Bulgarian Academy of Sciences. Българска академия на науките

Astrophysical Investigations, 6. Астрофизически изследвания, 6
Sofia. 1991. София

Influence of differential rotation on the degree of axysymmetry of magnetic fields of stellar objects

K.-H. Rädler

Key words: differential rotation, stellar magnetic fields, dynamomechanism.

Address: Zentralinstitut für Astrophysik der AdW der DDR, Rosa-Luxemburg-Str. 17a, Potsdam-Babelsberg, DDR-1591

The behaviour of magnetic fields which penetrate a body of electrically conducting fluid with differential rotation strongly depends on whether or not those fields are symmetric with respect to the axis of rotation. In both cases the effect of differential rotation consists in stretching and winding up the field lines. In the axisymmetric case the resulting field pattern is rather simple. The poloidal part of a field, as well as a purely toroidal field, remain unaffected by differential rotation; the only effect of differential rotation is that, if a poloidal part of the field exists, a toroidal part is generated. In the non-axisymmetric case, however, a field pattern is produced in which, as a rule, for strong differential rotation differently oriented field lines come close together. This, in turn, leads to enhanced dissipation.

In order to examplify these facts and to reach some conclusions of astrophysical interest some detailed investigations have been carried out (Rädler, 1981; 1986b). We consider here a spherical fluid body with radius R and magnetic diffusivity η. The differential rotation is supposed to be concentrated in a shear layer of the thickness L. Its influence on the magnetic fields may be described by a Reynolds number R_ω defined by

$$\mathrm{R}_\omega=\Delta\omega\ L^2/\eta \tag{1}$$

with a characteristic value $\Delta\omega$ of the change of the angular velocity across the shear layer. Any back-reaction of the magnetic field on the differential rotation is ignored.

We first deal with the decay of magnetic fields that initially have a characteristic length scale of the order of the radius R and that are poloidal.

In the axisymmetric case the length scale is not changed by differential rotation, and the decay of the field can be characterized by the time T_d given by

$$T_\mathrm{d}\approx R^2/\eta. \tag{2}$$

In the non-axisymmetric case the differential rotation produces a field pattern with smaller length scales. A simple estimate shows that for $R_\omega >> 1$, the decay of the field inside the shear layer can be characterized by the time t_d which is given by

$$(3) \qquad t_{\mathrm{d}} \approx \frac{2\pi}{\Delta\omega}\left(\frac{\mathrm{R}_\omega}{2\pi}\right)^{1/3} \approx \frac{L^2}{\eta}\left(\frac{\mathrm{R}_\omega}{2\pi}\right)^{-2/3} \ll T_{\mathrm{d}}.$$

After the decay of the field in the shear layer there will be still fields in the rigidly rotating parts of the fluid body. The characteristic times for their decay are then determined by length scales smaller than R, that is, are in any case smaller than T_{d}.

A field containing both axisymmetric and non-axisymmetric parts will, as a consequence of differential rotation, suffer some symmetrization in the course of time.

We secondly consider steady magnetic fields that are due to a mechanism generating poloidal fields in some inner part of the body. In the axisymmetric case the differential rotation produces no effect on the fields at the surface. In the non-axisymmetric case, however, a shear layer enveloping that inner part of the body has a screening effect. It corresponds to a reduction of the electromotive force generating the field by a factor f with

$$(4) \qquad f \approx \left(\frac{L}{R}\right)^2 \left(\frac{\mathrm{R}_\omega}{2\pi}\right)^{-2/3}.$$

When fields with axisymmetric and non-axisymmetric constituents are generated, again a symmetrization effect of differential rotation can be observed. The degree of symmetry of the field at the surface or in the outer space grows with R_ω.

For decaying fields, as well as for steady fields we consider the generation of a toroidal from a poloidal part. In the axisymmetric case the winding-up mechanism, with sufficiently strong shear, permits producing an arbitrarily strong toroidal part from a given poloidal part, i. e., for sufficiently large R_ω the ratio of their magnitudes assumes arbitrarily high values. In the non-axisymmetric case, however, this mechanism competes with an enhancement of dissipation, and the result is that this ratio is bounded. In the examples considered it is even less than unity for all R_ω.

That last finding is of great significance in dynamo theory. Contrary to the α^2-mechanism, the $\alpha\omega$-mechanism requires that, as an effect of differential rotation, the toroidal part of the mean magnetic field is much stronger that the poloidal part, i. e., the above ratio must be much larger than unity. This suggests that the $\alpha\omega$-mechanism, which is able to maintain axisymmetric fields, can hardly work with non-axisymmetric fields. This idea has been confirmed by detailed numerical investigations of spherical dynamo models (Rädler, 1980; 1986a).

The results presented here place remarkable constraints upon models of stellar objects. Generally speaking, the occurence of strongly non-axisymmetric magnetic fields of a body might be a hint that there is no noticeable differential rotation in the relevant layers of this body. In particular, the strongly non-axisymmetric fields observed at some Ap-stars are not quite compatible with a differential rotation in the outer layers of these objects. Furthermore, if one wants to explain these fields as a result of dynamo activity, an α^2-mechanism rather than an $\alpha\omega$-mechanism should be envisaged.

References

Rädler, K.-H. 1980. Mean-Field Approach to Spherical Dynamo Models, Astron. Nachr., **301**, 101-129.

Rädler, K.-H. 1981. A Remark on the Influence of a Differential Rotation of the Earth's Core on the Degree of Axisymmetry of the Magnetic Field. Veröffentl. Zentralinst. Physik d. Erde, Potsdam, Nr. 70, Teil 1, 85-90.

Rädler, K.-H. 1986a. Investigations of Spherical Kinematic Mean-Field Dynamo Models. Astron. Nachr., **307**, 89-113.

Rädler, K.-H. 1986b. On the Effect of Differential Rotation on Axisymmetric and Non-Axisymmetric Magnetic Fields of Cosmical Bodies. Plasma-Astrophysics (proceedings of the Joint Varenna-Abastumani International School and Workshop, Sukhumi, May 1986) ESA SP-251, 659-574.

Received 1. 12. 1987.