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A STATISTICAL VERIFICATION OF A HYPOTHESIS FOR QUASARS

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I. Introduction

The last two years a great number of hypotheses about the nature of quasars (quasi-stellar radio sources) were proposed. Accepting the classification of [1], here we shall also consider all hypotheses dividing them into scintillation and non-scintillation hypotheses. The variations of the light curve are explained by scintillation hypotheses with separate large changes in brightness which changes do not enclose the whole quasar. The scintillation hypothesis may be considered as originating from the ideas of Burbidge [2] and Shklovsky [3] about the supernovae explosions in the nuclei of some radiogalaxies. According to Burbidge the star density in the nuclei of some strong radiogalaxies is very large, $D > 10^5$ st/ps. The explosion of a star triggers the explosions of neighbouring stars and so on as "chain reaction".

The scintillation hypotheses, which are based on the explosions of the stars as supernovae, or analogous to supernovae (let us consider them as supernovaelike stars) are several — i. e. of Colgate and Cameron [4], Woltjer [5], Ulam and Walden [6], Field [7, 8] and some others.

Of course, other scintillation hypotheses can he offered, too, i. e. hypothesis [9]. Similar hypotheses are very particular and at first sight would be classified as non-scintillation ones.

Non-scintillation hypotheses are for example the hypotheses of type gravitational collapse — which have origin in the ideas of Oppenheimer [10—11] (cf. the review of Chiu [12]). Other non-scintillation hypotheses are for matter and antimatter, Terrell's hypothesis [13] — which is a development of Hoffman's idea [14], and others.

Some of the scintillation hypotheses explain very important peculiarity of quasars — e. g. the fluctuations in the light curve. Actually there exist other points of view also. Greenstein [15], Shklovsky [16] and Hoyle [17] (the last in discussion on the occasion of Woltjer's hypothesis [5]) think that the consequent erruptions can't explain the observed light curve. In this paper we will show, that consequent erruptions can explain fluctuations in the observed light curve.

Briefly, our purpose is the verification of the scintillation hypothesis — as if the explosions (of supernovae or supernovaelike) can explain the observed light curve. Unfortunately, more or less photometric observations of quasars (except of 3C273) haven't been made. That is why we should be satisfied with the data concerning variations of the light curve of 3C273 only.

The preliminary note of the present paper is published in [18]. Here we will profit of the occasion and shall make an essential correction in the final results of [18].

II. Method

We consider the hypothesis H_0: the variation in brightness of quasars is due to explosions of stars as supernovaelike. In order to verify the hypothesis H_0 (or the alternative one) we apply a method, based on the following assumptions:

1. The total emission of quasars is produced as superposition of the light curves of separate explosions.
2. The numbers of explosions in equal time intervals are distributed after Poisson.
3. The light curve of a separate explosion is known.

Are these assumptions justified? This question will be discussed in IV. Let us now examine the assumptions as postulates. Our purpose is to construct the model of quasars and to compare its light curve with such of a real object (in the case with 3C273).

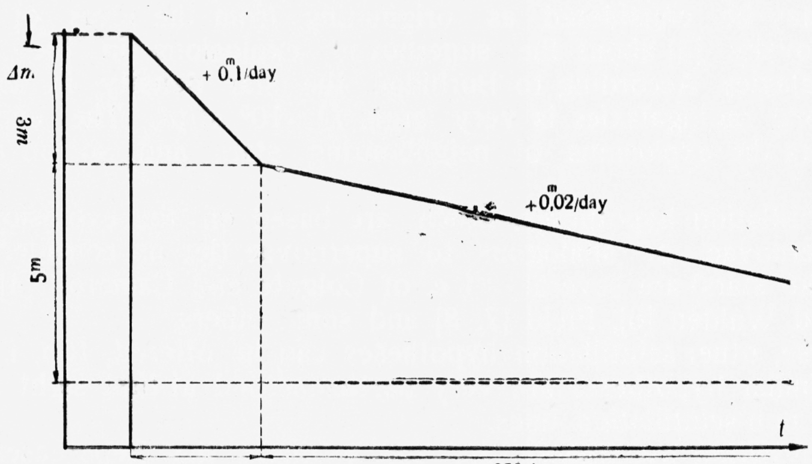

Fig. 1

The second assumption gives us the next: let for time interval Δt exploded an average N supernovae (everywhere under supernovae will be understood supernovaelike), which at the time intervals have Poisson distribution. Then for the determined value of N the corresponding probabilities $p(x)$, with which can be realised in the given time interval $N-k$, $k=1, 2, \ldots, N$, and $N+k$, $k=0, 3, \ldots,$ explosions

$$p(x) = \frac{e^{-N} N^x}{x!}, \quad x = 0, 1, 2, \ldots, \tag{1}$$

can be found easily.

If we accept some standard light curve (model) of supernovae after explosions, can be determined the mean (background) emission of object, the produced energy of which is liberated only by explosions of the supernovae. Therefore, if in the time interval Δt explose not N, but $N \pm k$, $k \neq 0$, stars, can be determined the corresponding change towards the background emission. Thus, by means of the distribution of the deviations from the background brightness for the theoretical model and the deviations of the mean brightness in the real object, can be made a verification of the hypothesis H_0.

For the variation of brightness of the supernovae we accept the light curve of Baade and Zwicky [19], reckoning that the supernovae instantly reaches the maximum brightness. The decreasing in the first 3^m after maximum is $+0^m.1$/day. For the next 5^m the decreasing is $+0^m.02$/day. The schematic curve is given on the Fig. 1. We examine the supernovae to 280th day after the explosions. In general outlines our model is in accordance to the great number of modern observations, i. g. [20, 21].

III. Results

Let us consider the stationary case for N expl/Δt (we will assume everywhere $\Delta t = 1$ day). Then the explosions are uniformly distributed in time. Here we should make some precisement. We will accept the background brightness as given from the points, which connected minimuma and maximuma at the moment before explosions of the following stars. The background brightness is a straight line, which is parallel to the time axis. For that reason there are several considerations. The first one is from the point of view of simplicity (in the contrary case, if the points represent the background brightness there is some probability density by the time axis and the problem becomes complicated). Moreover thus a widest range in changes of brightness is embraced and this is not insignificant.

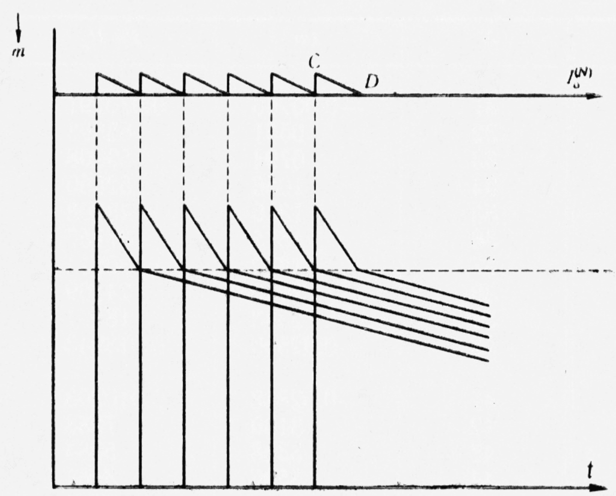

Fig. 2

In the stationary case (Fig. 2) the background brightness $I_0^{(N)}$, which is superposition of the brightness of all exploded supernovae to this moment is given with

$$I_0^{(N)} = A\left(\sum_{\nu=1}^{30N} \varrho^{-\frac{0.1}{N}\nu} + \varrho^{-3} \sum_{\mu=1}^{250N} \varrho^{-\frac{0.02}{N}\mu}\right), \tag{2}$$

where A is the real brightness of a supernova in maximum, and $\lg \varrho = 0.4$.

We shall verify 7 various hypotheses H_0, for which $N=30$, 10, 5, 3, 1, 0.5 and 0.1 expl/day. Now we already may evaluate the differences in stellar magnitudes between the points C and D (Fig. 2). The results of formulae (2) are given in Table 1. At the moment t_0-dt (before the explosion of the next star) we have a brightness $I_0^{(N)}/A$, and at the moment t_0+dt (in maximum) we have the brightness I. The differences in brightnesses are given in stellar magnitudes $\Delta m=\frac{\lg (I_0^{(N)}/I)}{0.4}$. The last column in Table 1 gives the differences $\Delta m'$ in stellar magnitude between the maximum of supernovae and the maximum points in the background brightness.

Table 1

N expl/day	$\frac{I_0(N)}{A}$	I	Δm	$\Delta m'$
30	406.216	407.216	0.0026	—6,5219
10	135.156	136.156	0.0080	—5,3271
5	67.3115	68.3115	0.0160	—4,5702
3	40.1913	41.1913	0.0267	—4,0105
1	13.0708	14.0708	0.0800	—2,7908
0.5	6.29652	7.29652	0.1600	—1,9978
0.1	0.928515	1.928515	0.7936	+0,0805

Table 2

30 stars/day

N	Δm	$P=P_S$	N	Δm	$P=P_S$
12	+0.04777	0.000104	31	—0.00266	0.070291
13	4512	0240	32	0531	65898
14	4249	0513	33	0794	59908
15	3984	1027	34	1056	52860
16	3720	1925	35	1316	45308
17	3456	3397	36	1575	37757
18	3190	5662	37	1832	30614
19	2925	8941	38	2088	24169
20	+0.02660	0.013411	39	2343	18591
			40	—0.02596	0.013943
21	+0.02394	0.019159			
22	2130	26126	41	—0.02848	0.010203
23	1864	34077	42	3100	7288
24	1598	42596	43	3348	5084
25	1332	51115	44	3596	3467
26	1066	58979	45	3843	2311
27	0800	65532	46	4089	1507
28	0534	70213	47	4331	0962
29	0267	72635	48	4575	0601
30	0.00000	0.072635	49	4816	0368
			50	—0.05057	0.000221
			51	—0.05297	0.000130

If N is given, after we have calculated the background brightness, we can determine the brightness in the cases, if $N\pm k$ stars exploded. We determine the corresponding change in the brightness in stellar magnitudes measured from the background brightness. The results for each studied hypothesis are given in Tables 2—8. The first column gives $N \pm k$, the second — Δm from the background brightness with the corresponding sign, and

Table 3

10 stars/day

N	Δm	P	P_s
0	+0.07921	0.000045	0.000015
1	7136	000454	00153
2	6349	002270	00767
3	5561	007567	02555
4	4770	018917	06388
5	3978	037833	12776
6	3186	063055	21294
7	2393	090079	30420
8	1596	112599	38025
9	0798	125110	42250
10	0.00000	0.125110	0.042250
11	—0.00793	0.113736	0.038409
12	1573	094780	32007
13	2340	072908	24621
14	3095	052077	17586
15	3839	034718	11724
16	4570	021699	07328
17	5290	012764	04310
18	5998	007091	02395
19	6696	003732	01260
20	—0.07382	0.001866	0.000630
21	—0.08058	0.000889	0.000300
22	8724	000404	00136
23	—0.09380	0.000176	0.000059

Table 4

5 stars/day

N	Δm	P	P_s
0	+0.07916	0.006738	0.001153
1	6345	033690	05764
2	4768	084224	14409
3	3184	140374	24015
4	1595	175467	30019
5	0.00000	175467	30019
6	—0.01572	146223	25016
7	3093	104445	17868
8	4567	065278	11168
9	5994	036266	06204
10	—0.07378	0.018133	0.003102
11	—0.08718	0.008242	0.001410
12	10018	3434	0587
13	11281	1321	0226
14	12506	0472	0081
15	—0.13696	0.000157	0.000027

Table 5

3 stars/day

N	Δm	P	P_s
0	+0.07905	0.049787	0.005183
1	5287	149361	15548
2	2674	224042	23323
3	0.00000	224042	23323
4	—0.02589	168031	17492
5	5042	100819	10495
6	7368	050409	05248
7	9578	021604	02249
8	11679	008102	00843
9	13678	002701	00281
10	—0.15582	0.000810	0.000084
11	—0.17398	0.000221	0.000023

Table 6

1 star/day

N	Δm	P	P_s
0	+0.07853	0.367879	0.013520
1	0.00000	367879	13520
2	—0.07323	183940	06760
3	13597	061313	02253
4	19018	015328	00563
5	23738	003066	00113
6	—0.27870	0.000511	0.000019

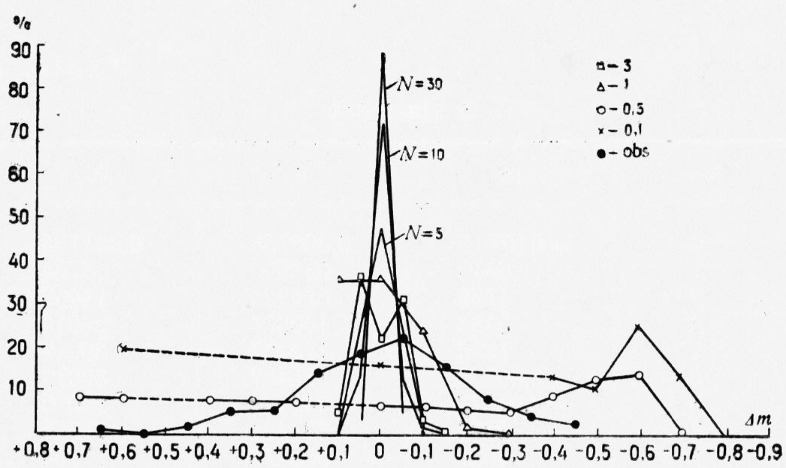

Fig. 3

the third — the probability P after (1). For Tables 2—6 P is the tabular probability by Janko [22], and for Table 7 and Table 8 it is computed for fraction argument with the help of Π-function using the property of Γ-function $\Pi(y)=\Gamma(y+1)=y\Gamma(y)$. It is clear in the Table 7 and the Table 8 the probabilities are not normalized.

The simple graphical plot of the hypotheses in the plain $P—\Delta m$ does not give a great information for the following reasons: 1. P in Table 7 and Table 8 are not normalized. 2. The distribution of Δm is not uniform. 3. The number of the points for various hypotheses are not equal. All this leads

Table 7

0.5 star/day

N	Δm	P	P_s
0.0	+0.69466	0.606531	0.002592
1	57173	594852	2542
2	44028	575075	2458
3	30079	548938	2346
4	15382	518069	2214
5	0.00000	483941	2068
6	—0.13471	447850	1914
7	23528	410905	1756
8	31237	374024	1598
9	37007	337953	1444
1.0	—0.41772	0 303266	0.001296
1.1	—0.45832	0.270387	0.001156
2	48902	239614	1024
3	51390	211130	0902
4	53418	185024	0791
5	55077	161314	0689
6	56436	139954	0598
7	57555	120854	0516
8	58477	103896	0444
9	59238	088935	0380
2.0	—0.59866	0.075816	0.000324
2.1	—0.60469	0.064378	0.000275
2	61046	54458	233
3	61600	45898	196
4	62131	38547	165
5	62640	32263	138
6	63129	26914	115
7	63598	22380	096
8	64098	18553	079
9	64480	15334	066
3.0	—0.64895	0.012636	0.000054
3.1	—.065293	0.010384	0.000044
2	65676	8509	36
3	66043	6954	30
4	66396	5669	24
5	66736	4609	20
6	67061	3738	16
7	67375	3024	13
8	67676	2441	10
9	67965	1966	8
4.0	—0,68243	0,001580	0,000007

to incomparable data. A possibility to avoid this inconvenience exists — the normalization of all curves. But in our case the normalization may be accomplished by two methods — by means of ordinates and areas. The second normalization is more acceptable. The areas of various hypotheses $\Sigma P\Delta(\Delta m)$

Table 8

0.1 star/day

N	Δm	P	P_s
0.0	+0.60795	0.904837	0.002422
1	0.00000	755491	2023
2	—0.38740	621795	1665
3	50992	505300	1353
4	55510	405992	1087
5	59129	322869	0864
6	62050	254372	0681
7	64422	198690	0532
8	06356	153972	0412
9	67938	118441	0317
1.0	—0.69237	0.000484	0.000242
1.1	—0.70306	0.068681	0.000184
2	71186	51816	139
3	71914	38869	104
4	72515	28999	078
5	73013	21524	058
6	73425	15758	042
7	73766	11688	031
8	74050	08554	023
9	74285	06234	017
2.0	—0.74480	0.004524	0.000012
2.1	—0.74642	0.003270	0.000009
2	74777	2355	006

are given in Table 9, together with the reduction coefficients r. The probabilities are normalized to the hypothesis $N=30$ expl/day, as worked up to the level $P=0.0001$.

The last column of Tables 3—8 gives the normalized probability P_s.

We may now already represent the results of the calculations graphically (Fig. 3).

Table 9

N expl/day	$P\Delta(\Delta m)$	v
30	0.002 630 000	1.000
10	0.007 787 948	0.337 701
5	0.015 372 953	0.171 080
3	0.025 264 270	0.104 100
1	0.071 562 240	0.036 751 2
0.5	0 615 409 157	0.004 273 58
0.1	0.982 312 206	0.002 677 36

For the hypotheses $N=5$, 10 and 30 expl/day the distribution is almost normal, as can be expected. The hypothesis $N=30$ expl/day may be rejected, as it requires variations in the brightness with in the range $<0^m.1$. When $N=3$ the distribution is very specific — there exist two maximuma and the range is $\sim 0^m.2$. By $N=1$ the specifics of the distribution appear the dispersion in observations is cut off at $+0^m.1$. For hypotheses $N=0.5$

and 0.1 the run of curves is a little unclear. By the way the hypothesis $N=0.1$ must be excluded, since it requires a large range, $\sim 1^m.5$. Since the big variation in Δm by $N=0.5$ and $N=0.1$ the run of the function between some separate points, which are connected with dotted line, is unknown.

Table 10

m	Number of cases	*m*	Number of cases	*m*	Number of cases
12.14	1	12.48	9	12.75	1
16	1	50	4	76	1
18	5	51	11	77	5
21	2	52	9	78	5
22	1	53	5	79	4
23	1	54	3	80	3
24	1	55	4	81	1
25	5	56	6	83	1
28	3	57	5	84	1
30	2	58	15	85	2
31	2	59	4	86	1
32	3	60	7	87	3
33	1	61	4	89	4
34	3	62	3	90	1
36	2	63	9	92	1
37	1	64	1	93	3
38	8	65	8	94	2
39	2	66	4	96	5
40	4	67	3	97	1
41	7	68	7	98	1
42	5	69	8	12.99	1
43	6	70	3	13.02	3
44	5	71	2	07	2
45	3	72	10	24	1
46	3	73	4	13.26	1
12.47	5	12.74	7		

But on the other hand, the intermediate probabilities are not equal to zero. More closely discrete values may be obtained, if the computation for P by $N=0.5$ and $N=1$ with a step by Δm, which is $<0^m.1$ is made. In these forms, however, the curves may be used for conclusions.

As we already mentioned, the only quasar, for which the verification of hypothesis H_0 is 3C273 may be made. Observations of its brightness have been made more than once. In the paper of Geyer [32] also observations of Smith and Hoffleit are given. The conclusion of [25], that 3C273 is able to change the brightness from $M_{pg}=-27^m$ to $-27^m.5$ for time of order 10^4-10^6 sec. is very important. The variability of 3C273 is undoubted.

All published observations can't be used because of its very small number (i. g. [23], [28]), because of great differences from the accepted photometric system (i. g. [27]), or because they are photometric ones ([31]). The total number of

Table 11

N	$\Delta m>0$	$\Delta m<0$
0.1	27.9%	72.1%
0.5	42.9	57.1
obs.	46.1	53.9
1	55.2	44.8
3	49.7	50.3
10	49.7	50.3

used by us observations is 291 (by Geyer [32] with the values of Smith and Hoffleit, Cessevitsch [29] and Kiperman [30]). All the values are given on Table 10.

The final frequency curve is tied to $\Delta m=0$ (the mean value of observations is $\bar{m}=12^m.60$) and is given in Fig. 3. One may see that the observing curve is placed between the graphics for the hypothesis $N=1$ and $N=0.5$. The corresponding frequencies from left and from right $\Delta m=0$ for the observations and $N=0.1$, 0.5, 1, 3 and 10 are given in Table 11. The observing frequency are located between the corresponding frequencies for $N=0.5$ and $N=1$.

The final result of the present paper is that the hypothesis H_0 is acceptable and $0.5 < N < 1$ expl/day.

IV. Discussion

We should compare the results of our paper with the results of other investigations by scintillation models.

According to the first paper of Field [7] (for 3C48) the number of exploding as supernovae stars for 1 year is $\sim 10^2$. In the second paper [8] it is not clearly determined, but in every case it is from several tens to several hundreds (200—300) yearly. (Our paper was finished before the publication of [8], but owing to circumstances over which we have no control it was not published at that time).

According to Woltjer [5] (in point of fact to Hoyle [17]) the number of the exploding stars is about $\sim 10^3$ for 1 year.

As it seems, our estimation for N is in accordance with other investigations.

If the energy of 3C273 is produced by the explosions of stars as supernovae, then, as we have shown, $0.5<N<1$. Since we have accepted, that the energetical output of the quasars is $10^{46}<E<10^{47}$erg/sec. then the energy of one explosion should be $\sim 5.8 . 10^{51}$ ergs. (In our previous paper we accepted an older estimation for the brightness and obtained an energy for one explosion $\sim 5.8 . 10^{50}$ ergs. Now we correct this value).

As it is known, a supernova gives the emission energy $10^{49}-10^{51}$ ergs integrated over the whole explosion. But kinetic energy of the envelope is the more conspicous: and it may reach $10^{52}-10^{54}$ ergs. We make use of the mechanism of Colgate and Cameron [4] for transformation of the kinetic energy in emission, which conforms that the energy reaches up to 10^{54} ergs. Therefore our evaluation for the energy of explosion of supernovae is not contrary to the contemporary picture.

We made certain assumptions by the statistical analysis. Let us consider their reflection to the final result.

In reality, it is possible that the total emission of quasars is not produced by the explosion of supernovae only, but there exists some proper background. But the size of the changes in the light curve is considerable, which shows that this background is not very high by brightness. Thus the brightness of the background does not change the order of the determined N.

The number of explosions in equal time intervals is distributed after Poisson. It should be very strange, if the distribution is not Poisson.

For the standard light curve reasonable objections can be made. The explosion of the star by no means becomes instantaneously. Moreover, we accepted for background brightness the points before the explosion. May be the light curve is very different from the accepted one by us, especially, as Zwicky shows (i. g. [34]), in all probability, there exist at least five types supernovae. Other important circumstance, which we do not mention up to now, is the seculary decreasing of brightness evaluating of $+0^m.1$/century to $+0^m.4$/century.

However, one easily conforms that all the effects mentioned above act contrary — in Fig. 3 some of these effects extend the theoretical curves by Δm, and others narrow them. Thus, there exists certain confidence in the order of the evaluation for N. The very strong support is the confidence in the order to the other evaluations, which are obtained by absoluteby different methods.

The object 3C273 is not the only variable quasar. The variation is registered and in 3C48 also. The fact, that Smith and Hoffleit [24] does not observe the fluctuations $>0^m.3$, immediately permits to evaluate by Fig. 3, that $N<3$. Analogous conclusions can be made also on the basis of evaluations of Sandage and Wyndham [35], but they would be very preliminary.

The proposed method for verification of scintillation hypotheses by quasars can be applied to the other objects.

An important application should be made for irregular variable stars. It is known by the treatment of the observations of irregular variable stars that the methods of mathematical statistics applied also — i. g. [36], where the detailed references are included. By the above method can make evaluations for the explosions of separate irregular variables. Especially interesting are the applications to the exploding stars.

Analogous reasons can have place in the studying of solar activity, specially for the statistical description of erruptions.

Certainly the assumptions should be absolutely different from these, with which is worked in the present paper.

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ВЪРХУ СТАТИСТИЧЕСКАТА ПРОВЕРКА НА ЕДНА ХИПОТЕЗА ЗА СВРЪХЗВЕЗДИТЕ

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(Резюме)

Направена е статистическа проверка на хипотезата, че вариациите в блясъка на свръхзвездите (квазарите) са причинени от избухвания на свръхнови (или свръхновоподобни) звезди. В същност проверката обхваща 7 модела за $N=30$, 10, 5, 3, 1, 0,5 и 0,1 взр. на денонощие. Крайният резултат, представен на фиг. 3, е, че $0{,}5<N<1$ взр. на денонощие и е в съгласие с оценките на други автори, намерени по независими методи.