БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ * BULGARIAN ACADEMY OF SCIENCES
ИЗВЕСТИЯ НА СЕКЦИЯТА ПО АСТРОНОМИЯ
BULLETIN OF THE SECTION OF ASTRONOMY
Том (Vol.) III

TIME DISTRIBUTION OF THE PERSEIDS

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INTRODUCTION

The study of time distribution of the meteors of a certain meteor stream is closely connected with the solution of one of the most important problems of meteor astronomy — the investigation of the stream structure.

The stream structure investigation of a meteor stream suggests the knowledge of the space coordinates and the velocities of its bodies. Nevertheless, as the number of the meteor bodies of a given stream is quite large and not all of them could be observed, the problem of the investigation of a stream is to a large extent a statistical problem, in which the moments of the meteor appearance are analysed. Here one has made use of the fact, that in first approximation the velocities of the stream meteor bodies are equal and their space coordinates are given with the coordinates of the moving Earth, which passes through the stream.

The different meteor streams should have different structure, which at the given moment would depend on the initial conditions and on the rate of the stream evolution. The evolution itself takes place under the action of the perturbations mainly from part of the Solar System planets, so that after a quite continuous interval of time one would expect the stream to have a Poisson character. The stream evolution is also influenced to a certain extent by the action of the Pointing-Robertson effect.

The stream Perseids is one of the streams, whose structure has been investigated most often after statistical analysis of the meteor time distribution. This fact is explained with the comparatively great density of the stream meteor bodies (a large hourly rate especially about the maximum), as well as with its continuous activity.

Millman [1] in 1936 pointed out, that by random distribution, the time intervals between two successive appearances of meteors should follow an exponential law. The analysis of the data of the visual observations of the Perseids during 1932—1935 show a predomination of the number of the small intervals in comparison with the theoretically expected ones. Nevertheless the author considers, that it depends rather on random variations and errors made when registering the meteors, than on the actual deviations

of the Perseid distribution from the random one. Fifteen years later Agadjanova with the help of the criterion χ^2 compared the distribution of the Perseids after Savruchin's observations with the random one and showed that the appearances of the stream meteors deviate considerably from the random distribution [2]. Kresak and Vosarova arrived to a contrary conclusion, when they analysed 1037 meteors, which belong to the Perseid stream and which have been observed during 21 hours from August 11 to August 16, 1952 [3]. They analyse besides the time intervals between the meteor appearances (after Millman) the number of the appearances of the n meteors in 5-minute intervals also, comparing this number with Poisson's distribution.

Recently the radar observations of the Perseids have been analysed too and at that controversal data about the distribution of the meteors of the stream have been obtained. For instance, Bowden and Davies, who besides the methods referred have made use of still two other ones, obtain that the radioechoes from the Perseids (on August 9—10, 1954) do not show a considerable deviation from the random distribution [4]. Nevertheless McKrosky, making use of the criterion χ^2, obtains a deviation for the Perseids from Poisson's distribution [5]. Poole's result is controversial [6].

The lack of common attitude about the structure of the Perseids, as obtained from the analyses of the distribution of the meteors from the stream, compelled us to return to this problem. Unlike other authors, we have analysed (after two methods) a quite extensive material, obtained from visual observations of the Perseids during 1956, 1958, 1959, 1961 and 1962. The data obtained during the first four years have been published partially [7—10], and those from 1962 are not published.

APPEARANCE OF METEORS AS A POISSON PROCESS. OBSERVATIONAL MATERIAL

While investigating the structure of the meteor stream Perseids and of the meteor background after the observations referred we checked up the hypothesis, that time appearance of the meteors is a Poisson process.

As it is well known, a process is called a homogeneous Poisson process (a simple stream), if it corresponds to the following three conditions:

A. Stationarity — meaning that for each group of an extreme number of non-overlapping time intervals, the probability of the realization of a definite number of events along every one of them, depends on this number and on the length of the intervals of the same magnitude. Particularly, the probability for the realization of k events during the interval of time $(T, T+t)$ does not depend on T, but appears as a function only from k and t.

B. Absence of consecution — the probability of the realization of the k events during the time interval $(T, T+t)$ does not depend on the probability of realization of the event for $\tau < T$.

C. Ordinariness — it expresses the requirement for the impossibility of the realisation of two or more events during the small enough interval of time h.

Let us consider in detail the condition of stationarity, which is of a substantial importance for the investigation of the time distribution of meteors.

As long as all terrestrial observations register only meteors (and beisdes not all), which are provoked by meteor bodies, crossing the orbit of the Earth, the stationarity would require an uniform space density of the streams, if the surveys are made for large intervals of time. It is clear, that this condition is realized for the background meteors, which (at least in the first approximation) have an uniform space density. According to modern conceptions the meteor bodies, which provoke these meteors, are formed usually by disruption of the meteor streams. And so the suggestion that the background meteors must have a Poisson time distribution is justified. By the shower meteors and especially by the Perseids a complex structure of the streams is being observed and besides some effects interfere, which proceed from the nature of the observations themselves, for example:

1. It is known that the microstructure of the Perseids shows an availability of characteristic space regions with dimensions conforming to the time of the order of 20—30 minutes which is necessary [13—14]. Therefore for these observations closed in intervals, which are smaller than this characteristic time the stream would not be stationary, especially when we have in mind, that in a similar interval quite a few meteors are being observed. We shall have stationarity in observations of the order of several hours.

2. The observations of meteor streams, which we consider stationary during one night would not show stationarity as the height of the radiant over the horizon changes. This effect leads to a change of the statistical characteristics which correspond to successive space regions.

3. Observations of streams even with a considerable cross-section, like the Perseids stream, carried out during several successive nights must absolutely show nonstationarity of the stream, because of changes (radial) of the mean space density.

Finally the Perseids are not a stationary stream, although they could be considered as such in first approximation for about one observational night. But under the condition that observations are carried out during almost hundred years and the mean change of the hourly rate of the meteors with the change of the longitude of the Sun is known, it could be pointed out, that when $\lambda_{\odot}$ is given, we have for the different years hourly rate distributions which do not change considerably. In this sense the stream is stationary.

From the observations diring the years, cited in the Introduction, we have selected only moonless nights with favourable atmospheric conditions: therefore we refuse the possibility of making use of different coefficients, which is done in [3]. These observations have not been corrected for the height of the radiant and for the number of the observers. The basic information about the observational material is given in Table 1.

ANALYSIS I

If we denote with n_i the number of the intervals, in which i meteors have been registred ($i=0, 1, 2, \ldots$) the number of all meteors is $\sum in$, and it means that the mean number of meteors in an interval is

$$\lambda=\frac{\sum in_i}{\sum n_i}.$$

Then the distribution of the meteor number must follow the Poisson law

$$P(i)=\frac{\lambda^i e^{-\lambda}}{i!}$$

where $P(i)$ is the probability that in a given interval there would be exactly i meteors.

Our basic hypothesis H_0 is that the distribution of the number of the meteors in 10-minute intervals is a Poisson one with a given confidence interval.

In order to verify this hypothesis we made use of the statistical criterion χ^2.

Table 1

Year; Observational Place	Data	Observational interval	Total number of the meteors	Number of the Perseids
1956 Vitosha	7/8. VIII	4h01m39s	55	36
	8/9	3 48 50	70	38
	9/10	3 06 19	64	29
	10/11	4 45 52	113	73
	11/12	5 19 38	235	160
Σ		21 02 18	537	336
1958 Astronomical Observatory	11/12	4 14 40	137	92
	12/13	6 03 35	262	202
	13/14	4 00 45	88	49
Σ		14 19 00	487	343
1959 Reservoir "Iskăr"	8/9	4 08 18	93	41
	10/11	4 19 55	78	40
	11/12	3 45 41	129	73
	12/13	3 47 23	206	160
Σ		17 00 57	506	314
1961 Astronomical Observatory	9/10	2 45 51	75	33
	10/11	3 28 09	92	42
	11/12	3 30 20	115	74
	12/13	4 57 26	204	125
Σ		14 41 46	486	274
1962 Vitosha	3/4	4 20 13	220	40
	4/5	4 18 46	225	42
	6/7	2 56 37	190	65
	8/9	2 44 35	177	62
	9/10	2 25 15	155	95
Σ		16 45 26	967	303
ΣΣ		83 49 27	2983	1570

It is known that the probability

$$P(\chi^2 > \chi_q^2) = q\,\%,$$

where χ_q^2 is the tabular value of the statistical criterion, when we accepted $q = 0.1$.

Table 2 gives the number of the meteors (Perseids — P, background — B) in successive 5-minute intervals for the respective years.

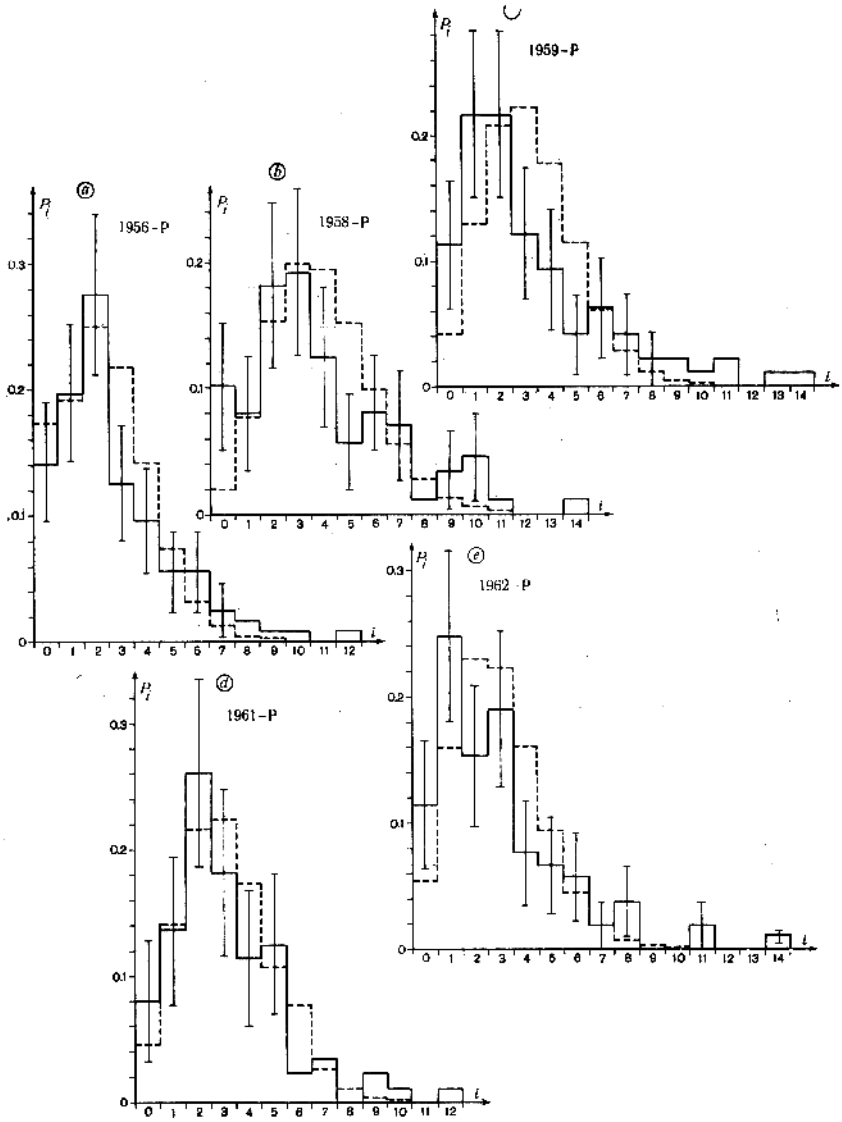

Fig. 1

Table 2

№	1956										1958						1959								1961								1962									
	7/8		8/9		9/10		10/11		11/12		11/12		12/13		13/14		8/9		10/11		11/12		12/13		9/10		10/11		11/12		12/13		3/4		4/5		6/7		8/9		9/10	
	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B	P	B
1	1	0	1	1	1	1	0	2	1	1	1	0	2	1	0	3	0	3	2	2	4	1	1	1	0	3	2	0	0	2	2	1	0	7	1	8	1	3	1	4	5	6
2	2	0	1	1	1	2	0	0	1	0	1	3	0	0	0	0	3	1	0	0	2	1	0	1	0	1	0	2	0	1	1	2	0	2	0	2	0	7	3	4	3	3
3	0	0	0	1	1	2	0	0	2	1	2	1	0	0	5	0	1	0	1	0	3	0	0	0	0	3	1	0	0	1	1	1	2	3	0	4	4	4	1	5	0	2
4	0	0	0	1	1	1	2	0	0	0	0	1	1	0	0	0	0	1	0	0	1	2	2	0	0	0	2	0	1	2	3	1	0	4	3	3	1	5	1	7	2	4
5	1	0	0	0	1	0	2	0	1	1	0	0	2	0	0	0	1	1	1	1	0	3	3	0	1	1	0	0	2	0	0	1	1	1	1	5	0	5	2	3	2	1
6	1	0	2	0	0	0	1	0	5	0	2	0	1	1	3	0	0	1	0	1	1	2	3	2	1	0	2	2	3	1	2	0	0	3	0	1	0	5	0	4	3	0
7	2	0	2	2	2	1	2	2	3	0	2	1	1	1	0	0	2	4	1	0	0	0	7	2	2	1	0	1	1	1	1	2	0	4	0	1	0	2	2	6	3	3
8	0	1	2	0	1	0	1	1	1	3	0	0	1	0	0	0	1	0	1	2	1	1	2	0	1	2	0	3	1	2	1	1	1	4	1	4	1	1	2	1	1	2
9	0	0	1	1	2	2	1	1	4	1	2	1	2	0	2	2	1	2	0	0	3	2	2	0	1	2	0	0	2	0	2	0	1	1	0	6	0	3	0	2	1	1
10	0	0	1	0	0	0	3	0	0	0	1	1	4	1	0	0	1	0	0	0	1	2	4	2	0	1	0	3	0	2	0	3	1	4	0	2	0	4	3	6	6	1
11	1	1	0	0	0	1	2	0	4	1	2	1	0	2	0	1	0	1	1	1	1	0	3	2	2	2	1	0	2	0	3	1	0	4	1	8	1	2	1	3	2	1
12	1	0	0	0	0	0	2	0	3	0	2	1	1	0	0	0	0	1	0	1	3	0	3	1	2	1	0	0	3	1	2	3	0	1	0	3	2	2	2	2		
13	0	0	0	1	1	1	2	0	4	1	2	1	1	0	0	0	2	1	1	1	1	1	1	0	0	0	1	0	0	1	1	1	0	3	0	5	0	5	1	3	4	2
14	0	1	2	1	3	0	0	2	0	0	0	1	2	1	1	0	0	1	0	1	2	0	1	2	0	0	2	1	1	0	0	1	0	6	0	1	0	3	0	4	4	0
15	0	2	1	0	0	3	1	0	3	0	7	0	0	1	3	0	2	0	1	0	2	2	4	3	1	1	0	2	1	1	1	0	2	3	1	4	0	3	2	5	4	2
16	1	1	1	1	0	0	1	0	1	0	0	0	1	2	2	0	0	2	0	1	2	4	9	0	0	1	2	1	1	3	3	2	0	4	0	8	0	6	2	3	4	3
17	1	1	0	2	1	0	0	1	2	1	0	1	1	0	0	1	0	0	0	1	2	4	3	0	1	0	1	1	2	0	4	0	0	4	3	3	3	4	2	2	2	4
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20	1	0	0	0	1	2	1	0	2	5	6	1	2	1	0	0	1	0	1	2	1	1	2	1	0	0	1	1	1	0	0	2	0	4	0	5	0	6	1	3	5	1
21	1	0	1	0	1	0	0	1	1	2	0	1	1	0	0	3	0	0	0	2	0	1	3	2	2	4	0	3	2	0	1	1	0	3	0	4	4	2	3	3	3	3
22	2	0	0	2	0	2	1	0	2	3	2	1	1	0	0	1	1	0	0	1	2	1	9	0	0	2	1	1	0	1	1	2	2	1	1	2	1	3	0	3	5	0
23	0	1	1	0	1	1	2	0	4	0	0	0	1	0	0	2	2	3	2	0	0	4	3	0	1	2	1	1	2	0	0	1	1	5	0	0	2	0	2	3	2	2
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34	0	2	0	0	3	0	2	3	0	4	3	1	4	1	2	3	1	1	0	1	0	0	5	0	1	0	3	2	3	2	2	2	0	3	2	1	2	0			
35	0	1	1	2	0	0	0	1	2	0	1	0	1	3	2	1	0	1	0	1	1	0	8	0			0	0	3	1	1	0	1	1	2	2	2	6			
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42	1	0	3	1			1	2	2	0	3	2	4	1	2	1	2	1	1	2	2	1	4	2			1	2	2	1	3	0	1	7	1	1					
43	2	0	1	0			0	0	6	0	5	2	2	1	2	1	0	0	1	0	2	1	4	3							2	0	1	3	1	3					
44	1	0	2	2			1	1	2	2	4	2	4	1	1	2	0	1	2	1	1	1	6	1							3	1	1	3	2	1					
45	0	1	0	1			1	1	3	2	1	1	6	4	0	0	0	1	2	0	2	0	0	1							1	2	3	4	2	3					
46	1	0	0	1			3	1	2	1	2	2	4	0	1	1	0	1	2	2	0	2									2	1	1	3	1	5					
47	2	0					2	0	4	5	3	1	2	0	0	1	1	3	2	1											3	3	1	7	1	2					
48							1	0	5	1	4	0	4	2	2	0	2	2	0	0											7	1	0	5	0	6					
49							2	1	5	2	0	0	5	0	0	1	3	0	0	0											2	4	2	3	0	3					
50							1	0	5	1	3	1	4	2					2	1											3	0	1	3	2	1					
51							3	0	2	1	2	2	3	1					0	2											6	3	1	1	3	4					
52							1	0	4	1			4	0					0	2											3	2	1	2	2	4					
53							3	0	2	0			3	0																	4	1									
54							1	1	3	0			4	1																	3	2									
55							1	1	2	2			3	2																	4	3									
56							0	2	1	0			8	0																	8	2									
57							1	2	3	1			4	1																	2	1									
58							0	1	3	1			6	3																	2	3									
59									4	2			4	0																											
60									2	0			4	2																											
61									0	2			3	1																											
62									2	0			2	1																											
63													3	0																											
64													4	1																											
65													5	3																											
66													9	0																											
67													3	0																											
68													7	0																											
69													3	3																											
70													2	0																											
71													4	2																											

In Figures 1—3 the empiric (with a solid line) and the theoretic (with a dotted line) Poisson distributions for 10-minute intervals for Perseids, background and Perseids + background are given, respectively. All distributions are normilized to 1.

In order to define the unknown probability $P(i)$ of the event, so that in a given 10-minute interval one could register exactly i meteors, we make

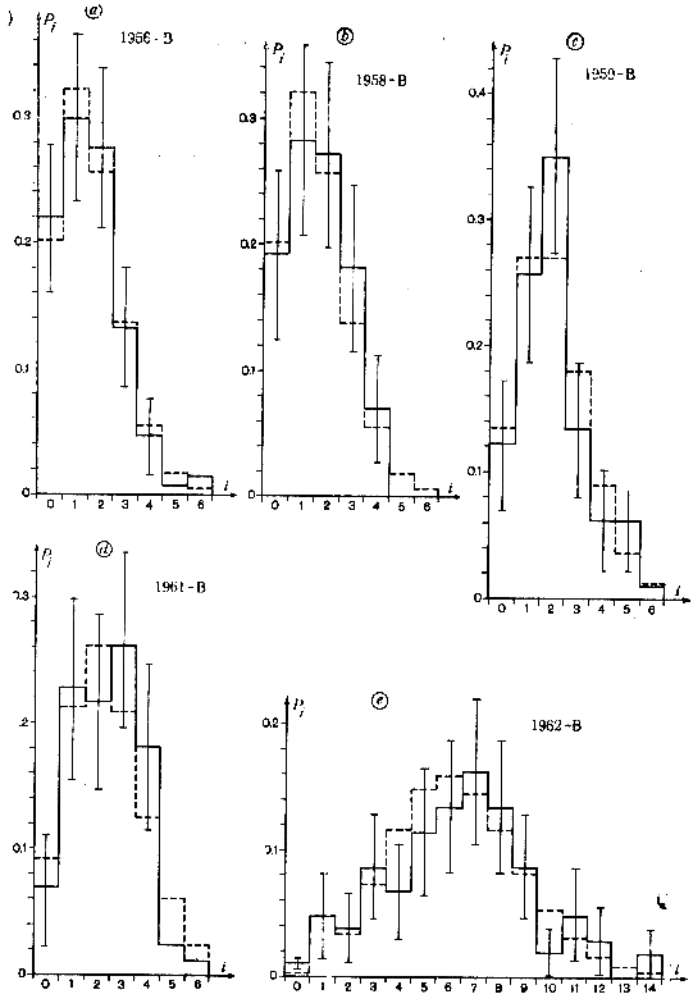

Fig. 2

use of the method of the confidence intervals of the unknown probability [15]. If we denote with $P_i^* = \frac{n_i}{n}$ a value, which is asymptotic normally distributed

$$N\left(P_i, \sqrt{\frac{P_i(1-P_i)}{n}}\right),$$

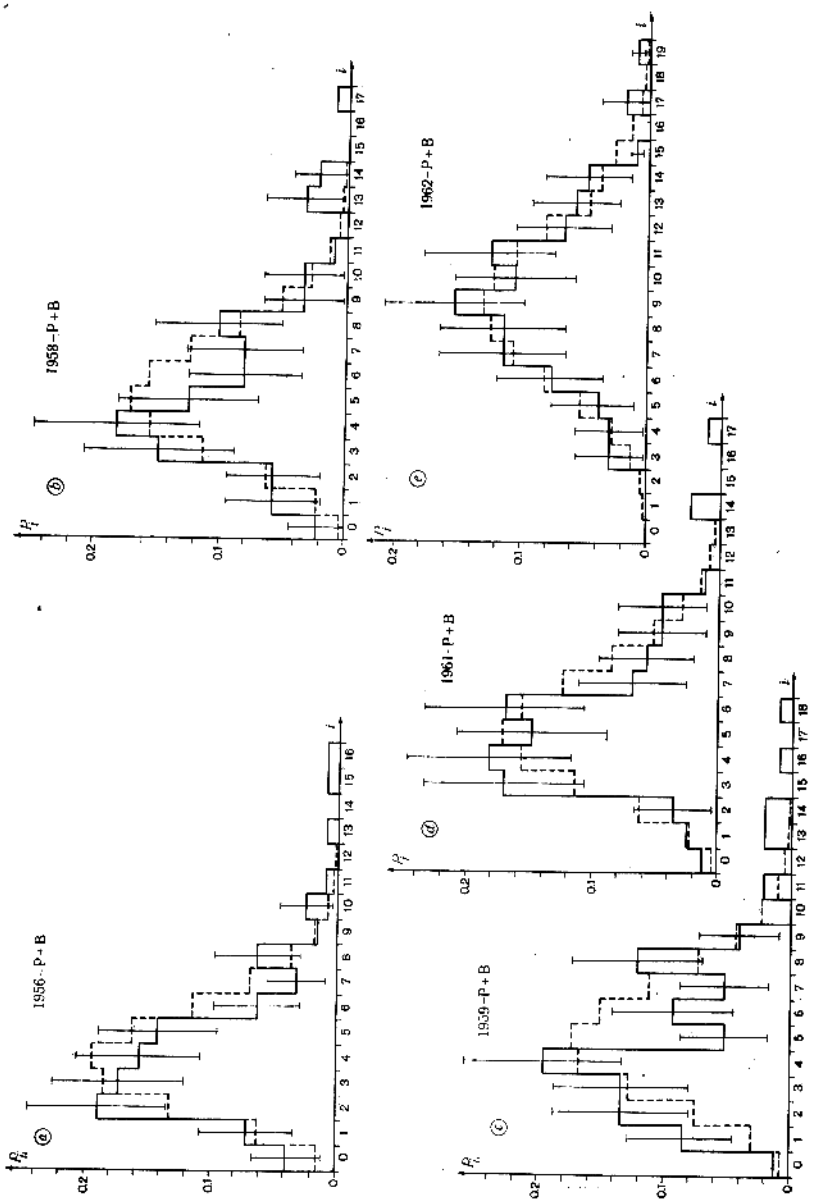

Fig. 3

Table 3

N.	1956		1958		1959		1962		1961	
	P	B	P	B	P	B	P	B	P	B
1	0.142 ± 0.060	0.220 ± 0.074	0.102 ± 0.064	0.192 ± 0.084	0.113 ± 0.064	0.122 ± 0.066	0.115 ± 0.064	0.010 ± 0.006	0.080 ± 0.058	0.068 ± 0.054
2	0.197 ± 0.070	0.300 ± 0.082	0.080 ± 0.058	0.284 ± 0.096	0.217 ± 0.084	0.258 ± 0.088	0.249 ± 0.085	0.048 ± 0.042	0.136 ± 0.074	0.227 ± 0.090
3	0.276 ± 0.080	0.276 ± 0.080	0.182 ± 0.082	0.272 ± 0.092	0.217 ± 0.084	0.351 ± 0.096	0.154 ± 0.070	0.038 ± 0.037	0.261 ± 0.094	0.216 ± 0.088
4	0.126 ± 0.058	0.134 ± 0.060	0.193 ± 0.084	0.182 ± 0.082	0.122 ± 0.066	0.134 ± 0.068	0.191 ± 0.078	0.086 ± 0.054	0.182 ± 0.082	0.261 ± 0.094
5	0.096 ± 0.052	0.047 ± 0.038	0.125 ± 0.070	0.070 ± 0.054	0.093 ± 0.060	0.062 ± 0.050	0.077 ± 0.052	0.067 ± 0.048	0.114 ± 0.068	0.182 ± 0.082
6	0.055 ± 0.040		0.057 ± 0.048		0.041 ± 0.040	0.062 ± 0.050	0.067 ± 0.048	0.115 ± 0.064	0.125 ± 0.070	
7	0.055 ± 0.040		0.080 ± 0.058		0.062 ± 0.050		0.058 ± 0.044	0.135 ± 0.066		
8	0.024 ± 0.024		0.070 ± 0.054		0.041 ± 0.040		0.019 ± 0.026	0.163 ± 0.072		
9							0.038 ± 0.037	0.135 ± 0.066		
0								0.086 ± 0.054		

the limits of the confidence intervals are presented with

$$\gamma_{1,2}=P^{*}\pm\lambda_{\varepsilon}\sqrt{\frac{P^{*}(1-P^{*})}{n}},$$

where λ_{ε} is 100 % meaning of the normal deviation. In Figures 1—3 the confidence intervals of the unknown probability $P(i)$ for $\lambda_{\varepsilon}^{*}=1.6$, are also presented which corresponds to a 89 % deviation from the normal distribution.

Table 3 presents the confidence intervals of the unknown probability $P(i)$, with $\lambda_{\varepsilon}^{*}=2$, which corresponds to a 95 % deviation from the normal distribution.

Table 4 gives the values of λ (the mean number of meteors in 10-minute intervals), the respective χ^2, the degrees of freedom ν and the level of the significance α.

ANALYSIS II

While investigating the length distribution of the intervals between the successive appearances of the meteors, we define a mean time interval $T=S/N$, where S is the common observational time and N — the number of all intervals registered in it.

Millman [1] points out, that the probability of an interval between two successive random appearances of meteors by random distribution having a value between t_1 and t_2 is

$$P(t_1<\eta<t_2)=\int_{t_1}^{t_2} e^{-t/T}\frac{dt}{T}=e^{-t_1/T}-e^{-t_2/T}.$$

The expected number of intervals with a length $\eta=t_2-t_1$ is $NP(t_1<\eta<t_2)$.

The hypothesis that our observational material gives the upper exponential distribution was verificated with the criterion χ^2, as well as with Kolmogorov's criterion. Let us suggest, that on the basis of a sufficiently large number of independent observations of the random value x, on the base of its meanings is composed

$$x_1 \leqq x_2 \leqq x_3 \leqq \cdots \leqq x_n .$$

Here the hypothesis we verify with these data co-ordinates with the assumption, that the considered random value has a definite continuous integral distribution function $F(x)$. Let us suggest, that the hypothesis is correct and let us denote with $\widetilde{F}_n(x)$ the step function of the summary frequencies of the observed order, i. e.

$$\widetilde{F}_n(x) = \begin{cases} 0 & x < x_1 \\ \frac{k}{n} & x_1 < x \leqq x_{k+1} \\ 1 & x > x_n \end{cases}$$

and we form the difference $|\widetilde{F}_n(x) - F(x)|$.

$$D_n = \max_{-\infty < x < \infty} |F_n(x) - F(x)|$$

presents namely the measure of disagreement. According to Kolmogorov's distribution, the limit probability of the event $(D_n \sqrt{n})$, will not surpass the preliminarily given number

$$\lim P(D_n \sqrt{n} \leqq \lambda) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2\lambda^2} = k(\lambda)$$

for each continuous function $F(x)$. (This criterion is inapplicable in Analysis I as the Poisson distribition there is discreet.)

The application of Kolmogorov's agreement criterion consists of the following: we find the largest difference D_n^0 between the values of the

Table 4

Year	P	B	P+B
1956	$\lambda=2.6$ $\chi^2=28.15$ $\nu=6$ $\alpha=0.00009$	$\lambda=1.6$ $\chi^2=0.73$ $\nu=3$ $\alpha=0.86$	$\lambda=4.2$ $\chi^2=20.82$ $\nu=7$ $\alpha=0.004$
1958	$\lambda=3.9$ $\chi^2=43.55$ $\nu=7$ $\alpha=0$	$\lambda=1.6$ $\chi^2=2.08$ $\nu=3$ $\alpha=0.55$	$\lambda=5.5$ $\chi^2=26.77$ $\nu=8$ $\alpha=0.0005$
1959	$\lambda=3.02$ $\chi^2=36.67$ $\nu=5$ $\alpha=0$	$\lambda=2$ $\chi^2=5.51$ $\nu=4$ $\alpha=0.24$	$\lambda=5.2$ $\chi_2=29.08$ $\nu=7$ $\alpha=0.0001$
1961	$\lambda=3.1$ $\chi^2=5.87$ $\nu=5$ $\alpha=0.31$	$\lambda=2.4$ $\chi^2=2.58$ $\nu=3$ $\alpha=0.46$	$\lambda=5.5$ $\chi^2=6.88$ $\nu=6$ $\alpha=0.33$
1962	$\lambda=2.9$ $\chi^2=35.65$ $\nu=6$ $\alpha=0$	$\lambda=6.4$ $\chi^2=11.39$ $\nu=9$ $\alpha=0.23$	$\lambda=9.3$ $\chi^2=1.46$ $\nu=8$ $\alpha=0.99$
Total without 1962	$\lambda=3.2$ $\chi^2=302$ $\nu=8$ $\alpha=0$	$\lambda=1.9$ $\chi^2=1.18$ $\nu=5$ $\alpha=0.94$	$\lambda=5.1$ $\chi^2=182$ $\nu=11$ $\alpha=0$
Total	$\lambda=3.1$ $\chi^2=234.5$ $\nu=8$ $\alpha=0$	$\lambda=2.8$ $\chi^2=531$ $\nu=8$ $\alpha=0$	

Intervals	1956															1958								
	7/8			8/9			9/10			10/11			11/12			11/12			12/13			13/14		
	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B
0—20		1	3	1		6	2	1	10	9	3	15	34	5	51	12		23	38	5	57	8	1	12
21—40	1		5		2	2	1	4	4	6	2	9	16	6	35	8	3	21	22	4	42	2	1	5
41—60	1	1	2	2	1	4	2	1	5	7	2	8	20	8	33	9	3	18	29	4	35	5	1	12
61—80				3	2	8	1	2	3	2		3	17	5	26	5	1	11	20	4	30	6	3	9
81—100	5	1	8	2		5	2		1	2	1	7	14	10	22	5	2	8	17	2	26	3	3	7
101—120	1	1	2	2		3	2	1	9	7	2	13	11	1	16	7	2	8	17	1	16	2	2	5
121—140	1		1	1	2	5	1	1	2	4	2	9	9		12	5	2	7	10		10	3	1	2
141—160	2	1	4	3	1	2		1	2	4		5	5	4	8	2	2	8	8	1	7	3		4
161—180				2	2	6		2	3	4	1	5	5	2	7	4		8	6		7	1	1	3
181—200	1		1	2	1	5		2	2	3	2	4	8	1	5	4	2	2	6	2	7	1	1	
201—220		1	3	2	2	4	1	2	3	3		4		3	3	3	2	5	7	2	7	1	2	4
221—240	4		3		1	2	2	1	4	4	2	5		4	3	2		4	1		3	2	2	5
241—260						3		1	2	1	1	5	2	1	1	1	2		1	1	4	2		2
261—280	1	1	1	1		3	1		1	3	2	7	3	1	2	2	1	2	1	2	1			
281—300					1	2		1		1	1	3	2	1	2	4	1		7	2	1		1	2
301—320	1	1	2						1				2		1	1	1	1	1	2	1			1
321—340	2		4	1	1	1		1	1		1	1	1	2	1	1		3		1		1		1
341—360	1		2	2	1	1					3	3		1	1	2	2	1		1		1		
361—380	1		2	2		1					1	1	2	1		1	3	2	1	4	2	1		
381—400			1		2	1		1	2			2	2	2	1	2		1	2	1	1		2	3
401—420	1			1				2	1		1	1		1			2					2		
421—440					2			1	1	1	1		1	2	2		1		2	3	1			2
441—460	1		1	1			1		1				1	1			1		2		1	1		1
461—480		1	1		1		2			1				1		1	1			1			1	
481—500									1	2										1				
501—520	1		1	1						1							2	1				1		
521—540	1				1				1	1						1				3				
541—560	1			1		1	2	2					1				1	1					2	
561—580								1	1	1			1	1	1									
581—600	1		2	1	1		1									1				1				
601—620				1			1				1			1			1			2				
621—640		1					1	2												1				
641—660							1	2			1			1		1	1							
661—680	1			1		1					1		1	1						1			1	
681—700					1					1						1		1					1	
701—720		1						1									1			2	1			
721—740	2	1	1	1						1	1			1		1					1			
741—760			1								1			1										
761—780	1						1																	
781—800		1	1				1				1			1			1							
801—820					1								1											
821—840															1					1				1
841—860																								
861—880																	1							
881—900										1														1
901—920		1	1	1						1	1			1										
921—940					1						1			1										
941—960							1									1				1				
961—980		1											1							1				
981—1000			1				1	1			1			1			1							
1001—1020																								
1021—1040																								
1041—1060				1																				
1061—1080		1																					1	
1081—1100																	1							
1101—1120					1					1														
1121—1140																								
1141—1160																								
1161—1180				1				1																
1181—1200		1																						

1959												1961										
8/9			10/11			11/12			12/13			9/10			10/11			11/12			12/13	
P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B
3	2	9	4	2	10	11	6	23	43		69	3	5	13	4	7	19	11	5	21	19	10
2	1	7	9	1	10	6	5	14	27	5	43	1		6	1	1	8	6	2	12	14	6
1	5	10	3	2	6	7	11	23	20	3	25	2	3	10	2	3	7	6	2	13	14	4
3	5	10	1	2	7	3	1	10	13	2	13	1	5	10	3	2	6	2		7	3	7
1	1	6	1		3	6	4	14	16	2	19	3	2	2	1	3	3	4		5	9	4
	3	7	1		2	1	2	5	9		9		1	3	1	3	6	5	1	9	11	2
1	1			3	5	4		6	3	4	7	1	2	3	1	3	9	7	2	7	9	2
2	4	6	1	2	4		3	3	3	1	5	1	3	4		5	7	7	1	8	10	7
1	2	9	1	1	4	5	2	4	6	4	3	3	1	3	1	1	2	1	1	7		1
	3	2			1	2	1	6	4	3	5	2	1	4	2		2	3	2	5	6	3
2		5		1	3	3	1	4	3	1	2	1	1	1	2	3	3	1	2	2	4	2
1		2	2	2	2	3	1	2	2	2	3		2	2	2	2	3		1	2	1	
2	1	1				1	1	1		1	1	2	1	3	1	1	3	3	2	3	4	2
2	4	3	1	2	4	4		1	4	1	2	2	2	1	1			3	1	2	1	3
1	1	1		2	1	3		2	1	1						1			2	3		4
	3	5	2	1	2	1		3		2			1		2		2	3	3	2	1	4
	1		1	1	2	1	1	1		1	1				3	1	2	1	1	1	1	1
1	1	1	1	2	1				1						1	1	2	1	1	1	4	1
			2				1	1				1		1		1		2	1	1	1	2
1	1			1		2			1				1	1	2	2	2	1		1	2	1
			1		2						1				1		1		1			2
1	1	3	1	1		1	1		1						1			1				1
	2				2	1	1						2	2		3	1	1				
2	1	1		2	1	1	2			2			1	1	1	1	2	1			3	1
	1			2	1				1	3	1	1			1			1	1			
1							1								1	1	1					
						1	1					1		1		1		2		1	1	1
		1					2						3	1	3				1			1
3							1					1									1	
	1						1						1	1					2		1	1
			1			1							1									
						1	1												1			
										2			2									
				1											1							
				1	1										1				1			
1	1						1															
1	1												1									1
																					1	
	1	1	1																			
	1					1													1			
										1												1
1	3		1		1	1				1				1					1			1
		1		1	1		1						1									
	1		1	1		1	1						1									
					1					1				1					1			1
	1	1	1			1															1	
							1							1								
				1																		
1	1																					
	1		1			1																
										2						1						
										1						1						
	1		1			1																
1	1																					
1	1			1												1						

Table 5

1962														
3/4			4/5			6/7			8/9			9/10		
P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B	P	B	P+B
3	31	47	3	40	51	9	28	55	9	27	52	25	7	42
2	35	46	1	32	47	9	17	37	4	18	34	16	8	30
1	29	34	4	19	35		21	29	7	9	26	6	7	28
	18	26	3	20	20	6	13	23	3	13	23	9	6	14
4	13	18		16	15	4	11	11	4	9	6	10	4	14
2	9	8	1	14	17	4	3	7	4	11	13	4	4	9
2	8	11	1	6	9	4	8	9	3	6	10	4	1	4
	8	11	3	4	6	4	4	4	1	4	3	4	1	3
2	8	6	1	5	7	2	1	3	4	3	4	2	1	3
1	2	1	2	5	4	4	5	2	3	4	2	2	3	4
1	3	2	2	7	4	2	1	1	2	5		3		
	1	2	1	2	1	2	2	2	3	1		3	1	
	6	2	2	1	1	2	1	1	2	1	2	1	2	
1	2	1		1	2	1	2	2		2	1		3	
1	1			1	1	2			2				1	1
1				3	2		1	1	3	2			1	
2	1	1		1		1	2	1	1				2	1
1										1		1	1	
	1			2		2	2						1	
1			1	1									1	
	1	1	1				1		2					
			2	1	1				1			1	1	
1						1	1							
		1	1											
1									1				1	
1														
1	1													
2			1									1		
			1									1		
						1			1					
									1					
1													1	
1														
			1											
							1							
			2											
			1			1								
1														
1			1											
			1											
1														
			2											
1														

Table 6

Yea	P	B	P+B
1956	$T=226$ $\chi^2=27.89$ $\nu=16$ $\alpha=0.03$	$T=379$ $\chi^2=15.28$ $\nu=16$ $\alpha=0.5$	$T=142$ $\chi^2=24.53$ $\nu=13$ $\alpha=0.03$
1958	$T=159$ $\chi^2=23.66$ $\nu=11$ $\alpha=0.01$	$T=393$ $\chi^2=11.57$ $\nu=16$ $\alpha=0.77$	$T=106$ $\chi^2=17.01$ $\nu=10$ $\alpha=0.07$
1959	$T=185$ $\chi^2=70.98$ $\nu=13$ $\alpha=0$	$T=301$ $\chi^2=18.17$ $\nu=14$ $\alpha=0.2$	$T=115$ $\chi^2=30.42$ $\nu=11$ $\alpha=0.004$
1961	$T=200$ $\chi^2=15.44$ $\nu=12$ $\alpha=0.22$	$T=255$ $\chi^2=15.58$ $\nu=15$ $\alpha=0.42$	$T=111$ $\chi^2=6.38$ $\nu=10$ $\alpha=0.78$
1962	$T=207$ $\chi^2=35.27$ $\nu=13$ $\alpha=0.0008$	$T=92$ $\chi^2=8.05$ $\nu=9$ $\alpha=0.53$	$T=63$ $\chi^2=5.62$ $\nu=7$ $\alpha=0.58$
Total without 1962	$T=196$ $\chi^2=94.88$ $\nu=21$ $\alpha=0$	$T=340$ $\chi^2=21.3$ $\nu=24$ $\alpha=0.62$	$T=121$ $\chi^2=52.43$ $\nu=14$ $\alpha=0$
Total	$T=199$ $\chi^2=190.8$ $\nu=19$ $\alpha=0$	$T=221$ $\chi^2=126.1$ $\nu=23$ $\alpha=0$	$T=156$ $\chi^2=2350$

Table 7

Year	P	B	P+B
1956	$N=336$ $D_n^0=0.0908$ $\lambda_0=1.66$ $\alpha=0.008$	$N=200$ $D_n^0=0.0536$ $\lambda_0=0.76$ $\alpha=6.61$	$N=536$ $D_n^0=0.0244$ $\lambda_0=0.57$ $\alpha=0.90$
1958	$N=324$ $D_n^0=0.121$ $\lambda_0=2.2$ $\alpha=0.0001$	$N=131$ $D_n^0=0.0688$ $\lambda_0=0.94$ $\alpha=0.34$	$N=478$ $D_n^0=0.0418$ $\lambda_0=0.92$ $\alpha=0.36$
1959	$N=311$ $D_n^0=0.1573$ $\lambda_0=2.76$ $\alpha=0$	$N=181$ $D_n^0=0.0742$ $\lambda_0=0.998$ $\alpha=0.27$	$N=503$ $D_n^0=0.0747$ $\lambda_0=1.67$ $\alpha=0.07$
1961	$N=267$ $D_n^0=0.056$ $\lambda_0=0.92$ $\alpha=0.36$	$N=207$ $D_n^0=0.6263$ $\lambda_0=0.381$ $\alpha=0.99$	$N=473$ $D_n^0=0.0254$ $\lambda_0=0.55$ $\alpha=0.92$
1962	$N=292$ $D_n^0=0.1027$ $\lambda_0=1.74$ $\alpha=0.004$	$N=655$ $D_n^0=0.0225$ $\lambda_0=0.58$ $\alpha=0.89$	$N=958$ $D_n^0=0.0124$ $\lambda_0=0.38$ $\alpha=0.99$

hypothetic function of distribution $F(x)$ and the stop function $\tilde{F}_n(x)$ and then we form $\lambda=D_n^0\sqrt{n}$. If at a definite level of agreement $\alpha=10\%$ the value of $1-k(\lambda_0)$ is smaller, i. e. if a not very probable event is realized, the hypothesis is rejected and vice versa, if the value of $1-k(\lambda_0)$ is sufficiently large, we have obviously to acknowledge the disagreement between the observed order and the hypothetic distribution as random and we

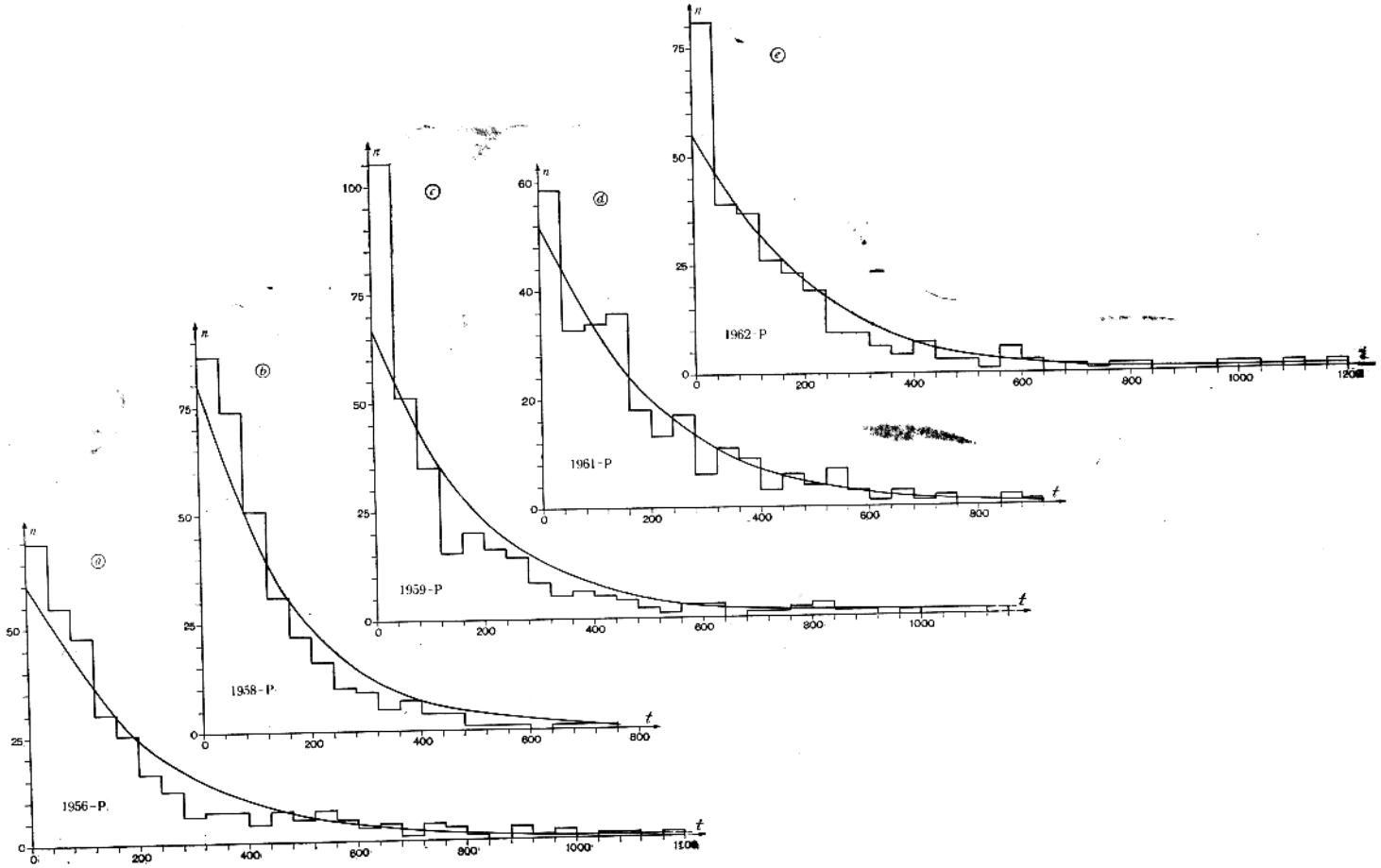

Fig. 4

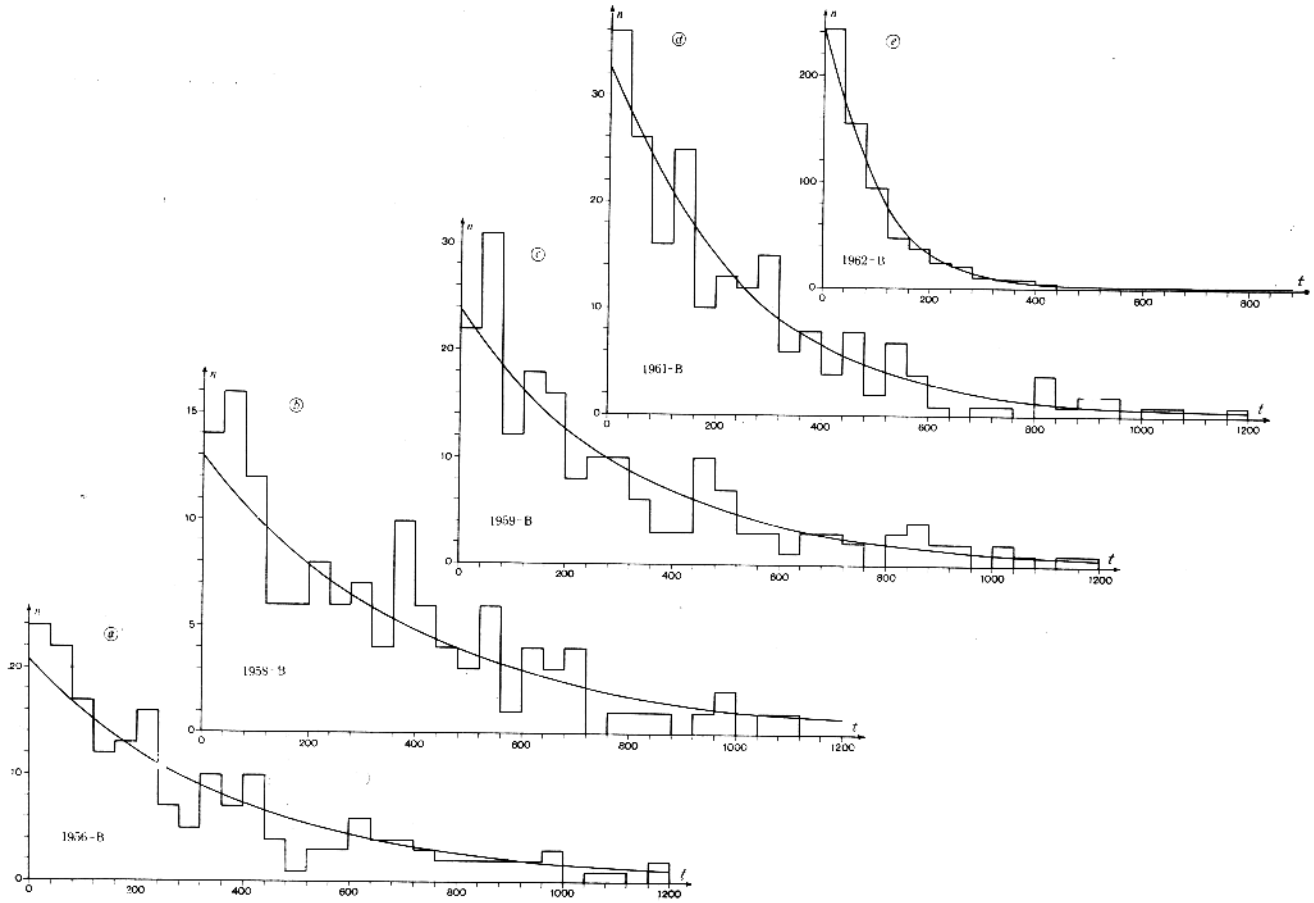

Fig. 5

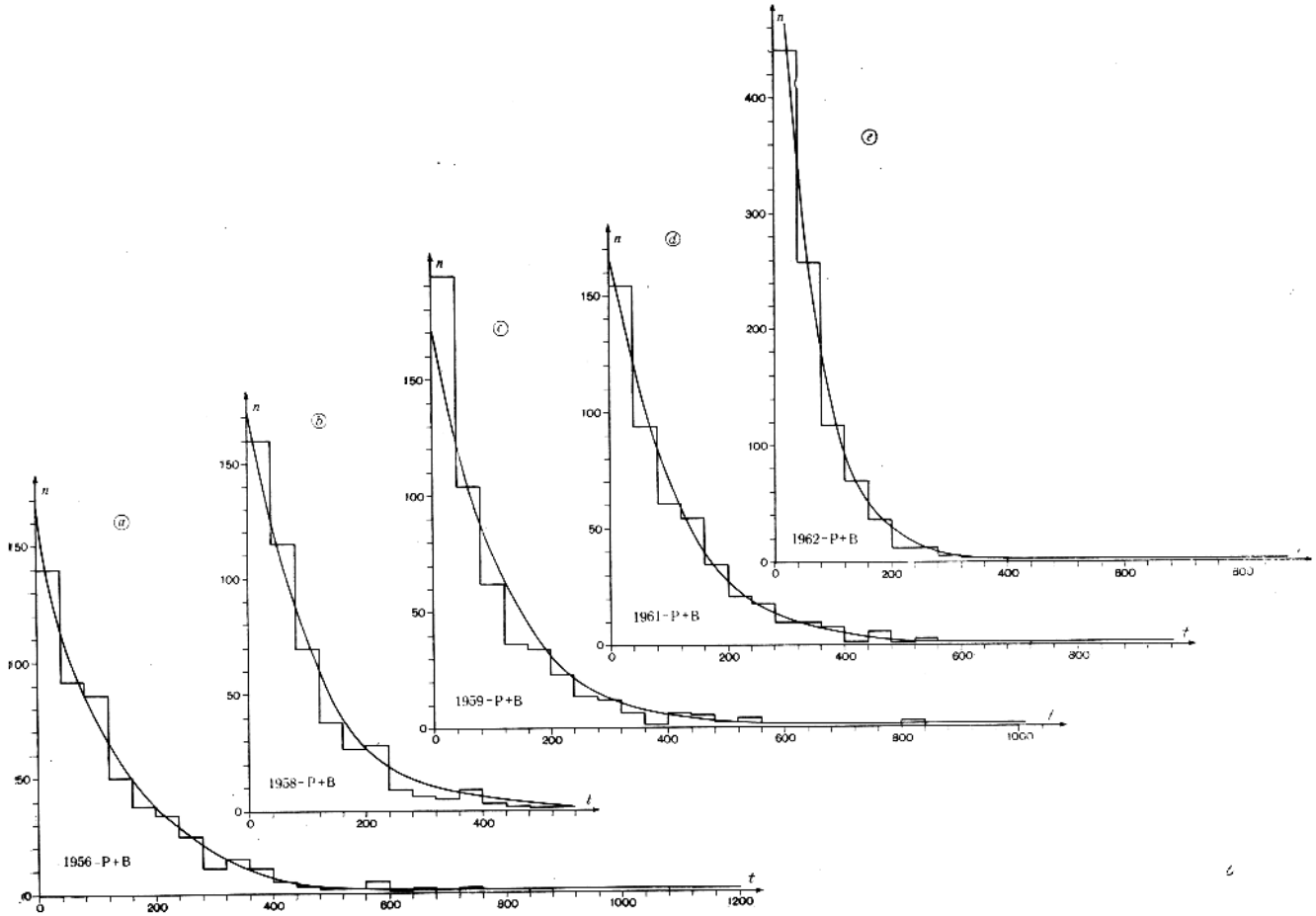

Fig. 6

can consider with this stipulation the distribution $F(x)$ coordinated with the observed distribution $\tilde{F}_n(x)$.

Table 5 presents the number of intervals between $m\delta t$ and $(m+1)\delta t$, where $\varepsilon t = 20^s$ for Perseids, background and Perseids + background. The intervals larger than 1200^s have not been taken into consideration. Table 6 presents the values of T (the mean interval of time) and the results of the application of the χ^2-criterion, while Table 7 gives the results of Kolmogorov's criterion.

Fig. 4 for Perseids, Fig. 5 for background and Fig. 6 for Perseids+background present the number distribution of the intervals between $m\delta t$ and $(m+1)\delta t$, where $\delta t = 40^s$. The continuous line presents the theoretic distribution.

DISCUSSION

The observational material is comparatively homogeneous and by virtue of it we can draw some conclusions about the general structure of the meteor stream Perseids from visual observations. Let ts consider the results year by year.

1956. Analysis I. The hypothesis of Poisson distribution of the number of the registered meteors in 10-minute intervals in the case of the Perseids is rejected ($\chi^2 = 28.15$, $\nu = 6$). For the background meteors the respective distribution is Poisson ($\chi^2 = 0.73$, $\nu = 3$).

Let us consider the distribution of all observed meteors (Perseids+background). For the given level of agreement $\alpha = 0.1$, from the obtained value $\chi^2 = 20.82$, $\nu = 7$, hence the hypothesis of the Poisson distribut on is rejected. The last result could be expected, as the number of the Perseids surpasses that of the background (Perseids—336; background—201).

Analysis II. The hypothesis of the exponential distribution of the time intervals between the successive appearances of the meteors for Perseids is rejected, but for the background meteors it is accepted.

By Perseids+background the results of the application of Kolmogorov's criterion and the graphic comparison of the theoretic and empiric distribution is in agreement with the hypothesis of the exponential distribution of the time intervals. The last result is not confirmed by the χ^2-criterion, but this difference is probably due to the grouping of the last intervals.

1958. Analysis I. The number distribution of the meteors in 10-minute intervals of the Perseid stream and Perseids+backg o nd shows considerable deviations from the Poisson distribution and that of the background meteors is a Poisson one ($\chi^2 = 2\ 08$, $\nu = 4$).

Analysis II. The distribution of the time intervals between the successive meteors presents results which are analogous to those from 1956 for Perseids, background and Perseids+background.

The basic conclusion for the time distribution of the background meteors shows that it is random, while that of the Perseids points to considerable deviations from the random distribution.

1959. Analysis I. The results of this method are analogous to those from 1956 and 1958 ($\chi^2 = 36.67, \nu = 5$; $\chi^2 = 5.51$, $\nu = 4$; $\chi^2 = 29.08$, $\nu = 7$).

Analysis II. The distribution of the time intervals between the successive meteors from the Perseid stream and Perseids+background presents consi-

derable deviations from the exponential distributions. The hypothesis for the respective distribution of the background meteors is accepted.

The final conclusion for the observations during 1956 points that the appearance of background meteors in time is a Poisson stream, while the time distribution of the Perseids is not a random one.

1961. Analysis I. The number distribution of the meteors in 10-minute intervals for Perseids, background and Perseids+background is a Poisson distribution ($\chi^2=5.87, \nu=5$; $\chi^2=2.58, \nu=3$; $\chi^2=6.88, \nu=6$).

The result obtained for the Perseids may be explained with the atmospheric conditions, as one has observed only meteors brighter than $+4^m, 5$. The statistic treatmeant of the moments of the appearance for the brighter Perseids must lead to the conclusion, that the distribution is a Poisson one. In general the larger meteor bodies which yield brighter meteors must have a more uniform space distribution in the stream, since there the Pointing — Robertson effect can be neglected. The microstructure of the stream (for observations in the order of several hours) can be established best while observing visually faint meteors from $5-6^m$, but not the faintest radiometeors, the larger part of which have a comparatively uniform space distribution.

Analysis II. The distribution of the time intervals between the successive meteors from the Perseids stream, the background and Perseids+background is exponential, which has to be expected taking into consideration the results of Analysis I.

The final conclusion of the observations for 1961 points out that the appearance of brighter meteors of the Perseids stream and the background meteors in time is a Poisson stream.

1962. Analysis I. The number distribution of the meteors in 10-minute intervals from the Perseid meteor stream is not a Poisson one ($\chi^2=35.65$, $\nu=6$), while that of the background meteors and Perseids+background is a Poisson one.

Analysis II. The hypothesis of the exponential distribution of the time intervals between the successive meteors of the Perseids is rejected. The respective distribution of the background meteors and Perseids+background is exponential.

The final conclusion from the observations during 1962 is that the appearance of background meteors in time is a Poisson stream, while for the Perseids this fundamental hypothesis is rejected.

General distribution of all stated above meteors. For all Perseids (1970) background ones (1413) and Perseids+background (2983) the numbers distribution of the meteors in 10-minute intervals is not Poisson.

Here are particularly involved the observations from 1962, when the hourly rate of the background meteors is very high in comparison with that of other years. If we exclude three observations, then we obtain that the respective distributions of Perseids and Perseids+background are not Poisson ones, and one of the background meteors is Poisson, as we had to expect it.

We received analogous results for the distribution of the time intervals between the successive meteors. For all meteors, except those observed during 1962, the respective distributions for the background meteors are

exponential. Nevertheless for Perseids and for Perseids+background the hypothesis for the exponential distribution is rejected.

The following are the most important conclusions of the present paper:

1. The meteor stream Perseids is not a Poisson one. This fact is established even when the mean hourly rate of the background is larger than the respective one for the Perseids. Hence the space structure of the meteor stream Perseids has a quite complex character.

This conclusion is in contradiction with some authors' results, quoted in the Introduction, which is probably due either to unfavourable observational conditions (Moon, clouds etc.) or to the registering of quite faint radiometeors, provoked by meteor bodies of insignificant masses. Obviously, by the latter, one has to observe a transposition of the stream.

2. The background meteors present a random space distribution, which is quite natural, when we take into consideration their origin.

The authors of the present paper express their profound gratitude to Dr. Boyan Penkov from the Mathematical Institute of the Bulgarian Academy of Sciences for his valuable advice, as well as for his participation in the discussion of the above stated results.

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Presented on September 7, 1967

РАЗПРЕДЕЛЕНИЕ ПО ВРЕМЕ НА МЕТЕОРИТЕ ОТ ПОТОКА ПЕРСЕИДИ

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(Резюме)

Върху наблюдателен материал, получен през 1956, 1958, 1959, 1961 и 1962 г. през време на действието на метеорния поток Персеиди, е анализирано разпределението на метеорите по време. Резултатите от

анализа са следните: 1. Метеорният поток Персеиди не е поасонов поток, което се установява даже и тогава, когато средното часово число на фона е по-голямо от съответното за Персеидите. Този извод е в противоречие с резултатите на някои автори, цитирани във въведението, което вероятно се дължи на неблагоприятни условия за наблюдение (Луна, облачност и др.) или пък на регистрирането на твърде слаби радиометеори, които са предизвикани от метеорни тела с нищожни маси. 2. Фоновите метеори имат случайно пространствено разпределение, което е напълно естествено, като се има пред вид техният произход.

РАСПРЕДЕЛЕНИЕ ПО ВРЕМЕНИ МЕТЕОРОВ ПОТОКА ПЕРСЕИД

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(Резюме)

На основе наблюдательного материала, полученного в 1956, 1958, 1959, 1961 и 1962 гг. во время метеорного потока персеид, анализировано распределение метеоритов по времени. Результаты анализа сводятся к следующему:

1. Метеорный поток персеид не является поасоновым потоком, что можно установить даже тогда, когда среднее часовое число фона больше соответствующего числа персеид. Этот вывод противоречит результатам некоторых авторов, цитированных в начале статьи, причиной чего, вероятно, являются неблагоприятные условия для наблюдений (Луна, облачность и др.) или регистрация очень слабых радиометеоров, которые вызваны метеорными телами с ничтожными массами.

2. Фоновые метеоры имеют случайное пространственное распределение, что вполне естественно, если иметь в виду их происхождение.