БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ • BULGARIAN ACADEMY OF SCIENCES
ИЗВЕСТИЯ НА СЕКЦИЯТА ПО АСТРОНОМИЯ
BULLETIN OF THE SECTION OF ASTRONOMY
Том (Vol.) III

ON THE COMPARISON OF THE OBSERVED AND THE RANDOM DISTRIBUTION OF GALAXY COUNTS

Marin Kalinkov and Bojka Veleva

The investigation of the surface distribution of the galaxies leads to the following question: are the counts of galaxies n_i, observed in squares z_i, randomly distributed?

Let us have N galaxies in the area

$$Z=\sum_{i=1}^{k} z_i, \quad z_1=z_2=\cdots=z_k.$$

If the probability for one of the galaxies to fall into the square z_i is p_i, we shall have

$$\sum_{i=1}^{k} p_i=1$$

and if n_i galaxies fall into this square,

$$N=\sum_{i=1}^{k} n_i.$$

As it may be proved the probabilities must have binomial distribution in order $n_1, n_2, \ldots, n_k$ galaxies to fall into the square z_i.

It is easy to compare the observed and the random distribution of counts of galaxies. The published papers on this question allow us to conclude that the observed distribution is non-randomly. But verifications for random distribution are not always correct because the non-random distribution of galaxy counts may be due to several effects. Consequently it is necessary to construct a theoretical model within the limits of which the hypothesis that the galaxy counts have a random distribution must be verificated. A model of this kind may be constructed very roughly because a general theory of the spatial distribution of the galaxies is not yet developed.

Our simple model however is based on three assumptions:

1. Clusters of I, II and so on orders do not exist and vice versa, if they exist their angular diameters d must be considerably greater than the angular diameter of the investigated area ($d \gg Z$). It is interesting to note that the existence of clusters, superclusturs etc. also could not essentially influence the distribution of n_i, only in case their corresponding angular diameters are $d \ll Z$. In other words, our model requires approximately homogeneously spatial distribution of the galaxies. And if we are interested in the distribution of the centers of the clusters then the above condition must be valid for the centers.

The disagreement between the observed distribution of counts of galaxies and the theoretical distribution for our model leads to the rejection of the present assumption.

2. The above characteristic is connected with the determination of the space density of the galaxies D in a volume unit. The space density must be a constant, at least for volumes, which are considerably greater than the corresponding volumes, responding to the examining squares.

3. It is necessary to assume the absence of selective effects, which include the peculiarities of the physical characteristics of galaxies and also the influence of appropriation for counting criteria.

Analogous questions about the distribution of galaxy counts in equal squares are examined by many authors, as for example Zwicky [1] and Scott [2].

According to Zwicky the distribution function is given with

$$P_i(n_i) = \frac{n!}{n_i!(n-n_i)!} p_i^{n_i} (1-p_i)^{n-n_i}, \tag{1}$$

where

$$p_i = \frac{1}{z} \tag{2}$$

in the case of equal squares. (1) is the binomial distribution, which is in the form

$$p_n\{x\} = \binom{n}{x} \theta^x (1-\theta)^{n-x}, \quad x = 0, 1, 2, \ldots, n. \tag{3}$$

The verification carried out by Zwicky for several cases (for example $z = 1296$ squares and $\sum n_i = 75,885$ galaxies) shows that the distribution of the galaxy counts is not binomial. The conclusions shown by Zwicky are qualitative only (for instance Fig. 17 of [1]).

Similar results follow from the counting of Shapley [3—6], Shane et al. [7—10] and others.

The sizes of squares are usually chosen conveniently for the direct counting of galaxies — $10' \times 10'$, $30' \times 30'$ or $1^\circ \times 1^\circ$. But when $z \to 0$ and for each limiting magnitude of the counting as may be shown, the distribution of n_i must be submitted to the binomial law independantly of the really surface galaxy distribution. Therefore, it is very important to determine these sizes of the squares, for which the observed distribution of n_i begins to resemble the binomial distribution. Lick counting of the galaxies is carried out on squares $10' \times 10'$, but a grouping by counts for $1^\circ \times 1^\circ$ is used for the plotting of the contour maps (isopleths) for the surface distribution of the galaxies as long as the limiting magnitude is $m_{\lim} \approx 18.4$.

One could expect that for zones, which have different populations, the limiting sizes of the squares for which we shall have an approximation of the binomial distribution, shall vary in a wide range.

It may be shown [11] that for a given value of θ with n increasing, the binomial distribution tends to the normal distribution, i. e. for large enough

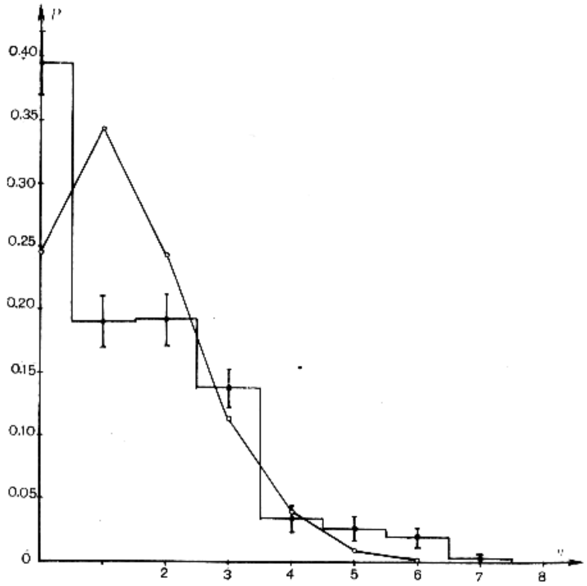

Fig. 1

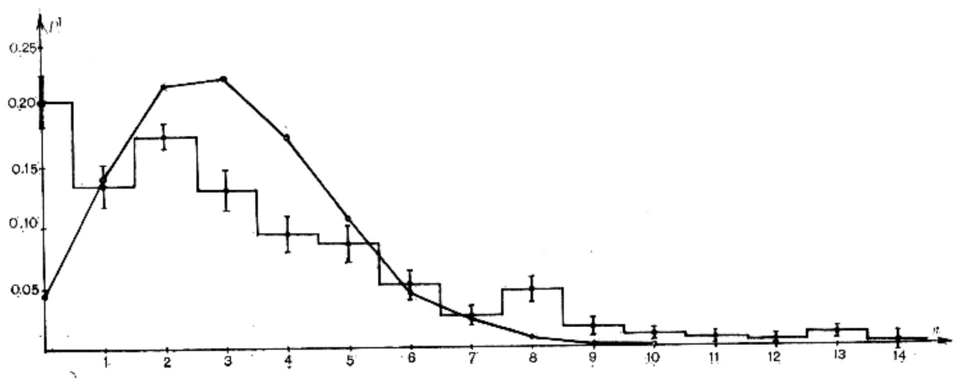

Fig. 2

values of n the binomial distribution may be approximated with the normal one which has a mean value $\xi = n\theta$ and a dispersion $\sigma^2 = n\theta(1-\theta)$ or

$$(4) \qquad p\{x\} = \binom{n}{x} \theta^x (1-\theta)^{n-x} \approx \frac{1}{2\pi n\theta(1-\theta)} \exp\left(-\frac{(x-n\theta)^2}{2n\theta(1-\theta)}\right).$$

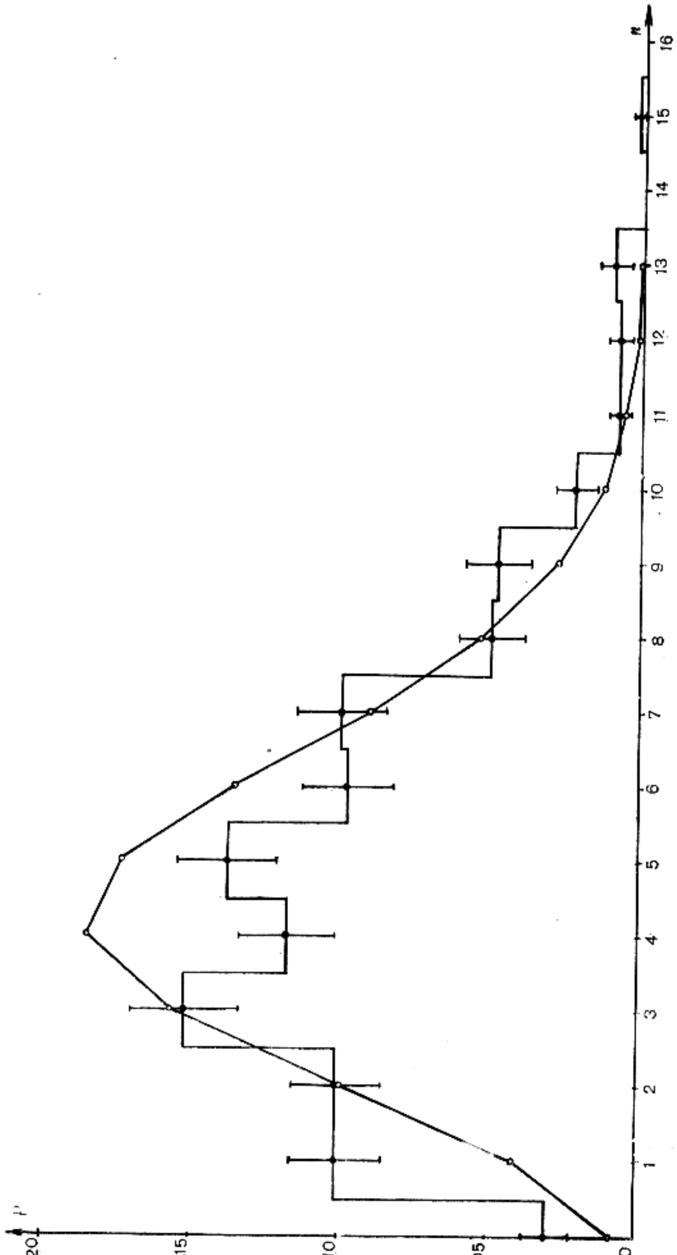

Fig. 3

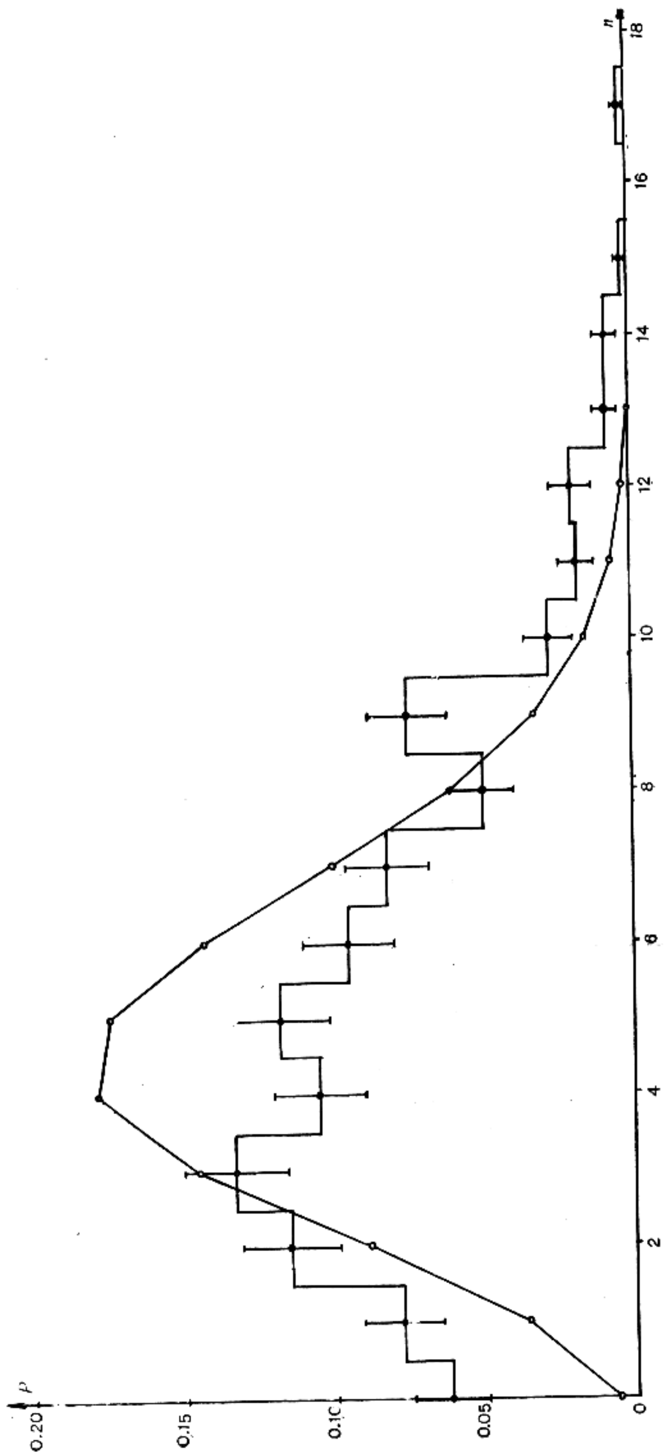

Fig. 4

If $\theta=\xi/n$, so that $\theta\to 0$ with $n\to\infty$ the binomial distribution tends to the Poisson distribution:

$$p\{x\}=\binom{n}{x}\theta^x(1-\theta)^{n-x}\to\frac{\xi x}{x}e^{-\xi} \tag{5}$$

for $n\to\infty$. It is usually accepted that the Poisson distribution may be examined as such approximating the binomial one when $\theta<0.1$.

A verification of the hypothesis H_0, that the distribution of galaxy counts of the Palomar atlas (some of the results of this counting are published in [12]) is the Poisson distribution, is presented in this paper.

We selected four areas on the print 83 E, which have various mean surface densities. The areas have sizes 20×20 squares and each square is $3'\times3'$. It was accepted that the galaxy counts n_i for each square given as the sum of the counts of galaxies according to both observers (who carried out the counting independently). In our case the real mean surface density of the squares is $n_i/2$. The observed and the theoretical distributions are presented in Fig. 1 — Fig. 4, where the mean-root-square errors are also marked. It may be seen that no agreement between the theoretical and observed distributions exists. This conclusion is supported with χ^2 test:

For area I $n=569$, $\xi=1.42$, $\chi^2=78.46$, for area II $n=1242$, $\xi=3.10$, $\chi^2>300$, for area III $n=1874$, $\xi=4.69$, $\chi^2=87.31$, for area IV $n=1974$, $\xi=4.93$, $\chi^2>300$.

Table 1

		Galaxies according to I observer						
		0	1	2	3	4	5	Σ
Galaxies according to II observer	0	158	14	4				176
	1	62	63	17				142
	2	11	30	17	7			65
	3	2	1	4	8	1		16
	4				1			1
	5							
	Σ	233	108	42	16	1		

Table 2

		Galaxies according to I observer										
		0	1	2	3	4	5	6	7	8	9	Σ
Galaxies according to II observer	0	82	20	6	1							109
	1	34	38	29	6	1						128
	2	6	20	24	15	4						69
	3	2	6	17	11	6	2					44
	4	1	1	6	3	14	1	1				27
	5			1	3	6	1	3				14
	6					1		1	1			3
	7				1			2	1			4
	8					1						1
	9					1						1
	Σ	125	105	83	40	34	4	7	2			

Table 3

Galaxies according to II observer	Galaxies according to I observer: 0	1	2	3	4	5	6	7	8	9	10	11	12	Σ
0	12	15	3											30
1	28	32	28	4	1									93
2	7	32	35	36	8	2								120
3	1	8	18	28	26	6								87
4			3	12	14	14	2							45
5				1	5	7	3							16
6							2	3						5
7						1	1		1					3
8														
9														
10														
11														
12													1	1
Σ	48	87	87	81	54	30	8	3	1				1	

Table 4

Galaxies according to III observer	Galaxies according to I observer: 0	1	2	3	4	5	6	7	8	9	10	11	12	13	Σ
0	25	1	3												29
1	26	36	21	2											85
2	8	34	35	24	4										105
3		6	22	26	11	2									67
4		1	8	22	15	16	1								63
5				3	13	8	4	2							30
6				1	2	1	3	2	1						10
7				1	2	2		2							7
8						1									1
9							1		1						2
10															
11															
12															
13										1					1
Σ	59	78	89	79	47	30	9	6	2	1					

The degrees of freedom for the four areas may be reckognized on Figs. 1—4.

The joint distributions of the counts, according to both observers are given in Table 1 — Table 4 for the four areas. Similar tables are given by Scott [2]. These tables show the assurance with which the counting of the galaxies is carried out. But we must take into consideration the fact that for smaller sizes of the squares (as in our case) a considerable dispersion could be expected while the grouping of the squares leads to a smaller distribution.

The total (summarized) joint distribution is presented in Table 5.

The distributions of the counts of the galaxies on the whole investigated print according to both observers are given in Fig. 5 and Fig. 6 (by squares $12' \times 12'$) together with the corresponding theoretical distributions ($\xi = 24.4$ and $\xi = 22.4$ respectively). The distinction between the theoretical and the

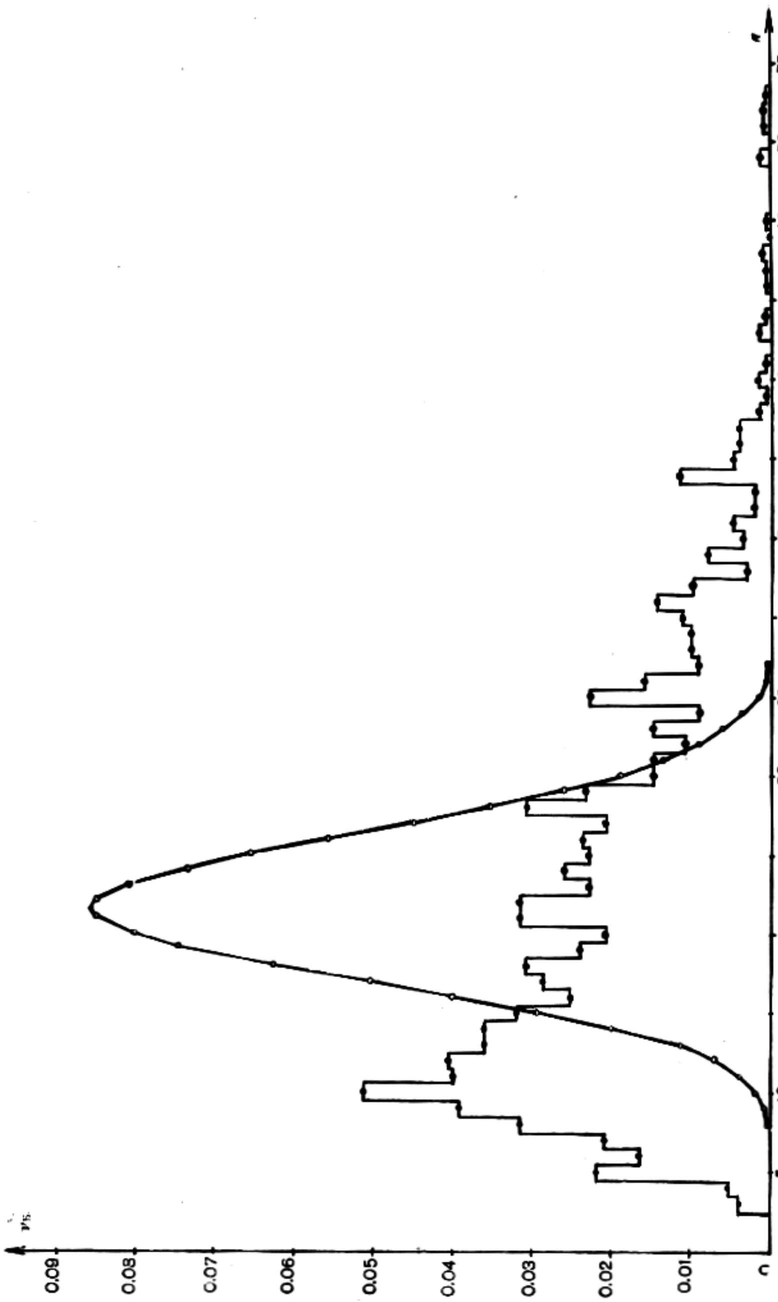
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75

Fig. 5

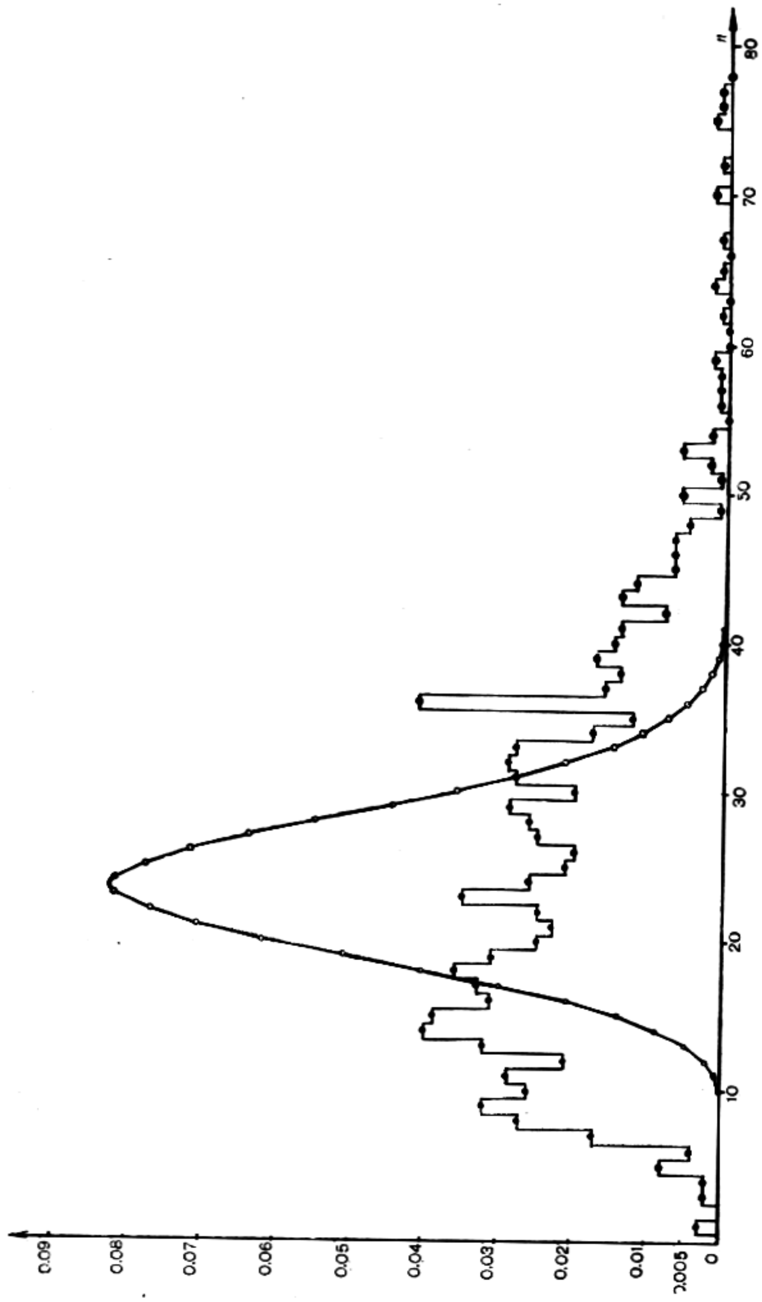

Fig. 6

observed distributions are also significant (all figures in the present paper are normalized).

Hypothesis H_0 must, therefore, be rejected. The rejection of the hypothesis that the surface distribution of the galaxies is non-randomly indicates that at least one of the fundamental assumptions for our model is incorrect. Because the corresponding regressions between the counts according to both observers are practically linear, assumption 3 is not very important. The assumption 2 is connected with assumption 1 and the last one is of the greatest interest.

Table 5

Galaxies according to II observer	Galaxies according to I observer																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Σ
0	277	50	16	1													344
1	150	189	95	12	2												448
2	32	116	11	82	16	2											259
3	5	21	61	73	44	10											214
4	1	2	17	38	43	31	4										136
5			1	7	24	16	10	2									60
6				1	3	1	6	6	1								18
7				2	2	3	3	3	1								14
8					1	1											2
9					1		1		1								3
10																	
11																	
12																	1
13													1				1
14										1							
15																	
Σ	465	378	201	216	136	64	24	11	3	1			1				

Therefore, on the basis of verifications, which are similar to the present paper it may be seen, that the clustering of the galaxies is a common law in the Metagalaxy.

The authors are greatly indebted to Dr. N. Nicolov, University of Sofia, for his valuable discussion on the present paper.

N o t e a d d e d i n p r o o f s. Some results of the present paper were reported on the XIIIth General Assembly of the International Astronomical Union (Prague, August 1967) and also on a Seminar in Meudon Observatory (Paris, December 1967). Some questions were discussed on a Seminar (October 1968) in the Astronomical observatory — Warsaw. (The reports were made by the former author.)

REFERENCES

1. Z w i c k y F., Morphological Astronomy, Berlin, 1957.
2. S c o t t E. L., In: Problems of Extra-Galactic Research, Ed. by G. C. Mcwittie, New York, 1962.
3. S h a p l e y H., Annals of the Astr. Obs. of Har. coll., 88, 1935, 5.

4. Baker R. H., Annals of the Astr. Obs. of Har. coll., **88**, 1937, 6.
5. Shapley H. and R. B. Jones, Annals of Har. coll. obs., **106**, 1938, 1.
6. Shaley H., Annals of the Astr. Obs. of Har. coll., **88**, 1950.
7. Shane C. D. and C. A. Wirtanen, AJ, **59**, 1954, 285.
8. Shane C. D., AJ., **61**, 1956, 292.
9. Shane C. D., C. A. Wirtanen and M. Steinlin, AJ, **67**, 1962, 370.
10. Neyman J., E. Scott and C. D. Shane, AJ, **117**, 1953, 92.
11. Хальд А., Математическая статистика с техническими приложениями (перевод с английского), Москва.
12. Veleva B. and M. Kalinkov, Bull. of the Astronomical Section, **2**, 1967, 151.

Presented on September 10, 1967

ВЪРХУ ПОВЪРХНОСТНОТО РАЗПРЕДЕЛЕНИЕ НА ГАЛАКТИКИТЕ ПО ПАЛОМАРСКИЯ АТЛАС

М. Калинков и Б. Велева

(Резюме)

По данните, получени при преброяванията на галактиките върху копие от Паломарския атлас, е направена проверка за случайното разпределение на броя на галактиките в равни квадратчета.

Разгледан е теоретичен модел, основан върху три предположения: 1. Купове от галактики (от I, II и т. н. порядък) не съществуват. 2. Пространствената плътност на галактиките е постоянна. 3. Отсъствуват селективни ефекти. Проверката показва, че предположение 1, както и свързаното с него предположение 2, не е в съгласие с наблюдаваното разпределение на галактиките.

О ПОВЕРХНОСТНОМ РАСПРЕДЕЛЕНИИ ГАЛАКТИК ПО ПАЛОМАРСКОМУ АТЛАСУ

М. Калинков и Б. Велева

(Резюме)

По данным, полученным при подсчете галактик на копии Паломарского атласа, сделана проверка случайного распределения числа галактик в одинаковых квадратиках.

Рассмотрена теоретическая модель, основанная на трех предположениях:

1) скоплений галактик (I, II и т. д. порядков) не существует;

2) пространственная плотность галактик постоянна;

3) селективные эффекты отсутствуют.

Проверка показывает, что предположение 1, как и связанное с ним предположение 2, не согласуется с наблюдаемым распределением галактик.