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ON THE DETERMINATION OF FUNCTIONS $N(m)$ AND $A(m)$

Venko Dobrichev and Donka Raikova

I. A great number of statistical investigations requires the knowledge of the integral distribution function $N(m)$ and the corresponding differential distribution function $A(m)$. The functions $N(m)$ and $A(m)$ represent the number of stars, brighter than stellar magnitude m and the number of stars of magnitude m respectively, which numbers are reduced to unit solid angle or to a square degree. These functions are connected by the well-known expression

$$N(m)=\int_{-\infty}^{m} A(m)dm.$$

Since in practice $N(m)$ and $A(m)$ are being determined for discrete value of m only, then the relationship between them is given by the equations

$$N(m_k)=\sum_{i=1}^{k} A(m_i), \tag{1}$$

$$A(m)=N\left(m+\frac{1}{2}\right)-N\left(m-\frac{1}{2}\right). \tag{1'}$$

Conventional methods for the determination of $N(m)$ and $A(m)$ inevitably include operations concerning each separate star. On account of this the detailed statistical investigations of the huge number of faint stars, which are available for modern telescopes, are very restricted. A method for the determination of $N(m)$ and $A(m)$, built on a new principle, was proposed by Butler [1]. This method is based on the change of the light beam passed through two parallel positive photographic plates of one and the same star field by displacement of the one plate towards the other in its own plane. The first variant of Butler's method leads to the solution of an integral equation and the second one is direct, but requiring very much labour.

II. In the present paper we consider in principle the possibility of determining functions $N(m)$, respectively $A(m)$, by using the television scanning technique and the results, which have already been attained in construction of special television apparatus for similar aims.

It is known in industry, biology, medicine and other branches of science, that the problem of speedy counting of microscopic particles has to a great degree been solved by using different methods of scanning [2-12]. The counting of equal particles is carried out comparatively easily, because they produce an equal number of pulses when scanned. Then the number of particles may be calculated directly from the total number of registered pulses. Methods of counting different in size particles have been developed also [3, 11]. The method proposed by Roberts and Young [2, 3] is of special interest. By this method the scanning is carried out simultaneously by two spots, formed by splitting the scanning spot of the cathode-ray tube with a birefringeant crystal. According to [2] the accuracy of this method is of the order of 1 per cent.

The star counting is usually carried out from a photographic plate. Since the diameters of the stellar images on the plates are a function of the magnitude, then the determination of $N(m)$ and $A(m)$ is reduced to differential counting of stellar images into different diameter ranges. Our idea consists in this: to use the television counting system for differential counting of stellar images, as the stars, fainter than a fixed magnitude, are consecutively eliminated.

To determine functions $N(m)$ and $A(m)$ it is possible to proceed by two methods.

1. A television counting system of the kind of [3, 11], which directly registers the number of the stellar images on the whole plate or a part of it may be used. In this case one obtains the number of stars up to the limiting magnitude m_k. This number, reduced to 1 square degree, gives $N(m_k)$. In order to determine $N(m_{k-1})$, $N(m_{k-2}), \ldots, N(m_1)$ respectively, it is necessary to eliminate the smaller stellar images on the plates consecutively by some method. This elimination may be attained as one works with two touching one another positive plates (transparent stellar images on the black background) of one and the same star field. In the position of correspondence, in which the images of each separate star on the two plates coincide, $N(m_k)$ is obtained. By consecutive displacements of the one towards the other plate in its own plane the black background covers the images of the brighter and brighter stars [1]. If the displacements are to distances $d_k, d_{k-1}, \ldots, d_2$, which are equal to the diameters of stellar images, corresponding to magnitudes m_k, $m_{k-1}, \ldots, m_2$, then $N(m_{k-1})$, $N(m_{k-2})$ etc. by scanning are obtained. The function $N(m)$ is determined by this method and on the basis of (1′) $A(m)$ is respectively determined.

2. To determine $N(m)$ and $A(m)$ one may also use a television counting system with progressive scanning, which registers all pulses, obtained by the multiple intercepts of the stellar images by the scanning spot. In this case one must also work with two plates by displacement of the first towards the second.

If σ is the distance between the scan lines, then in the position of correspondence each star of magnitude m_i will produce $\frac{d_i}{\sigma}$ pulses. In order to avoid the introduction of new signs, let us assume, that the scan covers 1 square degree. Then all stars of magnitude m_i will produce a total of $\frac{d^i}{\sigma}A(m_i)$ pulses. By displacement of the first plate towards the second at a distance x

in direction perpendicular to the scan lines, each star will produce x/σ pulses less than in the position of correspondence. Obviously, the stellar images with diameter $d<x$ will be completely covered by the black background and will produce no pulses (we assume, that there are no overlaps of different images). If in the position of correspondence and by the consecutive displacements at distances $d_k, d_{k-1}, \ldots, d_2$ towards this position, the total number of pulses is respecively $p_k, p_{k-1}, \ldots, p_1$, then we may put down the following system of equations:

$$\begin{array}{lllll}
\frac{d_1}{\sigma} A(m_1)+\frac{d_2}{\sigma} A(m_2) & +\ldots+\frac{d_{k-1}}{\sigma} A(m_{k-1}) & +\frac{d_k}{\sigma} A(m_k) & =p_k, \\
\frac{d_1-d_k}{\sigma} A(m_1)+\frac{d_2-d_k}{\sigma} A(m_2)+ & \ldots+\frac{d_{k-1}-d_k}{\sigma} A(m_{k-1})+ & 0 & =p_{k-1}, \\
\ldots & \ldots & \ldots & \ldots \\
\frac{d_1-d_2}{\sigma} A(m_1)+\quad 0 & +\ldots+\quad 0 & +\quad 0 & =p_1.
\end{array}$$

This system may easily be solved because the coefficients have a specific character. By subtraction of each equation from the preceding one and with the help of (1) we obtain

$$\begin{aligned}
N(m_k) &= \frac{\sigma}{d_k}(p_k-p_{k-1}), \\
N(m_{k-1}) &= \frac{\sigma}{d_{k-1}-d_k}(p_{k-1}-p_{k-2}), \\
&\ldots\ldots\ldots\ldots \\
N(m_2) &= \frac{\sigma}{d_2-d_3}(p_2-p_1), \\
N(m_1) &= \frac{\sigma}{d_1-d_2}\, p_1.
\end{aligned} \tag{2}$$

So the values of $N(m)$ are found directly from the number of the registered pulses, as the displacements of the first plate towards the second are known. From the values of $N(m)$ after (1′) may be obtained the values of $A(m)$ also.

III. It is clear from the above, that by means of the two variants of our proposed method $N(m)$ and $A(m)$ may be obtained without solving any integral equation or even a usual system, as distinguished from Butler's method. Moreover $N(m)$ and $A(m)$ are determined incomparably more speedily than by conventional methods.

The following more important factors influence the accuracy of our method:

1. The overlap of images of different stars by the displacement of the first plate towards the second introduces some error. This error is negligible for the high galactic latitudes or in the obscured regions of the Milky Way, because the distances between the separate images are larger than the diameters of images even of the brightest stars. That is why the overlap is an exceptionally rare case. The presence of very bright stars in the field requires a large displacement of the one plate towards the other and the probability

of overlap increases. But in as much as the bright stars are few in number, they may be blackened by ink on the plates and counted separately by conventional methods. But near the galactic equator the stellar images on the plates are disposed so closely, that the overlap makes the method inapplicable (especially if the plates are obtained by a short-focus camera or in the red spectral region).

2. The mechanical displacement of the one plate to a certain distance may introduce a considerable error in the determination of $N(m)$ and $A(m)$. This error may be minimised if one works with appropriately enlarged plates. But the great enlargement, favorable to avoid this error and in general to raise the accuracy, on the other hand leads to the decreasing of the scanning raster. That is why it is necessary to accept a reasonable compromise between the two requirements.

3. The most important factor for the accuracy of the method is the large resolving power of the television system.

Only an experimental verification will throw light upon the accuracy of the method proposed in the present paper. But this will be the object of another paper.

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ВЪРХУ ОПРЕДЕЛЯНЕТО НА ФУНКЦИИТЕ $N(m)$ И $A(m)$

В. Добричев и Д. Райкова

(Резюме)

В работата се разглежда принципно възможността за определяне на функциите $N(m)$ и $A(m)$ по фотоплаки, като се използва телевизионно броячно устройство. За последователно елиминиране на изображенията на звездите, по-слаби от дадена звездна величина, се използват две позитивни плаки, като едната се премества спрямо другата на определено разстояние в собствената ѝ равнина. По единия вариант на предлагания метод $N(m)$ се определя директно, а по другия — чрез елементарни пресмятания по формула (2). $A(m)$ се получава по (1′).