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PROOF THAT THE TIME MEASURED BY A FAR AWAY OBSERVER, DURING WHICH A TEST BODY FALLS IN THE CENTRAL SYMMETRIC FIELD OF THE TWO TEMPORAL THEORY OF RELATIVITY AND COMES NEAR THE CENTRE, IS FINITE

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In [1] and [2] we have proposed a new multitemporal theory of relativity, according to which at strong gravitational fields in natural phenomena new timelike variables begin to play a part. These variables allow to avoid some singularities inherent to the general theory of relativity.

We have considered in [3], [1] and [2] the static case with spherical space symmetry in the five-dimensional Riemannian space with the signature $+1, +1, +1, -1, -1$. The line element of this field can be written in the form

$$(1)\qquad -ds^2 = g_{ik}\,dx^i\,dx^k = -e^{\nu}(dx^4)^2 - e^{\varrho}(dx^5)^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + e^{\lambda}\,dr^2, \quad i,\ k = 1, \ldots, 5.$$

ν, ϱ and λ are functions of r only and tend to zero when r grows to infinity. Our co-ordinates are

$$(2)\qquad x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi, \quad x^4 = c_4 t_4, \quad x^5 = c_5 t_5.$$

t_4 — Einstein's relative time; c_4, c_5 — characteristic constants of our theory; t_5 is a new time manifesting according to [4] its action in very mighty gravitational fields. According to [3], [1] and [2] this new time will play a part at very strong gravitational fields, for example in the vicinity of Schwarzschild's sphere. c_5 and c_4 are connected by the equation

$$(3)\qquad c_5 \gg c_4.$$

The generalized Einstein's gravitational equations for the five-dimensional Riemannian space have according to [1] and [2] in the case of vacuum the form

$$(4)\qquad R_{ik} = 0, \quad i,\ k = 1,\ 2, \ldots, 5.$$

We obtain from (1) and (4) the relation

$$\varrho' = a\nu', \quad a = \text{const.} \tag{5}$$

From (1), (4) and (5) one gets the equations

$$\lambda' + (1+a)\nu' + \frac{1}{2}\, ar(\nu')^2 = 0, \tag{6}$$

$$e^{\lambda} = 1 + \frac{1}{2}\, r[(1+a)\nu' - \lambda'], \tag{7}$$

$$\nu'' + 2\nu'\left[\frac{1}{r} + \frac{1}{4}\left((1+a)\nu' - \lambda'\right)\right] = 0. \tag{8}$$

a is a constant expressing the influence of the fifth dimension on the gravitational field. According to [3] it manifests its existence especially in the regions where Schwarzschild's metric becomes singular.

For $a=0$ equations (6), (7) and (8) have the exact solution

$$\nu' = \frac{\frac{K}{r^2}}{1-\frac{K}{r}} = -\lambda', \quad k = \text{const}, \tag{9}$$

which is Schwarzschild's solution if we set

$$K = \frac{2fm}{c_4^2}, \tag{10}$$

f — Newton's gravitational constant, m — the mass of the central body with spherical symmetry.

We assume

$$K = \frac{2fm}{c_4^2} = 1,$$

K — radius of Schwarzschild's sphere.

At great distances from Schwarzschild's sphere, $r \gg 1$, the solution of the system of equations (6) — (8) must differ very little from Schwarzschild's solution (9), as this solution gives the Newtonian potential as well as the three effects of Einstein's general theory of relativity. On account of this we assume as boundary condition that for $r \gg 1$ the solution of the system (6) — (8) for ν' is equal to ν' from (9), i. e. for $r \gg 1$

$$\nu' \text{ from system (6) — (8)} = \nu' \text{ from (9)}. \tag{11}$$

From (6) and (8) we get

$$\nu'' = -2\nu'\left[\frac{1}{r} + \frac{1}{4}\left(2(1+a)\nu' + \frac{1}{2}\, ar(\nu')^2\right)\right]. \tag{12}$$

According to the investigations carried out in [3], a must be negative if the solution of the system (6) — (8) should be reqular on the Schwarzschild sphere. From the effect of Dicke follows (cf [4])

$$a \sim -0.1. \tag{13}$$

(The effect of Dicke refers to the influence of the oblateness of the sun on the motion of the perihelion of Mercury.)

For $\alpha=-0.1$ we (Jordan Denev and myself) integrated equation (12) with the boundary condition (11) using the computer of the Computing centre of the Mathematical Institute of Bulgarian Academy of Sciences [5]. In [5] we obtained the following result: the solution for ν' of the equation (12) at $\alpha=-0.1$ is regular for $\infty>r>0$. At $r=0$ we have a pole. $\nu'(r)$ is an increasing monotone function for $r\to 0$.

From the regularity of ν' in the interval $0<r<\infty$ follows according to equation (6) the regularity of λ' in the same interval. But then the integrals ν and λ shall also be regular in this interval. In this way we come to the exceptionally important result that the five-dimensional field given by the quadratic form (1) at $\alpha=-0.1$ is regular for the whole region $0<r<\infty$.

We accept the generalized Galilei's principle of inertia: a free particle moves along a geodesic in the Riemannian space. The equation of the geodesic in the five-dimensional Riemannian space has the form

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds}=0, \quad i,\ k,\ l=1,\ 2,\ 3,\ 4,\ 5. \tag{14}$$

Γ^i_{kl} — Christoffel's symbols. For $i=2$ we get from (14) according to (1) and (2) (cf. [3] and [4])

$$\frac{d^2\theta}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds}-\sin\theta\cos\theta\left(\frac{d\varphi}{ds}\right)^2=0. \tag{15}$$

If the particle moves initially in the plane $\theta=\pi/2$ then the initial conditions of movement are $d\theta/ds=0$, $\cos\theta=0$.

From (15) follows that

$$\frac{d^2\theta}{ds^2}=0.$$

Differentiating (15) we get

$$\frac{d^3\theta}{ds^3}=0, \quad \frac{d^4\theta}{ds^4}=0,\ldots$$

This shows that a particle originally moving in the plane $\theta=\pi/2$ will continue to do so throughout its motion.

Equations (14) for $i=1,\ 3,\ 4,\ 5$ can then be written in the form

$$\frac{d^2r}{ds^2}+\frac{1}{2}\lambda'\left(\frac{dr}{ds}\right)^2-e^{-\lambda}r\left(\frac{d\varphi}{ds}\right)^2+\frac{1}{2}e^{\nu-\lambda}\nu'\left(\frac{dx^4}{ds}\right)^2+\frac{1}{2}e^{\varrho-\lambda}\varrho'\left(\frac{dx^5}{ds}\right)^2=0, \tag{16}$$

$$\frac{d^2\varphi}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\varphi}{ds}=0, \tag{17}$$

$$\frac{d^2x^4}{ds^2}+\nu'\frac{dr}{ds}\frac{dx^4}{ds}=0, \tag{18}$$

$$\frac{d^2x^5}{ds^2}+\varrho'\frac{dr}{ds}\frac{dx^5}{ds}=0. \tag{19}$$

The last two equations can be immediately integrated and we obtain

$$\frac{dx^4}{ds}=D_1e^{-\nu}, \tag{20}$$

$$\frac{dx^5}{ds}=D_2 e^{-\varrho}, \tag{21}$$

D_1, D_2 — constants.

Instead of integrating equation (16) we can use equation (1) which here plays the part of the energy integral. It gives us

$$-e^{\nu}\left(\frac{dx^4}{ds}\right)^2-e^{\varrho}\left(\frac{dx^5}{ds}\right)^2+r^2\left(\frac{d\varphi}{ds}\right)^2+e^{\lambda}\left(\frac{dr}{ds}\right)^2=-1. \tag{22}$$

Let us consider the case of radial motion

$$\frac{d\varphi}{ds}=0. \tag{23}$$

From (20), (21), (22) and (23) we get

$$-D_1^2 e^{-\nu}-D_2^2 e^{-\varrho}+e^{\lambda}\left(\frac{dr}{ds}\right)^2=-1. \tag{24}$$

From (5) and from the condition that the metric for $r\to\infty$ must be pseudo-Euclidean (with signature $+1$, $+1$, $+1$, -1, -1), we get

$$\varrho=\alpha\nu. \tag{25}$$

Then from (24), (20), (21) and (25) we obtain

$$\left(\frac{dr}{ds}\right)^2=-e^{-\lambda}+D_1^2 e^{-\nu-\lambda}+D_2^2 e^{-\alpha\nu-\lambda}, \tag{26}$$

$$\left(\frac{dr}{dx^4}\right)^2=-\frac{1}{D_1^2}\, e^{2\nu-\lambda}+e^{\nu-\lambda}+\frac{D_2^2}{D_1^2}\, e^{2\nu-\alpha\nu-\lambda}, \tag{27}$$

$$\left(\frac{dr}{dx^5}\right)^2=-\frac{1}{D_2^2}\, e^{2\alpha\nu-\lambda}+\frac{D_1^2}{D_2^2}\, e^{2\alpha\nu-\nu-\lambda}+e^{\alpha\nu-\lambda}. \tag{28}$$

From (6) and (7) we get

$$e^{\lambda}=1+\frac{1}{2}\,r\left[2(1+\alpha)\nu'+\frac{1}{2}\,\alpha r(\nu')^2\right]. \tag{29}$$

Consequently, for $\alpha=-0.1$, e^{λ} is regular for the interval $\infty>r>0$ as ν is regular in the same interval.

From (26) one obtains

$$\pm\frac{dr}{ds}=(-e^{-\lambda}+D_1^2 e^{-\nu-\lambda}+D_2^2 e^{-\alpha\nu-\lambda})^{1/2},$$

$$\pm s(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}}(-1+D_1^2 e^{-\nu}+D_2^2 e^{-\alpha\nu})^{-\frac{1}{2}}\,dr+\text{const} \tag{30}$$

and from (27) and (28)

$$\pm\frac{dr}{dx^4}=\left(-\frac{1}{D_1^2}\, e^{2\nu-\lambda}+e^{\nu-\lambda}+\frac{D_2^2}{D_2^2}\, e^{2\nu-\alpha\nu-\lambda}\right)^{\frac{1}{2}},$$

$$\pm x^4(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}-\nu}\left(-\frac{1}{D_1^2}+e^{-\nu}+\frac{D_2^2}{D_1^2}e^{-a\nu}\right)^{-\frac{1}{2}}dr+\text{const}, \tag{31}$$

$$\pm\frac{dr}{dx^5}=\left(-\frac{1}{D_2^2}e^{2a\nu-\lambda}+\frac{D_1^2}{D_2^2}e^{2a\nu-\nu-\lambda}+e^{a\nu-\lambda}\right)^{\frac{1}{2}},$$

$$\pm x^5(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}-a\nu}\left(-\frac{1}{D_2^2}+\frac{D_1^2}{D_2^2}e^{-\nu}+e^{-a\nu}\right)^{-\frac{1}{2}}dr+\text{const}. \tag{32}$$

The function $e^{\lambda/2}$ enters as factor in the integrand of the integrals (30), (31) and (32). The same function is real if

$$e^{\lambda}\geqq 0. \tag{33}$$

We find from (29) the value of r for which $e^{\lambda}=0$

$$1+\frac{1}{2}r\left[2(1+a)\nu'+\frac{1}{2}ar(\nu')^2\right]=0$$

or

$$1+(1+a)\,\xi+\frac{1}{4}\,a\xi^2=0, \tag{34}$$

where we have set $r\nu'=\xi$.

For $a=-0.1$ from (34) we obtain the quadratic equation

$$40+36\xi-\xi^2=0. \tag{35}$$

The roots of (35) are $\xi_1=37.09$ and $\xi_2=-1.09$.

According to [5] $r\nu'=\xi>0$ for $\infty>r>0$, consequently only the first root ξ_1 will be relevant for our problem. From Table II of [5] we see that at $r=0.084$, $\nu'=440.85244$, $r\nu'=37.0314<\xi_1$. For $r=0.079$, $\nu'=469.80171$, $r\nu'=37.1142>\xi_1$. For all values of $r>0.084$ $r\nu'<\xi_1$ and for all values of $r<0.079$ $r\nu'>\xi_1$. Consequently $e^{\lambda}>0$ for $\infty>r>0.08$.

In the same interval $e^{\lambda/2}$ has real values.

The value $r=0.08$ is, of course, connected with the accuracy with which the calculation by the computer is performed. It is quite possible that at a finer step of the calculation this boundary value will move nearer to the pole $r=0$. This is obvious from the fact that in Table I of [5], where the calculation is carried out at a 10 times greater step, the boundary value for which $e^{\lambda}=0$ is $r=0.599$.

We assume that for ordinary particles and for photons which move in the central symmetric gravitational field $dx^5/ds=0$, i. e. according to (21)

$$D_2=0. \tag{36}$$

If we have in the beginning of motion $D_2=0$, then according to the integral (21) we shall have while the motion lasts $dx^5/ds=0$, i. e. the motion will be four-dimensional in a five-dimensional gravitational field which differs essentially from Schwarzschild's four-dimensional field.

N o t e. Our assumption that in the beginning of the motion of an ordinary particle $dx_5=0$ and $D_2=0$ does, of course, not mean that the line element is in general four-dimensional. There exist according to our theory particles for which $D_2\neq 0$. The interesting problem of the motion of a particle with $D_2\neq 0$ will be considered afterwards. We can, however, assume that for ordinary particles holds true (36).

Inserting (36) in (30) and (31) we obtain

$$\pm s(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}}(-1+D_1^2 e^{-\nu})^{-\frac{1}{2}}dr+\text{const}, \tag{37}$$

$$\pm x^4(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}-\nu}\left(-\frac{1}{D_1^2}+e^{-\nu}\right)^{-\frac{1}{2}}dr+\text{const}. \tag{38}$$

For $r\gg 1$ our solution differs very little from Schwarzschild's solution and we can set

$$e^{\nu}=e^{-\lambda}=1-\frac{1}{r} \tag{39}$$

for $r\gg 1$.

Let us assume that the fall of the test body starts from $r=r_0\gg 1$. We accept moreover that for $r=r_0$ $dr/dx^4=0$, i. e. at the beginning of the fall the radial velocity (and also the tangential velocity) is zero. Then from (27) and (39) we get

$$D_1^2=1-\frac{1}{r_0}. \tag{40}$$

With this value of D_1^2 (37) and (38) get the form

$$\pm s(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}}\left[-1+\left(1-\frac{1}{r_0}\right)e^{-\nu}\right]^{-\frac{1}{2}}dr+\text{const}, \tag{41}$$

$$\pm x^4(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}-\nu}\frac{1}{1-\frac{1}{r_0}}\left[-1+\left(1-\frac{1}{r_0}\right)e^{-\nu}\right]^{-\frac{1}{2}}dr+\text{const}. \tag{42}$$

According to [5] Tables I and II $\nu'(r)$ is for $r\to 0$ an increasing monotone function which has no singularity at $r=1$. For $r\gg 1$, however, $\nu'(r)$ differs very little from Schwarzschild's solution (9) for $K=1$. For $r\gg 1$ ν likewise will differ little from Schwarzschild's solution $\nu=\ln\left(1-\frac{1}{r}\right)$. For $\infty>r>1$ the function $\nu=\ln\left(1-\frac{1}{r}\right)$ is negative and its absolute value increases at $r\to 1$. For $r\to 1$ $\ln\left(1-\frac{1}{r}\right)\to-\infty$. Consequently, the solution of equation (12) for $r\gg 1$ must be negative and its absolute value must increase for $r\to 1$. But as ν' is an increasing monotone function for $r\to 0$, so the solution ν of (12) cannot have singularity for $r=1$, but its absolute value must increase for $r \to 0$. But then $e^{-\nu}$ will be an increasing monotone regular function in the interval $\infty > r > 0$ for $r \to 0$. The expression

$$-1+\left(1-\frac{1}{r_0}\right)e^{-\nu}, \tag{43}$$

which figures in the integrals (41) and (42) for $r \gg 1$, is approximately equal to

$$-1+\frac{1-\frac{1}{r_0}}{1-\frac{1}{r}}. \tag{44}$$

The last expression is for $r < r_0$ positive. When $r \to 0$ $e^{-\nu}$ grows, consequently the expression (43) remains at $r \to 0$ positive. (At $r=1$ (43) has no singularity in contrast to (44).)

Accordingly, in the interval $\infty > r > 0.08$ all factors in the integrand expressions of (41) and (42) are bounded regular functions. The same also holds frue for the integrals themselves. In this way we have proved that the proper time of falling $s(r)$ as well as the time measured by a far away observer $x^4(r)$ of a test body from the point $r=r_0$ to the vicinity of the pole $r=0$ (more precisely to $r=0.08$) is finite. The point $r=0.08$ is connected with the accuracy of the calculation performed by the computer using the step 0.005. If we choose a finer step this point will move nearer to the pole $r=0$.

For the photons which move with the velocity of light we shall assume likewise $D_2=0$. For motion with the speed of light $ds=0$, so that by (20)

$$D_1=\infty. \tag{45}$$

Inserting (45) in (37) and (38) we obtain

$$s(r)=0, \tag{46}$$

$$\pm x^4(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}-\frac{\nu}{2}}dr+\text{const.} \tag{47}$$

We can deduce equation (47) according to the footnote on page 127 of [6] directly from the equation of the geodesic (14) substituting in it the parameter s with some appropriate parameter p.

For $i=2$ we get again $d\theta/dp=0$, $\theta=\pi/2$. With these values we obtain for $i=3, 4, 5$ the equations

$$\frac{d^2\varphi}{dp^2}+\frac{2}{r}\frac{dr}{dp}\frac{d\varphi}{dp}=0, \tag{48}$$

$$\frac{d^2x^4}{dp^2}+\nu'\frac{dr}{dp}\frac{dx^4}{dp}=0, \tag{49}$$

$$\frac{d^2x^5}{dp^2}+\varrho'\frac{dr}{dp}\frac{dx^5}{dp}=0. \tag{50}$$

The integral of "energy" (22) obtains here the form

$$-e^{\nu}\left(\frac{dx^4}{dp}\right)^2-e^{\varrho}\left(\frac{dx^5}{dp}\right)^2+r^2\left(\frac{d\varphi}{dp}\right)^2+e^{\lambda}\left(\frac{dr}{dp}\right)^2=0. \tag{51}$$

From (48), (49) and (50) we get the integrals

(52) $$\frac{d\varphi}{dp}=\frac{K_1}{r^2},$$

(53) $$\frac{dx^1}{dp}=\bar{D}_1 e^{-\nu},$$

(54) $$\frac{dx^5}{dp}=\bar{D}_2 e^{-\varrho},$$

K_1, $\bar{D}_1$, $\bar{D}_2$ — constants of integration.

But as light moves with the velocity of light in the four-dimensional subspace of R_{3+2}, $dx^5=0$ holds true. Thus we get from (54)

(55) $$\bar{D}_2=0.$$

We consider the case of radial motion of light rays

(56) $$\frac{d\varphi}{dp}=0.$$

From (51), (53), (54), (55) and (56) we get

$$-\bar{D}_1^2 e^{-\nu}+e^{\lambda}\left(\frac{dr}{dp}\right)^2=0,$$

or

(57) $$-\bar{D}_1^2 e^{-\nu}+e^{\lambda}\left(\frac{dr}{dx^1}\right)^2\bar{D}_1^2 e^{-2\nu}=0.$$

From (57) we obtain

$$\pm x_4(r)=\int_{r_0}^{r} e^{\frac{\lambda}{2}-\frac{\nu}{2}}\,dr+\text{const.}$$

But this is exactly equation (47).

From equation (47) follows that the fall time of a photon, measured by a far away observer, in the gravitational field of a point mass starting from $r=r_0$ to the vicinity of the gravitational centre ($r=0.08$) is finite. (We assume that the gravitational radius is equal to 1.)

Consequently, a star which has collapsed under its gravitational radius, remains observable for a far away observer, contrary to the now generally accepted opinion.

Let us consider this question somewhat more in detail. We consider a number of similar atoms vibrating at different points in the region. Let the centres of gravity of the atoms be for a given interval of time t_4 at rest in our co-ordinate system (r, θ, φ, t_4, t_5). The test of similarity of the atoms is that corresponding intervals should be equal, and accordingly the interval of vibration of all the atoms will be the same.

Since the atoms are in rest we set $dr=0$, $d\theta=0$, $d\varphi=0$, $dx^5=0$ in the interval (1). The last condition $dx^5=0$ shows that the atoms do not move in the direction of the new dimension x^5, i. e. in this case we disregard the influence of the new time t_5 on the atomic processes. Then we get from (1)

(58) $$ds^2=e^{\nu}(dx^4)^2=e^{\nu}c_4^2\,dt_4^2.$$

Accordingly, the times t_4 of vibration (or the periods t_4 of the vibration) of the differently placed atoms will be inversely proportional to $e^{\nu/2}$.

For $r \gg 1$ we have $e^{\nu/2} \sim \sqrt{1-\frac{1}{r}}$. For $r \to 1$ and even for $r \to 0$ $e^{\nu/2}$ is different from zero and consequently a far away observer will be able to perceive the spectral lines belonging to the light radiated from atoms placed on the Schwarzschild sphere and even deeply beneath it.

APPENDIX

According to tables I and II in [5] the function ν' increases boundlessly at $r \to 0$. It is worthwhile to investigate the pole of the solution ν' of equation (12) at $r=0$. For this purpose we develop ν' in the series

(59) $$\nu' = \frac{a_{-1}}{r} + a_0 + a_1 r + a_2 r^2 + \cdots$$

From (59) follows

(60) $$\nu'' = -\frac{a_{-1}}{r^2} + a_1 + 2a_2 r + \cdots$$

Substituting (59) and (60) in (12) we obtain

(61) $$-\frac{a_{-1}}{r^2} + a_1 + 2a_2 r + \cdots = -2\left(\frac{a_{-1}}{r} + a_0 + a_1 r + a_2 r^2 + \cdots\right)$$
$$\times\left\{\frac{1}{r} + \frac{1}{4}\left[2(1+\alpha)\left(\frac{a_{-1}}{r} + a_0 + a_1 r + a_2 r^2 + \cdots\right)\right.\right.$$
$$\left.\left.+\frac{1}{2}\alpha r\left(\frac{a_{-1}}{r} + a_0 + a_1 r + a_2 r^2 + \cdots\right)^2\right]\right\}.$$

Comparison of the coefficients before $1/r^2$ gives

(62) $$\frac{1}{4}\alpha a_{-1}^2 + (1+\alpha)a_{-1} + 1 = 0.$$

The roots of equation (62) are

(63) $$(a_{-1})_{1,2} = -\frac{2(1+\alpha)}{\alpha} \pm \frac{2}{\alpha}\sqrt{1+\alpha+\alpha^2} = \frac{2}{\alpha}\left(-1-\alpha \pm \sqrt{1+\alpha+\alpha^2}\right).$$

As ν' increases at $r \to 0$ it is clear that $a_{-1} > 0$. Consequently at $\alpha = -0.1$ we shall have

(64) $$a_{-1} = \frac{2}{\alpha}\left(-1-\alpha-\sqrt{1+\alpha+\alpha^2}\right) > 0.$$

For $r \to 0$

(65) $$\nu' \sim \frac{2}{\alpha}\left(-1-\alpha-\sqrt{1+\alpha+\alpha^2}\right)\frac{1}{r},$$

(66) $$\nu \sim \frac{2}{\alpha}\left(-1-\alpha-\sqrt{1+\alpha+\alpha^2}\right)\ln r.$$

In this way we have determined the character of the pole of our solution ν' of equation (12) for $r=0$.

I wish to thank prof. Ivan Nedelkov for some stimulating discussions.

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ДОКАЗАТЕЛСТВО, ЧЕ ВРЕМЕТО, ИЗМЕРВАНО ОТ ДАЛЕЧЕН НАБЛЮДАТЕЛ, ЗА КОЕТО ЕДНО ПРОБНО ТЯЛО ПАДА В ЦЕНТРАЛНО СИМЕТРИЧНОТО ПОЛЕ НА ДВУВРЕМЕННАТА ТЕОРИЯ НА ОТНОСИТЕЛНОСТТА И ИДВА БЛИЗКО ДО ЦЕНТЪРА, Е КРАЙНО

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(Резюме)

В редица от работи, цитирани в настоящата работа, авторът разглежда едно петмерно обобщение на Айнщайновите гравитационни уравнения, като Римановото петмерно пространство има сигнатура 3,2. В сферичносиметричния случай тези уравнения могат сравнително лесно да се интегрират във вид на елементарни функции или с помощта на електронни сметачни машини. На голямо разстояние от сферата на Шварцшилд новите решения преминават в решенията на Шварцшилд. Новите решения зависят от една константа, която се определя от ефекта на Дике, свързан със сплеснатостта на Слънцето и преместването на перихелия на Меркурий. Изхождайки от принципа, че свободното движение на една материална точка се извършва по геодезична линия в петмерното пространство, авторът получава първите интеграли на движението от уравнението на геодезичната линия. Като се ограничава със специалния случай на радиално движение, авторът показва, че времето на падането на едно пробно тяло в сферичносиметричното поле до околността на центъра е крайно за разлика от теорията на Айнщайн, където това време е безкрайно голямо. В случая става дума за времето, мерено от далечен наблюдател. Този резултат е много важен, тъй като той показва, че едно тяло, колапсирало под своя гравитационен радиус, остава наблюдаемо за далечен наблюдател.

ДОКАЗАТЕЛЬСТВО, ЧТО ИЗМЕРЕННОЕ ДАЛЕКИМ НАБЛЮДАТЕЛЕМ ВРЕМЯ, ЗА КОТОРОЕ ПРОБНОЕ ТЕЛО ПАДАЕТ В ЦЕНТРАЛЬНОЕ СИММЕТРИЧНОЕ ПОЛЕ ДВУХВРЕМЕННОЙ ТЕОРИИ ОТНОСИТЕЛЬНОСТИ И ПРИХОДИТ БЛИЗКО К ЦЕНТРУ, ЯВЛЯЕТСЯ КОНЕЧНЫМ

Н. Калицин

(Резюме)

В ряде работ, цитируемых в настоящей статье, автор рассматривает пятимерное обобщение гравитационных уравнений Эйнштейна, причем риманово пятимерное пространство имеет сигнатуру 3,2. В сферично симметричном случае эти уравнения можно сравнительно легко интегрировать в виде элементарных функций или с помощью электронно-счетных машин. На большом расстоянии от сферы Шварцшильда новые решения переходят в решения Шварцшильда. Новые решения зависят от константы, которая определяется эффектом Дике, связанным со сжатием у полюсов Солнца и смещением перигелия Меркурия. Основываясь на принципе, что свободное движение материальной точки в пятимерном пространстве происходит по геодезической линии, из уравнения геодезической линии автор получает первые интегралы движения. Ограничиваясь специальным случаем радиального движения, автор показывает, что время падения пробного тела в сферично-симметричном поле до области центра конечно, в отличии от теории Эйнштейна, где это время бесконечно велико. В данном случае, речь идет о времени, измеренном далеким наблюдателем. Этот результат очень важен, так как показывает, что тело, колапсированное под своим гравитационным радиусом, остается наблюдаемым для далекого наблюдателя.