Mathematical Model of Star Formation in the Presence of External Perturbations

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Abstract. This paper investigates a mathematical model of a star formation system, including in-fall of gas from the local environment and out-flow of gas due to supernovae explosions and perturbations. In particular, the objective is to study how the variable density of interstellar components of the system namely atomic, molecular, and stellar components, interact in the star formation cycle. Under special parametric conditions, both limit cycle and stationary state behavior are observed. This indicates a stable star formation cycle in discrete episodes and an unstable star formation cycle converging to a stationary state. Observations of duty cycles under various parameters with varying intensity of supernovae shockwaves for dwarf galaxies, showed that the system adopts a self-regulatory oscillation state beyond a particular value. Analysis of giant galaxies showed decreased oscillatory periods for higher values. This is implied by the low production rate of supernovae in dwarf galaxies, increased production of cold gas, continuation of the cyclic behavior for a longer time. Stronger shockwaves increase the rate of dissipation of gases, resulting in shredding of mass, to get transferred into a precipitous lower mass. Star-forming rate (SFR) of high-mass star systems was found to vary against higher-order perturbations of supernovae shockwave but remained the same in the case of low-mass star systems.

Key words: Star formation, Mathematical modeling, Supernovae, Limit cycle, Stationary state

Introduction

In recent years, various fields of science have seen significant developments in the study of complex systems - in recognition of the distinction between simple and complex systems, ordered and chaotic systems, etc. Due to recent developments, many complex systems have been found that can be approximated or reduced to simpler systems and even simpler systems have been observed that exhibit complex behavior.

The theory of relativity was introduced with coordinate transformations between two reference frames and is now being used to study and explain the first few moments of creation after the Big Bang. Nicolis & Prigogine (1989) noted this and further explained how developments over the years have helped in understanding of the Universe which has further led to progressive research in this field. One such development has been in the understanding and modeling of star formation processes. In terms of modern research, a stellar system has been found to show a highly complex non-linear nature in an environment consisting of several intrinsic and extrinsic processes. However, this complex system can be further simplified to the basic interaction of three components of the Interstellar Medium (ISM). Ikeuchi et al. (1984) and Nozakura & Ikeuchi (1984) also noted this and made substantial contributions to the field by further proposing a model to subdivide the complex non-linear processes of a star-forming system into simple subdivisions. Due to recent technological advancements in the space sector, primarily in the domain of telescopes, several notable contributions have been made to the field owing to the availability of better computational data. This led to an advancement in the study of non-linear complex star-forming systems residing mostly in the locally centralized environments of galaxies or nebulae (Bodifee & Loore 1985). However, due to the complexity of the system, it is required to take some fundamental assumptions about the interaction between the trivial components, thereby directing the scope of study only to a finite number of processes.

In the ISM, the SFR varies from region to region in terms of the gravitational potential of the system and hence is non-uniform, as it is high in areas containing large quantities of interstellar component, and low for regions containing their small quantities (Barone et al. 2020; D'Eugenio et al. 2018). Observations of the Interstellar region through radio and optical telescopes also suggested that star formation activity is centralized in large clumps of baryonic mass throughout the Universe. These clumps consist of various kinds of ordered and disordered structures where regressive star formation processes take place, either locally in a neighborhood or throughout the whole structure (Adamo et al. 2020). A brief analysis of the observational data collected from telescopes resulted in observations of ordered structures across various regions of the galaxy. This suggests that the behavior of galactic self-organization can be found in these systems (Eden et al. 2015). However, different types of systems hosting these processes, can be observed depending on the nature of the ISM and neighborhood of the system.

There is a famous quote "Order comes out of chaos" often attributed controversially to German philosopher Friedrich Neitzche and this is quite evident in the case of star formation. Theoretical and observational data collected from star-forming regions through telescopes show that an ordered system with equilibrium conditions exists after a phase of non-linear processes (Adamo et al. 2020). The disorder or entropy of the non-linear structure tends to drive it towards a state of self-organization. This is caused by the continuous evolution of the system, guided by a phase of non-linear processes that tend to be irreversible in nature over time. Such regions are also referred to as dissipative structures, as shown by Bodifee & Loore (1985). A self-organization structure, therefore, comes to be seen after the non-linear processes, from which the system further converges to either a limit cycle or a stationary state. Sharaf et al. (2012) and Bodifee (1986) estimated the processes of a star-forming region as rates of transition of atomic gases to molecular gases, rate of spontaneous star formation, etc. In a similar work, Debsarma et al. (2016) studied the episodic model of a star formation region by assuming total masses of their corresponding systems as fractional components of the atomic, molecular and stellar components.

The non-linear processes of star-forming regions require huge volumes of hydrogen and dust clouds. These structures are usually concentrated along the spiral arms of the galaxy where they undergo condensation and gravitational collapses (Ikeuchi et al. 1984). Therefore, the localized baryonic regions suitable for star formation processes (SFP) are generally concentrated along these arms of the galaxy (Bodifee 1986). This baryonic mass consists of atomic, molecular, and stellar components of the ISM (Bodifee & Loore 1985). The basic interstellar components, when manifested in the local environment of the ISM, portray a very high degree of randomness along their paths. This degree of random behavior tends to decrease after some time, as clumps of gas begin to form in variable sizes. For a given system, to give rise to a stellar structure, certain conditions need to be satisfied. These conditions include the establishment of equilibrium rates between intrinsic and extrinsic processes, the presence of external perturbations in the neighborhood (Eg: Supernovae and Gamma-Ray Burst etc.), a continuous supply of gases from the external galactic environment, and removal of the components from the medium, etc. (Dubner & Giacani 2015). The external perturbations provided to the system, affect the individual atomic components, thereby provide small localized fluctuations to the system (Bodifee 1986). The magnitude of the fluctuations tends to increase over time and further amplifies the fluctuations in other parts of the system. This results in a positive feedback mechanism leading to many interactions.

The existence of particular processes such as solar flares, coronal mass ejections, etc removes the inactive/dormant material from the environment of the stellar structure. However, there exists a continuous supply of matter from the neighborhood against the dormant material being removed by various processes. A stellar system, therefore, exhibits the characteristics of an open system providing a continuous flow of mass and energy (Artymowicz & Lubow 1996). Under a careful choice of the established rates of transformation and magnitude of localized fluctuations, the non-linear structure transforms until it reaches equilibrium or a non-equilibrium state with a fair degree of stability or instability in the system (Cugliandolo 2013). The existence of the former governs the SFR of the structure, in terms of whether the processes guiding the star formation would be inexhaustive (Limit Cycle), or exhaust after some period of time (Stationary State). Analysis of data received from various observational sources suggests that processes such as supernovae explosions, magnetic fields and angular momentum can also influence the SFP (Bodifee & Loore 1985). Various other parameters such as density of the cloud structure, and the intensity of supernova shockwave, etc. also affect the characteristics of the system (Das et al. 2020).

Various authors have done works to formulate these mechanisms via mathematical modeling. Authors such as Das et al. (2020), Debsarma et al. (2016), De Boer et al. (2012), and Telles & Melnick (2018) proposed a model to study the star formation system by considering a system with episodic star formation conditions while Buonanno et al. (1999) proposed a system establishing the existence of a discrete episodic star formation system. Cameron & Truran (1977) and Slavin et al. (2017) considered star formation in the case of triggering star formation by external media. Other researchers, such as Aluzas (2014), Scowen et al. (2010), and Elmegreen (1987), have studied the effect of variable density of supernovae shockwaves on amplification or reduction of the variable processes in a star formation system and also its effects on the whole system.

Modeling of the phenomenon of star formation has been considered by Bodifee (1986), and further modified by Sharaf et al. (2012) and some particular systems have been studied in these works. Also, Ikeuchi et al. (1984), Kamaya & Hirashita (2000), Debsarma et al. (2016), and Das et al. (2020) have proposed some systems which analyze the limit cycle and stationary cycle processes of a star-forming structure. Motivated by the works done in literature by Ikeuchi et al. (1984), Inoue et al. (2001), and Das et al. (2020), the study of this system is focused on how varying density-dependent processes affect the SFRs in different classes of stellar structures.

This research article proposes a new mathematical model taking into consideration some of the physical phenomenon which are significant but have yet not been addressed in existing models.

1 Physical and Mathematical Model Formulation

1.1 Physical Formulation

Throughout the study, it has been assumed that the three components, namely atomic, molecular, and stellar material, are locally spaced, and distributed randomly throughout the ISM. The temperature of the atomic gas within the interstellar environment ranges from 30 to 80 K. Also, it has been assumed that there is a continuous supply of gas from the interstellar environment and at the same time, there is an active removal of the inactive material from the stellar structure by various processes (Adams & Myers 2001). When the system, particularly the atomic component interacts with external perturbations (like supernovae shockwave), or stellar remnants from an active star in the system (solar flares, coronal mass ejections, etc), the momentum of the incoming perturbation is transferred to that of the atomic clouds. This results in the generation of local density fluctuation inside the structure which tends to increase with time. Due to the increased fluctuations, huge clumps of gases begin to form, ranging from the sizes of a house to that of mountains and this tends to increase the gravitational potential of the system. Also, there is a corresponding increase in the temperature (Mestel 1972; Hennebelle 2012) of the system due to the increased collision of atomic components leading to the generation of more heat energy. Now, due to high temperature and the increased gravitational potential which tends to bind the atomic components together, the atomic components begin to fuse to make molecules and generate thermal energy. The released thermal energy is trapped inside the system due to gravitational bound components leading to a cyclic process of production of more and more molecules providing a positive feedback mechanism. The atomic, as well as molecular components, lie at the boundary of the structure providing a thick cloud-like structure. It acts as a shield for the stellar structure and preserves the trapped heat and local density fluctuations, regulating the atomic-molecular conversion rate by providing a continuous supply of the same from the interstellar space environment (Bodifee 1986). Due to the individual collisions between the molecular components, more thermal energy is generated which further leads to the generation of stellar components in the structure. The early generation of main sequence stars consisting of stellar material release solar flares, coronal mass ejections (CME) which ionize the molecular material present in the system. The ionized material is later converted into stellar material due to internal perturbations (Hennebelle 2012; Comeron et al. 2005; Dale et al. 2007). However, the stellar material released in the form of solar flares and CME's by the young stars is converted into the primary atomic state due to loss of heat to the medium.

When these young stars exhaust their supply of energy through continuous nuclear fusion, the outer parts of the stellar structure expand after some time.

The outer clouds provide a new supply of atomic material to the interstellar structure. In some cases, the stars after exhausting their fuel die releasing shockwaves in the space through supernovae and certain ejecta. Recently, brief studies have been conducted on how supernovae shockwave and ejecta released from a dying star affect a star-forming region (Scowen et al. 2010; Elmegreen 1997). Sarson et al. (2004) performed an independent work-study performed through numerical simulations using a multi-phase ISM for a 3-dimensional magneto-hydrodynamic model, consisting of movements driven by supernova explosions and showed a power law relation between the SFR and density. The latter result enabled us to relate how the variable random disturbances driven in the cloud structure guide the SFR of the galaxy (Slavin et al. 2017).

When these shockwaves encounter a star-forming body, some particular amount of gas is dissipated depending on the magnitude of the shockwave and gravitational potential of the star-forming body (Das et al. 2020; Debsarma et al. 2016; Katz 1992). Therefore, the temperature of the cloud increases due to the presence of frictional forces, generating more heat and thereby affecting the chemical structure of the gases (Matteucci et al. 2009). Due to this, some amount of molecular material converts into its basic form i.e. atomic component. However, depending on the gravitational potential or magnetic properties of the structure, the rate of dissipated gas varies from system to system. Zhang & Chevalier (2019) performed a 3-dimensional hydrodynamical simulation-based modelling on the various interactions between a supernova remnant and a turbulent molecular cloud medium. It was discovered that in a medium surrounding the interstellar cloud, various properties of supernovae remnants are governed by the density of the ISM. However, when the wave enters the turbulent medium with comparable higher intensity, the mean temperature, radial momentum, and X-ray emission are found to be less as compared in the case of low turbulence with the same density. These shockwaves when manifested into the system can induce star formation in the cloud (Assousa et al. 1977; McCray & Kafatos 1987).

1.2 Mathematical Formulation

In this subsection, a star formation model has been formulated assuming three active components of the ISM namely Atomic(A), Molecular(M), and Stellar(S) components. The masses of various fraction is in sun mass $M_{\odot}$; a sun mass (mass in astronomy) is approximately $2 \times 10^{30} kg$. In addition to the aspects that have already been studied in the available models in the literature, the proposed model considers the following important processes:

(i) Prevalence of atomic gas in molecular gas;
(ii) Evaporation of molecular gas embedded in stellar material;
(iii) Cooling of stellar material into atomic material;
(iv) Dissipation of molecular and atomic material due to supernova shock waves driven out by high temperature and pressure;
(v) Conversion of molecular material into atomic material due to the dissipation of gases.

A cluster of stars is the collapse of a cloud. On the H-R diagram, stars with different masses appear at different places. The different masses of stars form according to distinct evolutionary paths on the Hertzsprung-Russell (H-R) diagram (Airey & Eriksson 2019). The life cycle of stars is based on their mass; intermediate-mass (sun), high-mass and low-mass star having mass of four times of the sun mass, eight times of sun mass and one-twentieth that of the sun, respectively. A failed star or brown dwarf is a low-mass system that lacks the gravitational attraction necessary to start a nuclear reaction. The timescale for star formation is of 10^7 years.

Motivated by Bodifee & Loore (1985), Sharaf et al. (2012), Slavin et al. (2017), we now formulate a mathematical model taking into account the different phenomenon as discussed above: Let k_1 represents the magnitude of a gravitational force constant for a star-forming body, k_2 represents the rate of conversion of stellar material into atomic material and k_3 to be the rate of conversion of atomic material to molecular material. The parameters c and d represent the efficiency ratio of atomic and molecular cooling. Then the rate of change of atomic material can be mathematically represented as:

$$\frac{dA}{dt} = (k_1)M + (k_2)S - (k_3)A^{n_2}(M + cA)^{n_1}\,, \tag{1}$$

where $(k_1)M$ represents the inflow of dissipated material from supernovae shockwave due to the gravitational potential of the system, $(k_2)S$ represents the given volume of stellar material converted into atomic material due to various processes, and $[(k_3)(M + cA)^{n1}A^{n2}]$ represents the outflow of atomic material due to its conversion to molecular material. Here, n_1 and n_2 represent the parameters guiding the strength of conversion processes in the system.

Let k_4 represents the magnitude of force transferred by the front of supernovae shockwave to the cloud, and k_5 be the rate of conversion of molecular to stellar material which is precisely a parameter to monitor the triggered process. Now, the rate of change of molecular material with respect to time is given by:

$$\frac{dM}{dt} = (k_3)A^{n_2}(M + cA)^{n_1} - (k_4)(M + dA)^{n_3} - (k_5)S^2M\,. \tag{2}$$

Here, the first term $[(k_3)(M + cA)^{n_1}A^{n_2}]$ on the right-hand side of the above equation is due to the conversion of atomic material to molecular material, $(k_4)(M + dA)^{n_3}$ represents the outflow of molecular material from the system due to supernovae shockwave and $[(k_5)S^2M]$ represents the conversion of molecular material to stellar material. Here, n_3 is the parameter representing the strength of supernova shockwave.

Finally, the rate of change of stellar material is due to the dynamics of outflow from and the inflow into respective compartments given by:

$$\frac{dS}{dt} = (k_5)S^2M - (k_2)S\,. \tag{3}$$

In addition to the three compartments discussed above, the proposed model introduces a new compartment R, representing the variable amount of dissipated gas that has exited the system due to the supernovae shockwave. However, some amount of dissipated gas may return into the stellar body depending on the gravitational potential. Then, the rate of change of the dissipated gas R compartment with respect to time is given by:

$$\frac{dR}{dt} = (k_4)(M + dA)^{n_3} - (k_1)M \,. \tag{4}$$

Here $(k_4)(M + dA)^{n_3}$ represents the inflow of dissipated gas into the system due to the supernovae shockwave and the last term represents the corresponding dissipated gas that enters back into the system due to the gravitational potential of the star-forming body.

This mathematical model for the star-forming region considers the feedback mechanism. Throughout, it will be assumed that the SFR is proportional to the amount of molecular material present in the ISM. With a careful choice of parameters mentioned above, the establishment of an equilibrium state can be seen representing a corresponding structure, having an environment suitable for stellar processes. There also exists an interdependence between the proposed constraint values. The primary objective of this research is to study and analyze the non-linear processes which stimulate the final developmental regime of a star formation system, namely :

(i) Evolution towards a Stationary State: In this case, the system begins with some periodic oscillations which then get damped afterwards until the system reaches a constant value.

(ii) Evolution towards a Limit Cycle: In this case, the system rather than converging to a finite value enters into an oscillating stable state.

2 Graphical Analysis

By varying the value of one constraint in each system, while keeping other constraints fixed, a critical analysis of the components has been conducted through a 2-dimensional graphical analysis. This has been done to gain better understanding about the environment of such a stellar structure under different conditions.

2.1 System 1

Consider a stellar structure consisting of thick dense clouds of atomic and molecular components ($n_1 = n_2 = 2$) with a suitable rate of conversion between each of the components ($k_2 = 1, k_3 = 10, k_5 = 15$). The values of c and d are taken to be 0.01 and 0.005. In this system, the intensity of the perturbation, that is, the supernovae shockwave has been varied for systems of both high-mass and low-mass stars with the value of k_4 being suitably fixed as 17 and k_1 is taken to be 11.

For Low-Mass Stellar Body [$a_0 = 0.25, m_0 = 0.25, s_0 = 0.18$]

By varying the intensity of a supernovae shockwave (n_3) to 2, 3 and 4 in smaller stars, it is implied by the period and amplitude of the oscillations that the SFR of the system is same with time in (d), (e), (f) of Figures 1, 2, 3. This suggests that the SFR in the smaller stars, where there exists a lower density of the gases, remains almost the same against the higher-order intensity of the supernova shockwave (Debsarma et al. 2016). This can also be seen in Vilchez

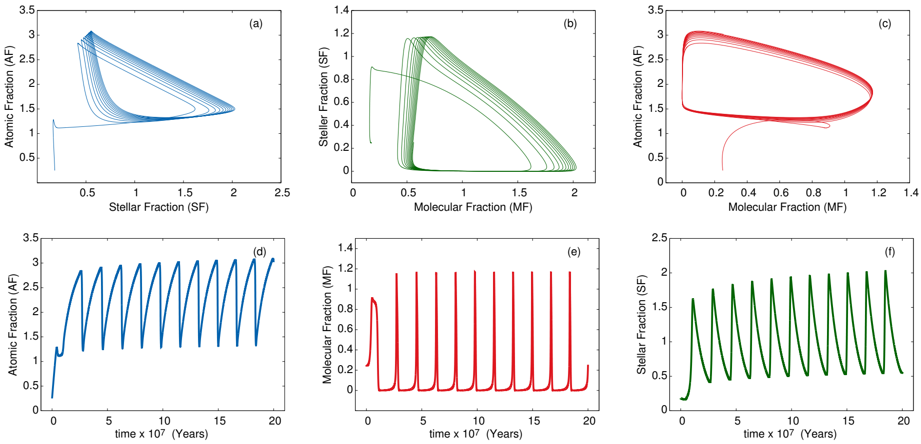

Fig. 1. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/ sun mass $M_{\odot}$) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time (10^7years) at $n_3 = 2$ (Low-Mass).*

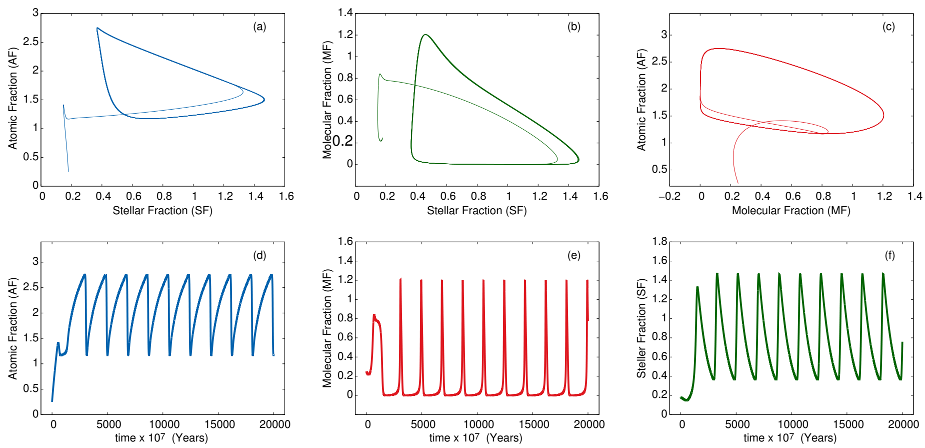

Fig. 2. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/ sun mass $M_{\odot}$) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time (10^7years) at $n_3 = 3$ (Low-Mass).*

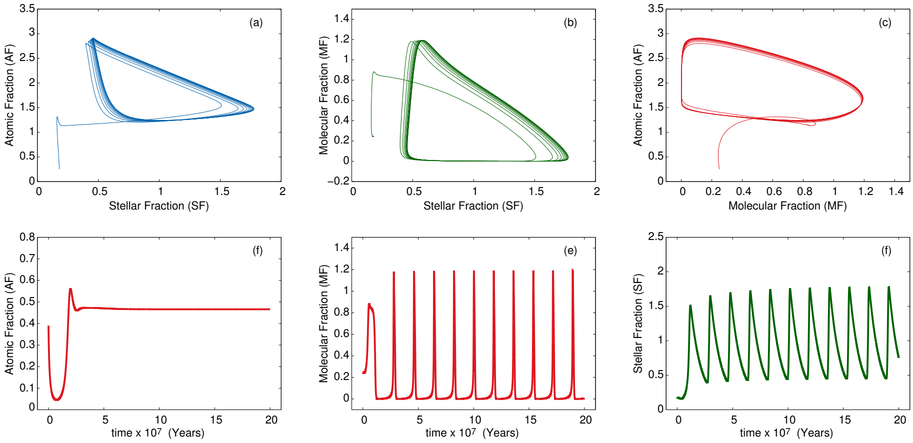

Fig. 3. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/sun mass $M_\odot$) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time (10^7years) at $n_3 = 4$ (Low-Mass).*

(1995), and Iglesias-Paramo & Vilchez (1997), where it has been suggested that in a stellar environment consisting of lower density of gases, the SFR is very high. This may be implied by the fact that due to the lower density of the system, the increased amount of heat transfer by the higher-order supernovae shockwave does not propagate efficiently throughout the system owing to the relatively larger distances between the components of the ISM. Therefore, the rate of local density fluctuations increases in the short term but does not produce any notable change in its behavior in the long term. Also, in graphs (a), (b), and (c) of Figures 1, 2, 3, a limit cycle behavior can be observed such that the combined interactions of the system cause the SFR to remain the same.

<u>For High-Mass Stellar Body</u> $[a_0 = 0.40, m_0 = 0.39, s_0 = 0.36]$

For a high-mass star system, when the density of the supernovae shock wave is low, there exists a correspondingly lower density of the gaseous components in the system. Analysis of the oscillatory cycle suggested that the SFR is very high for this system which was also implied by the spectroscopy analysis of Vilchez (1995) and Iglesias-Paramo & Vilchez (1997). The latter would also be confirmed in the second system, where the density of the system increases, showing lower star-forming activity with a higher density of gaseous components due to a high-intensity supernovae shockwave.

By subjecting the system to a supernovae shockwave of intensity 1.5, 3, and 4 in high-mass stars, the period and amplitude of the oscillation in each of the components is seen to vary significantlty with time. The amplitude was observed to be low for $n_3 = 1.5$ in (d), (e), (f) of Figure 4, which increased for $n_3 = 3$ in (d), (e), (f) of Figure 5 and even further increased for $n_3 = 4$ in (d), (e), (f) of Figure 6. This suggests that the volume of atomic, molecular, and stellar components produced is low for $n_3 = 1.5$ and the system also adopts

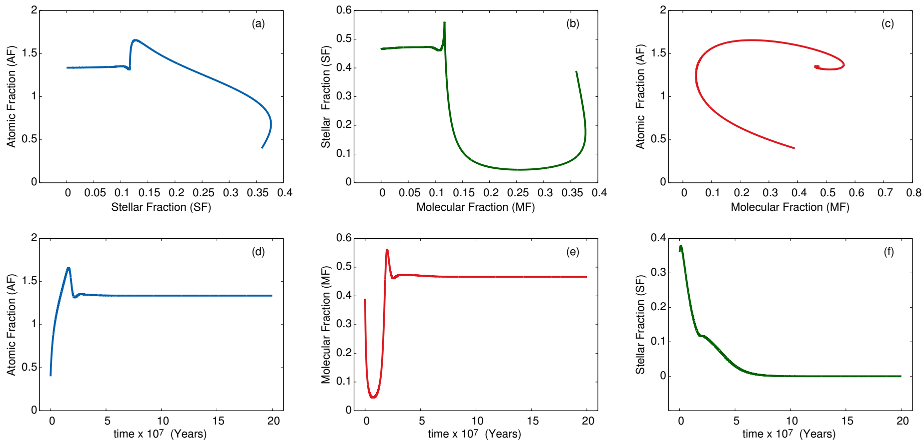

Fig. 4. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/ sun mass $M_\odot$) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time (10^7years) at $n_3 = 1.5$ (High-Mass).*

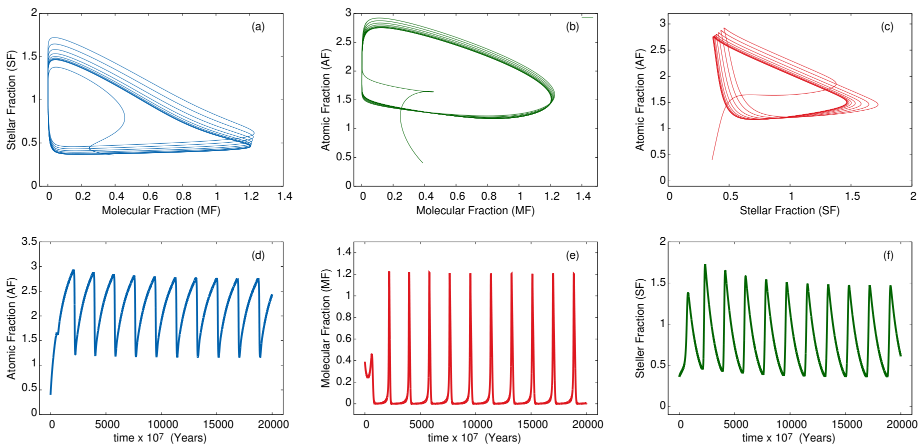

Fig. 5. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/ sun mass $M_\odot$) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time (10^7years) with $n_3 = 3$ (High-Mass).*

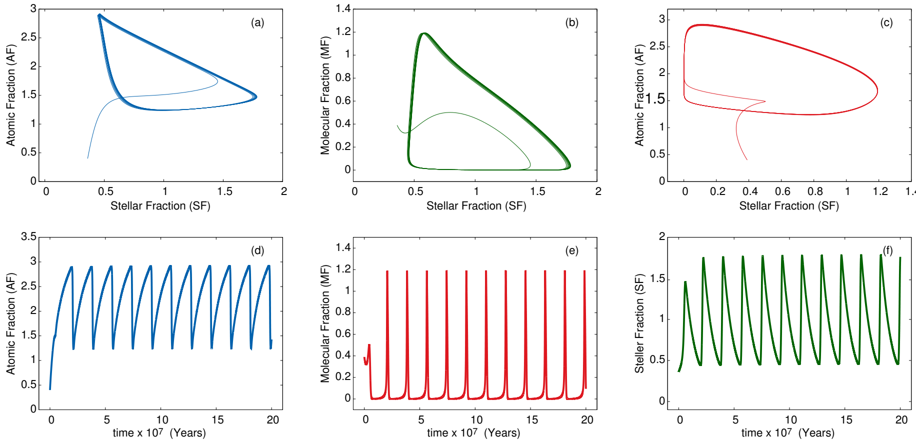

Fig. 6. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/ sun mass* $M_\odot$*) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time* (10^7*years) with* $n_3 = 4$ *(High-Mass).*

a stationary state. For $n_3 = 3$, the volume of each of the components is more than that of the previous case, however, the volume remains the same for $n_3 = 4$. The system adopts an established limit cycle state for $n_3 = 3$, 4 and an established stationary state for $n_3 = 1.5$. This suggests that when subjected to increasingly higher-order perturbations, the momentum transferred by the supernovae shockwave to a cloud produces local density fluctuations such that the system first has a stationary state, then a limit cycle which is thereafter followed again by a limit cycle in (a), (b), (c) of Figures 4, 5 and 6. Therefore, the SFR of a high-mass star varies for higher-order perturbations. This may be due to an increased density of interstellar components resulting in a relatively lower separation distance between the individual components. Due to this, the local density fluctuations are now more efficiently distributed throughout the system in this compact structure. Therefore, the rate of fluctuations varies in the long term and it gives rise to different types of behavior as compared to the case of smaller stars.

2.2 System 2

Consider a stellar structure consisting of clouds of atomic and molecular components with a suitable rate of conversion between each of the components ($k_2 = 3$, $k_3 = 9$, $k_5 = 13$). The values of c and d are taken as 0.02 and 0.003. The intensity of the perturbation, that is, the supernovae shockwave (n_3) is taken as 3, and the value of k_4 is taken as 17. The value of k_1 is taken to be 12 and the gravitational potential of the system corresponding to the initial masses a_0, m_0, and s_0 are respectively taken as 0.45, 0.35, and 0.18. In this system, the density of the atomic (n_1) and molecular cloud (n_2) have been varied together from 1 to 2 for a given set of constraints to see how a change in the density of the components affects the SFR.

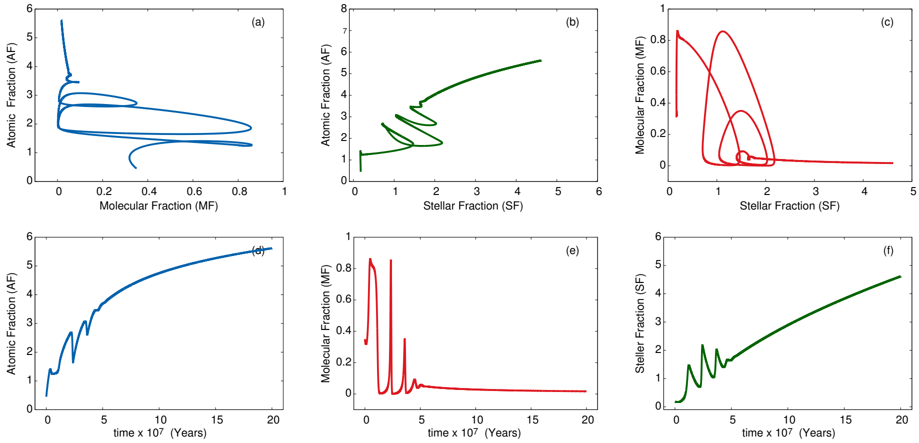

Fig. 7. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/ sun mass $M_\odot$) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time (10^7years) with $n_1 = n_2 = 1$.*

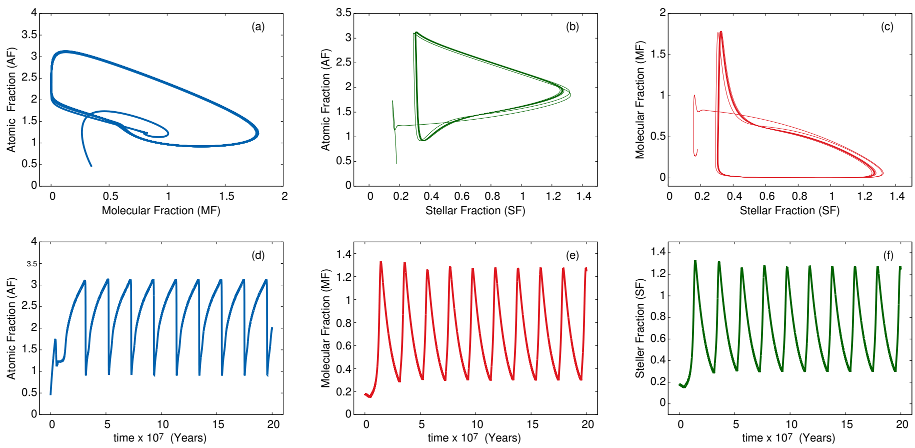

Fig. 8. *Phase diagram between (a) atomic fractions v/s stellar fractions, (b) stellar fractions v/s molecular fractions, (c) atomic fractions v/s molecular fraction (masses are in solar/ sun mass $M_\odot$) of star formation model, and (d) atomic fractions, (e) molecular fractions, and (f) stellar fractions as functions of time (10^7years) at $n_3 = 2$ (Low-Mass) with $n_1 = n_2 = 2$.*

By varying the intensity of molecular and atomic cloud, the individual components of the body are seen to evolve from an established stationary state in (a), (b), (c) of Figure 7 to a limit cycle state in (a), (b), (c) of Figure 8. The increased density of the clouds preserves the rate of local density fluctuations, or perturbations inside the system more efficiently because of the existence of a thicker cloud at the boundary of the system. The newly formed dense structure shields the system more efficiently against the perturbations of the interstellar environment. This results in an increased production of atomic, molecular, and stellar components, increasing the mass per density volume of the structure. This is also implied by the decreasing amplitude of oscillations in (d), (e), (f) of Figure 7, while there exists oscillations of equal amplitude in (d), (e), (f) of Figure 8 due to the increased production of the interstellar components. Therefore, the rate of local density fluctuations also increases, providing a positive feedback mechanism. This happens until an equilibrium is achieved between the local density fluctuations and the mass per density volume such that the system doesn't achieve a stationary state but a limit cycle is obtained.

2.3 System 3

Consider a stellar structure with a variable density of atomic and molecular cloud having a suitable rate of conversion between the components ($k_2 = 1, k_3 = 8, k_5 = 19$). The intensity of supernovae shockwave (n_3) is taken as 2 with the value of k_4 to be 18 which indicates the existence of a high-intensity supernovae shockwave. The value of c and d have been taken as 0.02 and 0.005 respectively here. The density of both atomic and molecular clouds has been varied for both high-mass and low-mass systems to study and evaluate how the dissipation of gases varies across different systems. Varying values of these parameters in SFM, we observe the following behavior:

Higher-Mass System Initial masses a_0, m_0, and s_0 of the interstellar components are chosen to be 0.35, 0.38, and 0.30, respectively, such that the system has characteristics of a high-mass stellar structure. Therefore, there exists a higher gravitational potential of the system, and keeping in mind the strong correlation between k_1 and the values of masses; k_1 is taken as 12.

In Figure 9(a), where the value of n_1 and n_2 are taken as 1, the existence of an atomic and molecular cloud with low density can be observed. The graph appears in the 4^{th} quadrant where it is steadily decreasing. This implies that the rate at which dissipated gas is being pulled back into the system due to the gravitational pull of the high-mass system is more than the rate at which dissipated gas appears exiting the system due to the supernovae shockwave. The graph decreases continuously suggesting that all the dissipated gas due to supernovae shockwave is pulled back into the system.

In Figure 9(a), where the value of n_1 and n_2 are taken as 2, the existence of an atomic and molecular cloud with increased density can be observed. The graph clearly indicates that it first decreases at a very high rate and finally achieves a stationary state after a slight oscillation. This implies that the rate at which dissipated gas is pulled back into the system is greater than that the rate at which gas exits the system. The occurence of oscillations indicate

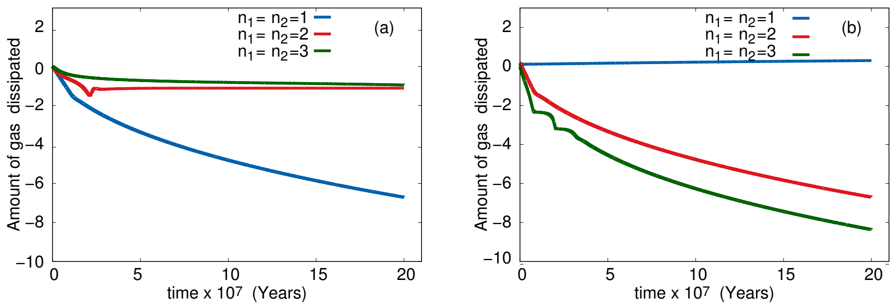

Fig. 9. *The variation of amount of gases dissipated from the system and time. (a) n_1, n_2 for High-Mass system (atomic fraction=0.35, molecular fraction=0.38, stellar fraction=0.30), and (b) n_1, n_2 for Low-Mass system (atomic fraction=0.15, molecular fraction=0.18, stellar fraction=0.16).*

that the rate at which gas exits the system increases instantaneously and then decreases. The graph tends to oscillate until the system achieves a stationary state.

In Figure 9(a) again, where the value of n_1 and n_2 are both taken as 3, a thicker and denser molecular and atomic cloud structure is observed. The graph of the dissipated gas in the 4^{th} quadrant decreases at a very high rate and then achieves a stationary state, converging to a particular value for the same reasons as mentioned for the case with $n_1 = n_2 = 1$.

Therefore, in a high-mass star-forming system, when the density of the atomic and molecular cloud is increased for a given set of conditions, all the dissipated gas which exits the system due to the supernovae shockwave comes back to the system in all cases because of the strong gravitational pull of the system on the gas particles. Also, as can be seen, the graph for $n_1 = n_2 = 3$ converges to a stationary state faster as compared to the case with $n_1 = n_2 = 1$. This suggests that as the density of the clouds increases, the rate of dissipation of gas from the stellar structure increases. So, the increased rate of dissipation in the high-mass systems causes the graph for $n_1 = n_2 = 3$ to achieve a stationary state faster in the negative axis than the case for $n_1 = n_2 = 1$.

Lower-Mass system The initial masses a_0, m_0, and s_0 of the interstellar components are chosen to be 0.15, 0.18, and 0.16, respectively, so that the system has a lower gravitational potential. Since the given system has a lower gravitational potential, the value of k_1 is taken to be 7, ensuring a strong correlation between k_1 and the value of masses.

In Figure 9(b), where the value of n_1 and n_2 is taken as 1, the existence of an atomic and molecular cloud with low density is observed. The graph of the dissipated gas versus time, lies entirely in the first quadrant. The graph is first increasing and then achieves a stationary state, converging to a particular value. This suggests that the rate at which gas is dissipated from the system is more than the rate at which gas is being pulled back into the system. This may be implied by the lower-mass of the system resulting in a lower gravitational pull of the system on the individual dissipated gas particles.

In Figure 9(b), it can be seen that when the value of n_1 and n_2 is taken as 2, indicating the existence of an atomic and molecular cloud with a thicker density is observed. Initially, the graph of the dissipated gas lies in the first quadrant where it first increases and then decreases, approaching the 4^{th} quadrant where it converges to a stationary state. This suggests that in the increased density of the molecular and atomic cloud, the relative amount of dissipated gas exiting the system first increased, then the amount started decreasing to a point where all the individual dissipated gases entered the system again. The given behavior of the system can be seen until the system approaches a given value.

Again from Figure 9(b), it can be seen that when the value of n_1 and n_2 are both taken as 3, the existence of an atomic and molecular cloud with a thicker density can be observed. The graph of the system first decreases, intersects the *x-axis*, and then increases. This suggests that in this system due to higher density, the rate of dissipation first decreases, and then starts increasing. This may be implied due to the lower gravitational potential of the system where the dwarf system cannot hold the dense structure and the gas tends to dissipate in case of supernova perturbations. This implies that in a lower-mass stellar structure with dense clouds of atomic and molecular gases, the rate of molecular production is higher compared to a case of lower density. Therefore, the SFR of such a system tends to be higher as compared to the case of a lower density system.

3 Conclusion

In this research work, an episodic model of star formation has been proposed to study the dissipative stellar structure in both high-mass and low-mass stellar systems. The study has been executed based on an analysis of the transition rates representing various physical processes guiding the star formation scenario in galaxies. These physical processes include transitions of gases between individual components, the outflow of gas due to supernovae explosion, interstellar gas ionized by the neighborhood stellar sources, tidal forces exciting the interstellar components, thereby influencing the star formation by processes due to the interaction with neighborhood galaxies. Taking into account the available mathematical models on star formation and the specific processes mentioned above, we propose a new model for the said process where the dissipated gas is also taken into consideration. This gas exists as a result of combination of interactive forces between the density-dependent supernova shockwave and gravitational potential of the system. The present study features the following findings:

(i) The SFR of low-mass stellar systems, against both, lower and higher-order density-dependent perturbations of supernovae shockwave, was found to remain the same. However, the rate varies for high-mass stellar structures for different densities, suggesting that SFR is not uniform in such systems.

(ii) The SFR for low densities of atomic and molecular mass for a given star-forming system is low as compared to higher densities of such gases where there exists a limit cycle rather than a stationary state.

(iii) In a high-mass star-forming system, the dissipated gas due to supernovae shockwaves enters back into the system due to the high gravitational potential of the system in all cases. However, in the case of low-mass systems, the rate of dissipation of the gases varies for different systems. This suggests that in a low-mass stellar system, the rate of outflow of gas due to supernovae shockwave and inflow of gas due to gravitational potential of the system greatly varies as compared to a high-mass system.

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