |
| $36.7932,-16.4133$ | L1 | -36.24 | $-61.75^{*}$ | -31.17 |
| $37.0459,25.6272$ | L0 | -39.56 | -14.08 | -0.38 |
| $38.9997,-23.5224$ | L1 | -12.45 | -8.45 | -10.38 |
| $42.8121,-3.8794$ | L3 | 15.68 | $-120.09^{*}$ | -14.14 |
| $58.8474,11.5621$ | L5 | -6.29 | -22.19 | -14.79 |
| $75.0875,3.5139$ | L4 | -4.82 | -20.41 | -16.43 |
| $80.9093,-14.0506$ | L2.5 | -16.27 | -3.26 | 3.3 |
| $84.9667,-0.9839$ | L5 | -21.86 | 3.31 | 13.84 |
| $90.6269,39.1831$ | L1 | -11.9 | -26.51 | -2.47 |
| $103.1280,47.1763$ | L4.5 | 7.38 | 5.97 | -4.08 |
| $105.1527,31.9574$ | L3.5 | 42.13 | -28.97 | -15.62 |
| $109.3178,57.0953$ | L3 | 13.88 | -2.03 | -8.07 |
| $116.6773,20.0089$ | L0.5 | -55.63 | -15.04 | -1.74 |
| $127.1425,-13.1555$ | L2 | -38.71 | -13.49 | -20.68 |
| $128.9273,-8.3233$ | L5 | -33.22 | -14.77 | 0.89 |
| $131.8697,-15.5437$ | L2 | 15.84 | -10.68 | 1.79 |
| $140.3088,-21.0791$ | L1.5 | 19.43 | $-94.59^{*}$ | 5.77 |
| $155.7009,58.4293$ | L1 | -65.37 | $60.85^{*}$ | -46.79 |
| $161.3500,-1.8327$ | L1 | -35.71 | -14.22 | -12.83 |
| $162.1784,1.1994$ | L1 | -24.91 | -34.5 | -1.43 |
| $167.1284,68.5047$ | L1 | -41.59 | -36.59 | -17.93 |
| $168.1070,35.8036$ | L4.5 | -20.75 | -25.17 | -13.83 |
| $178.9147,-37.4597$ | L2 | 35.42 | -50.55 | -22.97 |
| $180.9922,0.2639$ | L3 | -83.82 | $-67.49^{*}$ | -29.92 |
| $185.3654,2.9555$ | L0 | 51.86 | -23.29 | -27.25 |
| $195.1773,19.2098$ | L1 | -3.87 | -98.51 | -26.4 |
| $196.4175,-25.6850$ | L2 | -20.27 | -19.39 | 6.24 |
| $216.3666,-36.8397$ | L3 | -5.05 | -22.47 | -10.26 |
| $219.8682,19.4875$ | L1 | -85.11 | $-42.92^{*}$ | 17.75 |
| $226.7267,13.3517$ | L3 | -41.64 | -48.84 | 36.85 |
| $226.9487,-16.4607$ | L5 | -26.41 | -17.1 | -39.67 |
| $228.7535,48.7949$ | L6 | -107.23 | $-30.47^{*}$ | -2.76 |
| $234.9245,-5.3452$ | L4 | 37.58 | 35 | -4.76 |
| $238.2461,29.8135$ | L0 | -11.09 | -22.92 | -3.75 |
| $238.8155,-9.9349$ | L1 | 52.11 | 3.19 | -57 |
| $251.3421,-13.3310$ | L1.5 | 33 | -46.83 | -0.08 |
| $254.5158,70.4504$ | L1 | 24.56 | -42.29 | 22.43 |
| $262.8739,27.3565$ | L0 | -5.28 | -29.71 | -13.28 |
| $271.8164,50.2588$ | L1.5 | 7.46 | -0.67 | -5.4 |
| $309.0132,10.8582$ | L3 | 24.26 | 4.73 | -5.27 |
| $316.0621,-10.6269$ | L2.5 | -38.79 | -29.37 | -33.36 |
| $336.1825,-1.9811$ | L4.5 | -9.62 | $-63.97^{*}$ | -6.01 |
|  |  |  |  |  |

## 2. Computational methods

Consider the average space velocities along the Galactic coordinates are ( $\bar{U}, \bar{V}, \bar{W}$; $\mathrm{km} \mathrm{s}^{-1}$ ), i.e.

$$
\begin{equation*}
\bar{U}=\frac{1}{N} \sum_{i=1}^{N} U_{i} ; \bar{V}=\frac{1}{N} \sum_{i=1}^{N} V_{i} ; \bar{W}=\frac{1}{N} \sum_{i=1}^{N} W_{i} \tag{1}
\end{equation*}
$$

where $(N)$ is the total program UCDs. Figure (2) shows the distribution of spatial space velocity components along the Galactic coordinates. For our program UCDs have space velocities $(\bar{U}, \bar{V}, \bar{W})$, the Solar elements $\left(S_{\odot}, l_{A}, b_{A}\right)$ with spatial velocity considered may take the following:

$$
\begin{array}{r}
S_{\odot}=\sqrt{\bar{U}^{2}+\bar{V}^{2}+\bar{W}^{2}} \\
l_{A}=\tan ^{-1}(-\overline{V U}) \\
b_{A}=\sin ^{-1}\left(-\bar{W} S_{\odot}\right) \tag{4}
\end{array}
$$

Where $\left(S_{\odot}\right)$ is the absolute value of the Sun's velocity relative to our program UCDs, $\left(l_{A}\right)$ is the Galactic longitude and $\left(b_{A}\right)$ is the Galactic latitude of the Solar apex respectively.


Fig. 2. The vector space velocity distribution $(\mathrm{U}-\mathrm{V}),(\mathrm{U}-\mathrm{W})$, and $(\mathrm{V}-\mathrm{W})$ of our program UCDs

## 3. Galactic rotation kinematics

The disk of our Galaxy is in a state of differential rotation around an axis through the Galactic center. At any specified distance $(R)$ measured from the center in the Galactic plane, there is a particular velocity, called the linear rotation velocity $(V)$ of the Galaxy at that radius (Mihalas \& Binney 1981). On the other hand, it's known that the Sun is moving around the Galactic center on a nearby circular orbit of radius ( $R_{\circ}$ ) with $8.20 \pm 0.10 \mathrm{kpc}$ as mentioned recently by Bland-Hawthorn (et al. 2019), whose linear rotation velocity ( $V_{\circ}$ ) of the Galaxy at $\left(R_{\circ}\right)$ is about $247.5 \mathrm{~km} \mathrm{~s}^{-1}$ (Oort 1927a \& 1927b), a direct measurement of $\left(V_{\circ}\right)$; based on radial velocities of globular clusters or
spheroidal component stars in our Galaxy or external galaxies in the Local Group, yield in the range ( $200 \leq V_{\circ} \leq 300 \mathrm{~km} \mathrm{~s}^{-1}$ ) (Mihalas \& Binney 1981) and a "best" estimate of ( $V_{\circ}=250 \mathrm{~km} \mathrm{~s}^{-1}$ ), $204 \mathrm{~km} \mathrm{~s}^{-1}$ ) (Bovy 2017), 213.7 $\mathrm{km} s^{-1}$ (Chengdong et al. 2019), and recently we determined $\left(V_{\circ}\right)$ is 221.2 km $\mathrm{s}^{-1}$ (Nouh \& Elsanhoury 2020) with space and radial velocities of 1,583 red giant stars collected from the SEGUE-1 and SEGUE-2 surveys. The angular rotation rate $\left(\omega_{\circ}\right)$ of the Galaxy at a radius $\left(R_{\circ}\right)$ is given like:

$$
\begin{equation*}
V_{\circ}=\omega_{\circ} R_{\circ}=(A-B) R_{\circ} \tag{5}
\end{equation*}
$$

where $A$ and $B$ are Oort's constants in $\left(\mathrm{km} \mathrm{s}^{-1} \mathrm{kpc}^{-1}\right)$. Some kinematics and velocity ellipsoid parameters (VEPs) in terms of the matrix elements $\left(\mu_{\mathrm{ij}}\right)$ concerning the dispersion velocities [i. e., $\sigma_{\mathrm{j}}=\sqrt{\lambda_{\mathrm{i}}} ; \forall_{j}=1,2,3$ ] along axes of velocity ellipsoid may be deduced by Elsanhoury et al. (2015). The main major quantity we can devote to dwarf's kinematics is the relation between the ratio of $\left(\sigma_{2} / \sigma_{1}\right)$ with the Oort's constants, i.e.

$$
\begin{equation*}
\left(\frac{\sigma_{2}}{\sigma_{1}}\right)^{2}=\frac{-B}{A-B}, \quad \text { or } \quad \frac{-B}{A}=\frac{1}{\left(\sigma_{1} / \sigma_{2}\right)^{2}-1} \tag{6}
\end{equation*}
$$

## 4. Discussion and conclusion

Based on the model described in the previous section, we construct a Mathematica code to serve us for computing the kinematics of our first program UCDs (49 points; UCDs 49). On the other hand, eleven high-velocity UCDs have been detected in Table (11) of Blake et al. (2010), and ten of them are considered in our sample of Table (1) (with asterisk) and consequently in our calculations. Since, Objects with high velocities may be subdwarfs from an older, lower metallicity thick disk or halo population (Black et al. 20210). Now, we return to exclude those in our sample, therefore we have second program UCDs (39 points; UCDs 39), then we again analyze the Galactic rotation kinematics to show their influence on velocity dispersion and Oort's constants.

Our originality numerical results will be listed in Table (2) for different kinematical parameters like: dispersion $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)$ and Solar $\left(S_{\odot}\right)$ velocities, ratios $\left(\sigma_{2} / \sigma_{1}\right)$ and $\left(\sigma_{3} / \sigma_{1}\right)$. We point out attention here to the comparison between our two programs UCDs 49 and UCDs 39 based on our computations and other authors.

From Table (2), we imply that the dispersion velocity $\sigma_{0}=\sqrt{\sigma_{1}^{2}+\sigma_{2}^{2}+\sigma_{3}^{2}}$ for UCDs, 49 is about $60.00 \pm 7.72 \mathrm{~km} \mathrm{~s}^{-1}$ and that for UCDs 39 is about 43.26 $\pm 6.58 \mathrm{~km} \mathrm{~s}^{-1}(\sim 72 \%$ of first program $)$, Solar velocity $S_{\odot}$ is about $(32.13 \pm$ 0.18 UCDs $49 \& 21.73 \pm 0.22$; UCDs 39 ) in $\mathrm{km}^{-1}$ units ( $\sim 86 \%$ of UCDs 49 ), ratios $\left(\sigma_{2} / \sigma_{1}\right)$ are about $0.78 \pm 0.06$ and $0.79 \pm 0.05$ for UCDs 49 and UCDs 39 respectively, while $\left(\sigma_{3} / \sigma_{1}\right)$ are about $0.41 \pm 0.04$ and $0.51 \pm 0.03$ also for our two programs respectively. We consider in Table (3) for our programs UCDs

|  | $79^{\circ} 0$ | TL ${ }^{\circ} 0$ | $\varepsilon ¢ \%$ | 7．91 | L＇\＆z | Z＇IE | 9W－8N |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| （L96I）пегчәМ | $69^{\circ}$ | $99^{\circ}$ | \＆L 1 | 8.81 | LZ | I＇Z¢ | 7N－8Y |  |  |  |  |  |  |  |
|  | ¢0．0 干 99．0 | 900 $\square^{\circ} 29.0$ |  |  |  |  | $06<{ }^{\wedge} W$ |  |  |  |  |  |  |  |
| （996ז）рлемря $ช$ ләКオ | 70．0 干 79.0 | 50．0 干 $2 L^{\circ} 0$ |  | 0＇I 干 I＇LZ | İI 干 $\chi^{\prime} \downarrow \square$ | $\mathrm{G}^{\circ} \mathrm{L}$ 干 $8^{\circ} \mathrm{E}$ ¢ | $8<{ }^{\wedge} W<6.8$ |  |  |  |  |  |  |  |
|  |  | $90^{\circ} 0$ 干 $78^{\circ} 0$ | － | $0 \cdot \mathrm{I}$ 干も゙0Z |  | も． L 干 008 | ${ }^{\wedge}$ W88＞ |  |  |  |  |  |  |  |
| （LL6I）＇「е ұә Кә［IOOM | $67^{\circ} 0$ | $79^{\circ} 0$ | 00\％ 28 |  |  | － | sృгтм У рие $\$  \hline （8L6I）บәл．̊d $\cap$ | 970 | $89^{\circ} 0$ | $00^{\circ} 98$ | － |  |  |  |
| （ 266 L ）¢セ ұә иәля才 $\cap$ | $99^{\circ} 0$ | L9．0 | 06.8 干 $0 L^{\circ} 99$ | － |  |  | s．aełs N pue＇Y |  |  |  |  |  |  |  |
|  | $89^{\circ} 0$ | 29.0 | $9{ }^{\circ} \mathrm{E}$ 干 $0 \cdot 8 \%$ | $Q^{\prime} \checkmark \mp \mathcal{E}^{\prime} L \mathcal{L}$ | $9 \cdot 7$ F 6.98 | $q^{\circ} \mathcal{E} \mp 0 \pm 9$ |  |  |  |  |  |  |  |  |
| （986L）оечZ पе！ | DL．0 | $29^{\circ}$ | 00．It |  |  |  | S．te7s ədK7－ |  |  |  |  |  |  |  |
|  | ［600 | 9 ${ }^{\circ} 0$ | \＆\％＇¢\％ | 01．0才 | 78． $8 ¢$ | 90＇ti | đM VG－uon |  |  |  |  |  |  |  |
|  | E80 | $69^{\circ}$ | 82．8I | 96． LE $^{\text {c }}$ | \＆9＊97 | \＆9．88 | ©M VG |  |  |  |  |  |  |  |
| （9L0z）＇⿺辶 ұә Кınoчuest＇̇ | 980 | 72．0 | L6．LZ | L6．98 | 66.08 | $87^{\circ} 77$ | © ${ }^{\text {PIOO }}$ |  |  |  |  |  |  |  |
|  | $08^{\circ}$ | $09^{\circ}$ | $87^{\prime}$ LI | LE＇\＆\％ | 09．2I | 01＇67 | CM ${ }^{+0} \mathrm{H}$ |  |  |  |  |  |  |  |
| Крnұs quәлınด | $80^{\circ} 0$ 干 $49^{\circ} 0$ | $90^{\circ} 0$ 干 $6 L^{\circ} 0$ | $77^{\circ} 0$ 干 $\mathcal{L} L^{\prime} \mathrm{L} Z$ | 80＇t 干 \＆\％${ }^{\circ} 91$ |  | 89 9 F $99^{\circ} \mathrm{LE}$ |  |  |  |  |  |  |  |  |
| イpn7s quәıın○ | 70．0 干 LT＊ 0 | 90．0 干 8 2.0 | 8L＇0 干 \＆${ }^{\circ} \mathrm{Z}$ \％ | 97＇t 干 91＇8I | $6{ }^{\circ} \mathrm{G}$ 干 76. ¢ | 0L＇9 干 8 $L^{\prime}$ 切 | 67 sGDด Uex．80．${ }^{\text {d }}$ |  |  |  |  |  |  |  |
| әЈиәәјәу | （ $10 / 8_{\rho}$ ） | （ $10 / z_{0}$ ） | ${ }^{\circ} \mathrm{S}$ | 80 | $z_{0}$ | ${ }^{1}$ |  |  |  |  |  |  |  |  |

49 and UCDs 39 that the obtained results of Oort's constants ( $A, B ; \mathrm{km} \mathrm{s}^{-1}$ $\mathrm{kpc}^{-1}$ ) are in the same manner about ( $11.94 \pm 0.29,-18.55 \pm 0.23$; UCDs 49 \& $11.46 \pm 0.30,-19.03 \pm 0.24 ;$ UCDs 39$)$ and the angular velocity $(|A-B| ;$ $\mathrm{km} \mathrm{s}^{-1} \mathrm{kpc}^{-1}$ ) is about ( $30.49 \pm 5.53$; UCDs $49 \& 30.49 \pm 5.52$; UCDs 39) and comparison as well as with different authors. Thus, we may conclude that the dispersion, Solar velocities, $\left(\sigma_{2} / \sigma_{1}\right)$, and ( $\sigma_{3} / \sigma_{1}$ ) are quite comparable for these samples of dwarfs and subsamples.

Therefore, the effects of those UCDs with high negative velocities may be slightly important for dispersion and the Solar velocities but not for both Oort's constants and the angular velocity.

According to Equations (3) and (4), we have computed the Solar elements $\left(l_{A}, b_{A}\right)$ for our two programs and the results are (65.47, 17.38) and ( $70.00,21.92$ ) for UCDs 49 and UCDs 39 respectively.

Mihalas \& Binney (1981) stated that the longitude of the vertex $\left(l_{2}\right)$ of the velocity ellipsoid shows that the principle axis points nearly in the direction of the Galactic center for later spectral types, but it shows larger and larger departures from this direction for early types. This departure is called the vertex deviation. This expectation is confirmed by analysis of an ellipsoidal velocity distribution with the equations of stellar dynamics(see Chapter 14; Mihalas \& Binney 1981) and they adopted that the longitude of the vertex $\left(l_{2}\right)$ often differs from zero.

To specify the orientation of the velocity ellipsoid, we need to determine only the Galactic longitude along which the principle axis lies (i.e., the longitude of the vertex $\left(l_{2}\right)$. Our code gives the longitude of the vertex $\left(\left(l_{2}=\right.\right.$ $-0^{\circ} .8967$; UCDs $49 \&-0^{\circ} .4454$; UCDs 39 ), where this obtained result is in line with that of Mihalas \& Binney (1981) and others e.g. -0 $0^{\circ} .006394$ and $0^{\circ} .630221$ (Elsanhoury and Al-Johani 2023); $-0^{\circ} .5410,-0^{\circ} .4937$, and $-0^{\circ} .9495$ (Elsanhoury et al. 2021); $-0^{\circ} .92,-0^{\circ} .906$, and $-0^{\circ} .697$ (Nouh \& Elsanhoury 2020); - $0^{\circ} .412,-0^{\circ} .171,-0^{\circ} .642,-0^{\circ} .441$, and $-0^{\circ} .684$ (Elsanhoury 2020); $0^{\circ} .0 .83,-0^{\circ} .75,-0^{\circ} .41,-0^{\circ} .05$, and $-0^{\circ} .39$ (Elkhateeb \& Elsanhoury 2019); $0^{\circ} .87$ and $-0^{\circ} .91$ (Elsanhoury et al. 2018)

Table 3. The rotation constants devoted to our program UCDs and recently computed ones

| $\sigma_{2} / \sigma_{1}$ | $\left(A ; \mathrm{km} \mathrm{s}^{-1} \mathrm{kpc}^{-1}\right)$ | $\left(B ; \mathrm{km} \mathrm{s}^{-1} \mathrm{kpc}^{-1}\right)$ | $A-B \mid ; \mathrm{km} \mathrm{s}^{-1}$ | References |
| :--- | :---: | :---: | :---: | :---: |
| $0.78 \pm 0.06$ | $11.94 \pm 0.29$ | $-18.55 \pm 0.23$ | $30.49 \pm 5.53$ | Current work |
| $0.79 \pm 0.05$ | $11.46 \pm 0.30$ | $-19.03 \pm 0.24$ | $30.49 \pm 5.52$ | Current work |
| 0.72 | $12.90 \pm 0.28$ | $-13.16 \pm 0.28$ | 26.06 | Elsanhoury \& Al-Johani(2023) |
| 0.74 | $16.06 \pm 0.68$ | $-19.43 \pm 1.37$ | 35.50 | Elsanhoury et al. (2021) |
| - | $16.31 \pm 0.61$ | $-12.00 \pm 0.79$ | 28.30 | Wang et al.(2021) |
| 0.71 | $15.60 \pm 0.04$ | $-13.90 \pm 1.80$ | 29.50 | Nouh \& Elsanhoury (2020) |
| - | $15.73 \pm 0.32$ | $-12.67 \pm 0.34$ | 28.40 | Krisanova et al.(2020) |

## Acknowledgment

The authors are deeply thankful to the referee for his/her valuable and constructive comments that improved the original manuscript.

## References

Bailer-Jones C. A. L., 2004, Astron. Astrophys., 419, 703
Basri G., Mohanty S., Allard F., et al., 2000, Astrophys. J., 538, 363
Blake C.' H., Charbonneau D., and White, R. J., 2010, Astrophys. J., 723, 684
Blake C. H., Charbonneau D., White R. J., Marley M. S., and Saumon D., 2007, Astrophys. J., 666, 1198

Bland-Hawthorn J., Sharma, S., et al., 2019, Mon. Not. Roy. Astron. Soc., 486, 1167
Bochanski J. J., Hawley S. L., Covey K. R., et al., 2010, Astron. J., 139, 2679
Bochanski J. J., Munn J. A., Hawley S. L., et al., 2007, Astron. J., 134, 2418
Bovy J., 2017, Mon. Not. Roy. Astron. Soc., 468, L63
Burgasser A. J., 2004, Astrophys. J. Supp., 155, 191
Burgasser A. J., 2009, IAU Symp., 258, 317
Casewell S. L., Jameson R. F., and Burleigh M. R., 2008, Astron. Astrophys., 390, 1517
Chabrier G., 2003, Pub. Astron. Soc. Paci., 115, 763
Chabrier G., and Baraffe, I., 1997, Astron. Astrophys., 327, 1039
Charbonneau D., Berta Z. Ǩ., Irwin J., Burke C. J., Nutzman P., Buchhave L. A., Lovis C., Bonfils X. et al., 2009, Nature, 462, 891
Chengdong Li, Gang Zhao, and Chengqun Yang, 2019, Astrophys. J., 872, 205
Dyer and Edward R. Jr, 1995, Astron. J., 61, 228
Elkhateeb M. M. and Elisanhoury W. H., 2019, 29, 5
Elsanhoury W. H. and Al-Johani, Amnah S., 2023, Astrono. Nachr., 344, Issue 7, article id. e20230047
Elsanhoury W. H., 2020, Bulg. Astron. J., 32, 83
Elsanhoury W. H., Nouh M. I., and Abdel-Rahman H. I., 2015, Rev. Mex. Astron. Astrofís., 51, 197
Elsanhoury W. H., Nouh M. I., Branham R. L., and Al-Johani A. S., 2021, Astron. Nachr., 342(989), 989
Elsanhoury W. H., Postnikova E. S., Chupina N. V., Vereshchagin S. V., Sariya Devesh P., Yadav R. K. S., Jiang Ing-Guey, 2018, Astrophys. S. Sci., 363, 58
Epchtein N., de Batz B., Capoani L. et al., 1997, Msngr, 87, 27
Faherty J. K., Burgasser A. J., Cruz K. L. et al., 2009, Astron. J., 137, 1
Fuchs B., Dettbarn C., Jahreiß H., and Wielen R., 2001, Dynamics of Star Clusters and the Milky Way, 228, 235
Gaidos E., Haghighipour N., Agol E., Latham D., Raymond S., and Rayner J. 2007, Science, 318, 210
Jameson R. F., Casewell S. L., Bannister N. P., et al., 2008, Astron. Astrophys., 384, 1399
Kaiser N., Burgett W., Chambers K., et al., 2010, Proc. SPIE, 7733, 77330E
2Kirkpatrick J. D., 2005, Ann. Revi. Astron. Astrophys., 43, 195
Krisanova O. I., Bobylev V. V., and Bajkova A. T., 2020, Astron. Lett., 46(6), 370
Laughlin G. and Adams F. C., 1997, Astrophys. J., 491, 51
Lunine J. I., Macintosh B., and Peale S., 2009, Physics Today, 62, 050000
McLean I. S. et al., 1998, Proc. SPIE, 3354, 566
Mihalas D. and Binney J., 1981, Galactic Structure "Structure and Kinematics", 2nd. Edition W. H. Freeman and Company - San Francisco

Nouh M. I. and Elsanhoury W. H.., Astrophys., 2020, Vol. 36, No. 2, pp. 179-189
Oort J. H., Bull. Astron. Inst. Netherl., 3, 275, 1927a
Oort J. H., Bull. Astron. Inst. Netherl., 4, 91, 1927b
Pineda J. S., West A. A., Bochanski J.' J., and Burgasser, A. J., 2013, Astron. J., 146, 50
Pirzkal N., Burgasser A. J., Malhotra, S., et al., 2009, Astrophys. J., 695, 1591
Ratnatunga K. U. and Upgren A. R., 1997, Astron. J., 476, 811
Schmidt S. J., Cruz K. L., Bongiorno, B. J., Liebert, J., and Reid, I. N., 2007, Astron. J., 133, 2258
Tian K.-p. and Zhao, J.-l., 1986, Chin. Astron. Astrophys., 10, 98
Upgren A. R., 1978, Astron. J., 83, 626
Upgren A. R.,' Ratnatunga K. Ü., Casertano S., and Weis E., 1997, in: Hipparcos-Venice '97, eds. R. M. Bonnet, E. Hog, P. L. Bernacca, et al., Proceedings of the European Space Agency (ESA) Symposium 'Hipparcos - Venice 97 ' (Venice, Italy), Vol. 402, 583
Wang F., Żhang H. W., Huang Y., Chen B. Q., Wang H. F., and Wang C., 2021, Mon. Not. Roy. Astron. Soc., 504(1), 199
Wehlau A. W., 1957, Astron. J., 62, 169
Wielen R., 1977, Astron. Astrophys., 60, 263
Woolley R. V. D. R., Martin W. L., Penston M. J., Sinclair J. E., and Aslan S., 1977, Mon. Not. R. Astron. Soc., 179, 81
Wright E. L., Eisenhardt P. R. M., Mainzer, A. K. et al., 2010, Astron. J., 140, 1868
York D. G., Adelman J., Anderson J. E., et al., 2000, Astron. J., 120, 1579
Zapatero Osorio, M. R., Martín, E. L., Béjar, V. J. S., et al., 2007, Astrophys. J., 666, 1205

