Star formation under explosion mechanism in a magnetized medium

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Abstract. The model of star formation has been developed as a result of explosive phenomena in the central region of Our Galaxy. The shock wave generated as a result of such explosion, during its propagation, cools and compresses the ambient medium in a thin layer, which subsequently fragments into molecular clouds. These clouds finally form star clusters or field stars under gravitational collapse. We primarily consider the central region of Our Galaxy under the influence of a magnetic field for modeling such explosive phenomena and derive the minimum Jeans mass for gravitational collapse. It is found that under an inverse variation of temperature with density, a wide range of fragments can be formed. The mass range is enhanced in the presence of a constant as well as varying magnetic field. Under suitable physical conditions a burst of star formation is possible. It is also found that rotation of such fragmented clouds of the order of few km s^{-1} kpc^{-1}, might lead to a stable structure.
Key words: galaxies: star formation — physical data and processes: shock waves — physical data and processes: instabilities

Introduction

Star formation remains a tantalizing problem in modern astronomy. Most of the astronomers now believe that the star formation is triggered by the gravitational collapse of dense molecular clouds but very little information has been gathered about the long path leading from diffuse interstellar matter to emerging main sequence stars (McKee & Ostriker, 2007; Kennicutt & Evans, 2012). Various theories arising out of the observations and as well as from simulations have been developed to demonstrate the mechanism of the growth and evolution of these clouds leading to the formation of star clusters or field stars (Lada & Lada, 2003; Khoperskov et al., 2013). It is believed that different activities occurring in the nuclei of galaxies including Our Galaxy has immense influence on the growth and evolution of these molecular clouds (Burbidge, 1970; Morris & Serabyn, 1996).

The evidence of such violent activities is revealed by the photographs of galaxies like M82, M87, NGC5128 etc. (Lynds & Sandage, 1963; Solinger, 1969). Besides photographic studies, the spectroscopic observations also reveal similar violent phenomena in the nuclei of some Seyfert galaxies. From the nuclei of these galaxies huge clouds of gas is ejected with a speed of the order of two thousand kilometers per second. NGC1068 is a good example of such high velocity ejection. Similar types of observations have been found by other authors (Van der Kruit, 1970 & 1971; Sanders & Prendergast, 1974; Defouw, 1976). Molecular clouds are formed due to those ejected materials through propagating shock fronts from the Galactic center (Alūzas et al., 2012 & 2014). Also numerical studies of cloud formation in dense shells through gravitational fragmentation have been done by several authors (Dale et al. 2009, Dale et al. 2011, Iwasaki et al. 2011).

The actual reason of such explosion has not been established yet but many authors have suggested different mechanisms. One such mechanism is "Black hole hypothesis" (Vanbeveren, 1978). Some authors (e.g. Sargent et al., 1978) have proposed that the existence of a black hole of mass $5 \times 10^9 M_\odot$ in the nucleus of M87 is responsible for the high energy activities near the center of M87. It is now well accepted that the formation of Galactic disc including the molecular rings and spiral arms is closely associated with the generation of shock waves as a result of nuclear explosion (Kato, 1977; Saitō & Saito, 1977; Sofue, 1994) and shock induced star formation in shock compressed molecular clouds have been modeled by several authors (Roberts, 1969; Kutner et al., 1977; Elmegreen & Lada, 1977 & 1978; Lada et al., 1978; Whitworth et al., 1994; Koyama & Inutsuka, 2000; Balfour et al., 2015). Molecular rings found in the central region of Our Galaxy have been modeled by various authors as a result of explosion shocks (Saitō & Deguchi, 1980; Bhattacharyya & Basu, 1982; Saha et al., 1985; Basu & Kanjilal, 1989).

In the present work we have modeled the shock induced star formation as a result of central explosion in Our Galaxy including magnetic field and rotation for a shock compressed medium which is more realistic than the previous models. In Section 1 we have described the model in presence of a constant magnetic field. Section 2 includes a constant magnetic field with constant rotation of molecular clouds. Section 3 includes a varying magnetic field with constant rotation of molecular clouds. Final conclusions are given in Section 4.

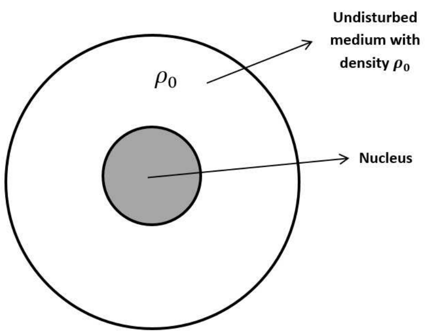

Fig. 1. Schematic diagram of central part of a galaxy before explosion

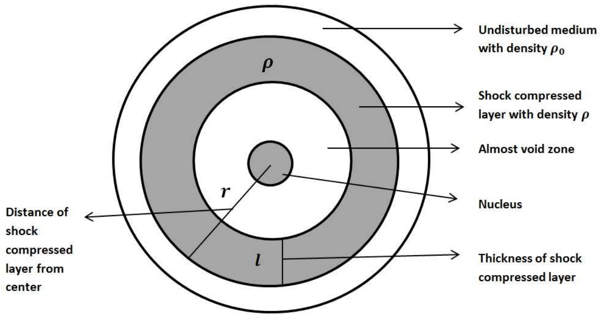

Fig. 2. Schematic diagram of central part of a galaxy after explosion

1 Mathematical Model: Presence of a constant magnetic field

Let us consider a spherical distribution of matter near the Galactic center. When a central explosion occurs shock waves are generated and as the shock moves gradually outwards it compresses the gas within a thin spherical shell of thickness l (say) (Figs. 1 and 2). Let r be the distance of a shock front from the Galactic center at time t. We assume that negligible amount of matter is blown away. Then from the conservation of mass we get

$$\int_0^r 4\pi r^2 \rho_0 dr = \frac{4}{3}\pi[r^3 - (r-l)^3]\rho. \quad (1)$$

Initial distribution of radial mass density for the unperturbed disk is taken as, $\rho_0 = sr^{-\alpha}$ (Saitō & Deguchi, 1980). Where s, α are constants and s is determined from the boundary condition $\rho_0 = 460\ cm^{-3}$ at $r = 1$ kpc (Saitō & Deguchi, 1980). For strong non-radiating shock we take $\alpha = 1.8$ (Saitō & Deguchi, 1980). Since $r \gg l$ then from equation (1)

$$\int_0^r 4\pi r^{2-\alpha}\ s\ dr \simeq 4\pi r^2 l\rho,$$

which gives

$$\rho = \frac{sr^{1-\alpha}}{(3-\alpha)l}$$

$\Rightarrow$

$$l = \frac{r}{3-\alpha}\frac{\rho_0}{\rho} = \frac{r}{3-\alpha}\frac{(\gamma-1)}{(\gamma+1)}, \quad (2)$$

where

$\frac{\rho_0}{\rho} = \frac{(\gamma-1)}{(\gamma+1)}$, (Zel'dovich & Raizer, 1966).

We take adiabatic constant, $\gamma = \frac{4}{3}$, then from (2) $l \simeq 0.119\ r$. Temperature distribution in the shock compressed layer is taken as $T = \bar{B}\rho^{-\beta}$ (Basu & Kanjilal, 1989), where $\bar{B}$ and β are constants and $0 < \beta < 1$ as we expect a moderate rate of change of temperature with density. $\bar{B}$ is determined from the boundary condition $T = 80$ K at $\rho = 460\ cm^{-3}$ (Saitō & Deguchi, 1980) for a specific value of β. Variation of temperature with respect to thickness of shock compressed layers for different values of β are shown in Figure 3. This shows that temperature steadily increases with respect to the thickness of the shock compressed layer for different values of β and temperature falls rapidly for higher values of β.

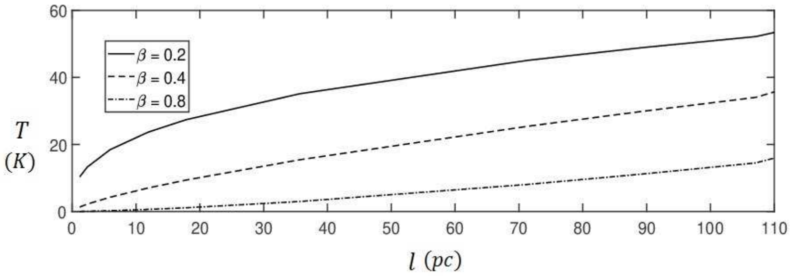

Fig. 3. Variation of temperature (T) with respect to thickness of shock compressed layers (l) for different values of β

Let the spherically symmetric gaseous cloud at rest having ρ_0, p_0, Φ_0, B_0 as density, pressure, gravitational potential and magnetic field respectively, undergoes compression with small perturbation under the explosive phenomenon and after that density, pressure, velocity, gravitational potential and magnetic field become ρ, p, $\boldsymbol{v}$, Φ, B respectively, where
$\rho = \rho_0 + \rho_1$, $\boldsymbol{v} = \mathbf{0} + \boldsymbol{v_1}$, p = $p_0 + p_1$, $\Phi = \Phi_0 + \Phi_1$, $B = B_0 + B_1$ and ρ_1, $\boldsymbol{v_1}$, p_1, Φ_1, B_1 are respective perturbed quantities.

Magnetic field strength is the fundamental quantity to estimate the dynamical importance for evolution of galaxies. The average strength of the magnetic field for a sample of 74 spiral galaxies lies in the range $B_{\text{tot}} = 9$ μG - 11 μG (Nicklas & Beck, 1997). Gas rich spiral galaxies with high star formation rates like M31, M83 and NGC6946 and starburst galaxies like M82 (Adebahr et al., 2013), nuclear starburst regions in NGC4038 (Heesen et al., 2011) and barred galaxies (Beck et al., 2005) etc. have total field strength in the range $B_{\text{tot}} = 20$ μG - 30 μG. Spiral galaxies like Milky Way have weaker total magnetic field of about $B_{\text{tot}} = 10$ μG at $r = 3$ kpc (Beck, 2003). Also magnetic field of Milky Way's disk has component only along azimuthal (ϕ) direction (Ferrière, 2015), i.e. $\boldsymbol{B}$=(0,0,B) where $B \equiv B(r)$.

The governing equations i.e. equation of continuity, equation of motion and Poisson equation are

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{v}) = 0, \tag{3}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}.\boldsymbol{\nabla})\boldsymbol{v} = -\frac{\boldsymbol{\nabla p}}{\rho} + \frac{1}{\rho}(\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla}\Phi + (f(r), 0, 0), \tag{4}$$

$$\frac{1}{r^2 \sin\theta}[\sin\theta \frac{\partial}{\partial r}(r^2 \frac{\partial \Phi}{\partial r}) + \frac{\partial}{\partial \theta}(\sin\theta \frac{\partial \Phi}{\partial \theta}) + \frac{1}{\sin\theta}\frac{\partial^2 \Phi}{\partial \phi^2}] = 4\pi G \rho. \tag{5}$$

After linearization equations (3) and (4) become

$$\frac{\partial \rho_1}{\partial t} + \rho_0(\boldsymbol{\nabla}.\boldsymbol{v_1}) = 0, \tag{6}$$

$$\frac{\partial u_1}{\partial t} = -\frac{\partial \Phi}{\partial r} - \frac{dp}{d\rho_1}\frac{1}{\rho_0}\frac{\partial \rho_1}{\partial r} - \frac{B_0}{\rho_0}\frac{\partial B_1}{\partial r} + f(r), \tag{7}$$

$$\frac{\partial v_1}{\partial t} = -\frac{1}{r}\frac{\partial \Phi}{\partial \theta}, \tag{8}$$

$$\frac{\partial w_1}{\partial t} = -\frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}, \tag{9}$$

where $f(r)$ is the force per unit mass due to shock along radial direction and u_1, v_1, w_1 are the perturbed components of velocity along r, θ and ϕ directions respectively. The perturbation is of the form

$$x_1 = k_1 e^{i(\frac{2\pi r}{\lambda} - \omega t)}, k_1 = constant. \tag{10}$$

1.1 Jeans Mass for Gravitational Collapse

The equation of state for gaseous atmosphere is given by

$$p = R\rho T,\ R = \text{universal gas constant.}$$

With the help of temperature distribution, equation of state become

$$p = R\bar{B}\rho^{1-\beta},$$

from which

$$\frac{dp}{d\rho} = R\bar{B}(1-\beta)\rho^{-\beta},$$

as $0 < \beta < 1$ then $\frac{dp}{d\rho} > 0$. This indicates shock pressure gradually increases with density. Rewriting equations (7), (8) and (9) as

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial u_1}{\partial t}) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial \Phi}{\partial r}) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{dp}{d\rho_1}\frac{1}{\rho_0}\frac{\partial \rho_1}{\partial r}) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{B_0}{\rho_0}\frac{\partial B_1}{\partial r})$$

$$+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f(r)), \tag{11}$$

$$\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta\frac{\partial v_1}{\partial t}) = -\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\Phi}{\partial\theta}), \tag{12}$$

$$\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}(r\sin\theta\frac{\partial w_1}{\partial t}) = -\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}. \tag{13}$$

After adding equations (11), (12) & (13) and using Poisson equation (5) we get

$$\boldsymbol{\nabla}.(\frac{\partial \boldsymbol{v_1}}{\partial t}) = -4\pi G\rho - \boldsymbol{\nabla}.(\frac{dp}{d\rho_1}\frac{1}{\rho_0}\frac{\partial\rho_1}{\partial r},0,0) - \boldsymbol{\nabla}.(\frac{B_0}{\rho_0}\frac{\partial B_1}{\partial r},0,0) + \boldsymbol{\nabla}.(f(r),0,0).$$

Substituting equation (6) in the above equation we obtain

$$-\frac{1}{\rho_0}\frac{\partial^2\rho_1}{\partial t^2} = -4\pi G(\rho_0+\rho_1) - \frac{\partial}{\partial r}(\frac{R\bar{B}(1-\beta)(\rho_0+\rho_1)^{-\beta}}{\rho_0}\frac{\partial\rho_1}{\partial r}) - \frac{B_0}{\rho_0}\frac{\partial^2 B_1}{\partial r^2} + f'(r)$$

for small perturbations $\frac{\rho_1}{\rho_0} < 1$
$\Rightarrow$

$$-\frac{1}{\rho_0}\frac{\partial^2\rho_1}{\partial t^2} \simeq -4\pi G\rho_0 - \frac{R\bar{B}(1-\beta)(1-\beta\frac{\rho_1}{\rho_0})}{\rho_0^{1+\beta}}\frac{\partial^2\rho_1}{\partial r^2}$$
$$+\frac{R\bar{B}\beta(1-\beta)(1-\beta\frac{\rho_1}{\rho_0}-\frac{\rho_1}{\rho_0})}{\rho_0^{2+\beta}}(\frac{\partial\rho_1}{\partial r})^2 - \frac{B_0}{\rho_0}\frac{\partial^2 B_1}{\partial r^2} + f'(r)$$

as $0 < \beta < 1$ then $0 < 1 - \beta\frac{\rho_1}{\rho_0} < 1$
$\Rightarrow$

$$-\frac{1}{\rho_0}\frac{\partial^2\rho_1}{\partial t^2} \simeq -4\pi G\rho_0 - \frac{R\bar{B}(1-\beta)}{\rho_0^{1+\beta}}\frac{\partial^2\rho_1}{\partial r^2} + \frac{R\bar{B}\beta(1-\beta)}{\rho_0^{2+\beta}}(\frac{\partial\rho_1}{\partial r})^2 - \frac{B_0}{\rho_0}\frac{\partial^2 B_1}{\partial r^2} + f'(r).$$

Substituting perturbation form of magnetic field and density (from equation 10) in the above equation, we get the dispersion relation as

$$w^2\frac{\rho_1}{\rho_0} = \frac{4\pi^2}{\lambda^2}[\frac{B_0}{\rho_0}B_1 + R\bar{B}(1-\beta)\frac{\rho_1}{\rho_0^{1+\beta}} - R\bar{B}\beta(1-\beta)\frac{\rho_1^2}{\rho_0^{2+\beta}}] + [f'(r) - 4\pi G\rho_0]$$

$\Rightarrow$

$$w^2 = \frac{4\pi^2}{\lambda^2}[\frac{B_0}{\rho_1}B_1 + R\bar{B}(1-\beta)\rho_0^{-\beta} - R\bar{B}\beta(1-\beta)\rho_0^{-\beta}\frac{\rho_1}{\rho_0}] + \frac{\rho_0}{\rho_1}[f'(r) - 4\pi G\rho_0]. \tag{14}$$

For real wave propagation $w^2 > 0$. Also for small perturbations $\frac{B_1}{B_0} < 1$. Hence equation (14) gives rise to a critical length (Jeans length) for gravitational contraction as (after neglecting the third term on RHS which is a second order perturbation term)

$$\lambda^2 < \lambda_J^2 = \frac{\pi R\bar{B}(1-\beta)\rho_0^{-\beta-1} + \pi\frac{B_0^2}{\rho_0^2}}{G(1-\frac{f'(r)}{4\pi G\rho_0})}.$$

The system is unstable for a linear growth of perturbation if $\lambda^2 \geq \lambda_J^2$, where λ_J is the Jeans length for molecular clouds under a shock force, $f(r) = \frac{P}{l\rho}$, where P is the shock pressure given by $P = \frac{8}{25}\frac{1}{(1+\gamma)}\frac{E}{r^3}$, (Zel'dovich & Raizer, 1966). Hence

$$f(r) = \frac{8}{25}\frac{(3-\alpha)}{(1+\gamma)}\frac{E}{sr^{4-\alpha}}.$$

Therefore the Jeans mass for gravitational collapse of molecular clouds at a distance r from the Galactic center is given by

$$M_J = \tfrac{\pi}{6}\rho_0\lambda_J^3$$

$\Rightarrow$

$$M_J = \frac{\pi}{6}\rho_0\{\frac{\pi R\bar{B}(1-\beta)\rho_0^{-\beta-1} + \pi\frac{B_0^2}{\rho_0^2}}{G(1-\frac{f'(r)}{4\pi G\rho_0})}\}^{\frac{3}{2}}.$$

Table 1. Jeans masses in presence of a constant magnetic field, for various values of parameters

r (pc)	B_0 (μG)	E (erg)	l (pc)	β	T (K)	M_J ($M_\odot$)	r (pc)	B_0 (μG)	E (erg)	l (pc)	β	T (K)	M_J ($M_\odot$)
20	1	10^{54}	2.4	0.2	13.26	4.24	50	1	10^{54}	6	0.2	18.44	15.92
				0.4	2.20	0.33					0.4	4.25	2.08
				0.8	0.06	0.0013					0.8	0.22	0.02
20	10	10^{54}	2.4	0.2	13.26	4.80	50	10	10^{54}	6	0.2	18.44	24.24
				0.4	2.20	0.60					0.4	4.25	7.05
				0.8	0.06	0.12					0.8	0.22	3.10
20	1	10^{56}	2.4	0.2	13.26	3.82	50	1	10^{56}	6	0.2	18.44	15.46
				0.4	2.20	0.30					0.4	4.25	2.02
				0.8	0.06	0.0012					0.8	0.22	0.02
20	10	10^{56}	2.4	0.2	13.26	4.33	50	10	10^{56}	6	0.2	18.44	23.54
				0.4	2.20	0.54					0.4	4.25	6.84
				0.8	0.06	0.10					0.8	0.22	3.02
100	1	10^{54}	12	0.2	23.66	43.58	150	1	10^{54}	18	0.2	27.38	78.96
				0.4	7.00	8.39					0.4	9.37	19.13
				0.8	0.61	0.22					0.8	1.10	0.82
100	10	10^{54}	12	0.2	23.66	111.62	150	10	10^{54}	18	0.2	27.38	322.01
				0.4	7.00	57.67					0.4	9.37	213.55
				0.8	0.61	37.29					0.8	1.10	159.79
100	1	10^{56}	12	0.2	23.66	43.10	150	1	10^{56}	18	0.2	27.38	78.46
				0.4	7.00	8.29					0.4	9.37	19.01
				0.8	0.61	0.22					0.8	1.10	0.81
100	10	10^{56}	12	0.2	23.66	110.37	150	10	10^{56}	18	0.2	27.38	319.97
				0.4	7.00	57.02					0.4	9.37	212.20
				0.8	0.61	36.88					0.8	1.10	158.78
500	1	10^{54}	60	0.2	42.24	496.99	1000	1	10^{54}	119	0.2	54.21	1.64×10^3
				0.4	22.30	241.93					0.4	36.73	1.18×10^3
				0.8	6.22	41.56					0.8	16.87	412.32
500	10	10^{54}	60	0.2	42.24	1.38×10^4	1000	10	10^{54}	119	0.2	54.21	1.52×10^5
				0.4	22.30	1.30×10^4					0.4	36.73	1.50×10^5
				0.8	6.22	1.21×10^4					0.8	16.87	1.46×10^5
500	1	10^{56}	60	0.2	42.24	496.41	1000	1	10^{56}	119	0.2	54.21	1.64×10^3
				0.4	22.30	241.65					0.4	36.73	1.17×10^3
				0.8	6.22	41.52					0.8	16.87	412.14
500	10	10^{56}	60	0.2	42.24	1.38×10^4	1000	10	10^{56}	119	0.2	54.21	1.52×10^5
				0.4	22.30	1.30×10^4					0.4	36.73	1.50×10^5
				0.8	6.22	1.20×10^4					0.8	16.87	1.46×10^5

2 Mathematical Model: Presence of a constant magnetic field along with constant rotation of molecular clouds

Here basic assumptions are same as in Section 1. Additionally here we consider constant rotation ($\boldsymbol{\Omega}$) of molecular clouds, which has component only along the azimuthal direction i.e. $\boldsymbol{\Omega} = (0,0,\Omega)$.

Let $\boldsymbol{v}$ be the velocity in rotating frame and u, v, w are the component of $\boldsymbol{v}$ along r, θ and ϕ directions respectively and corresponding perturbed components are u_1, v_1 and w_1 respectively. The governing equations in rotating frame are

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.\{\rho(\boldsymbol{v} + \boldsymbol{\Omega} \times \boldsymbol{r})\} = 0, \tag{15}$$

$$\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}.\boldsymbol{\nabla})\boldsymbol{v}+2(\boldsymbol{\Omega}\times\boldsymbol{v})+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{r}) = -\frac{\boldsymbol{\nabla p}}{\rho}+\frac{1}{\rho}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}-\boldsymbol{\nabla}\Phi+(f(r),0,0), \tag{16}$$

$$\frac{1}{r^2 \sin\theta}[\sin\theta \frac{\partial}{\partial r}(r^2 \frac{\partial \Phi}{\partial r}) + \frac{\partial}{\partial \theta}(\sin\theta \frac{\partial \Phi}{\partial \theta}) + \frac{1}{\sin\theta}\frac{\partial^2 \Phi}{\partial \phi^2}] = 4\pi G\rho. \tag{17}$$

After linearization equations (15) & (16) become

$$\frac{\partial \rho_1}{\partial t} + \rho_0(\boldsymbol{\nabla}.\boldsymbol{v_1}) = 0, \tag{18}$$

$$\frac{\partial u_1}{\partial t} = -\frac{\partial \Phi}{\partial r} - \frac{dp}{d\rho_1}\frac{1}{\rho_0}\frac{\partial \rho_1}{\partial r} - \frac{B_0}{\rho_0}\frac{\partial B_1}{\partial r} + f(r) + \Omega^2 r + 2\Omega v_1, \tag{19}$$

$$\frac{\partial v_1}{\partial t} = -\frac{1}{r}\frac{\partial \Phi}{\partial \theta} - 2\Omega u_1, \tag{20}$$

$$\frac{\partial w_1}{\partial t} = -\frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}. \tag{21}$$

Combining above three equations (19), (20) & (21) and using Poisson equation (17) we get

$$\boldsymbol{\nabla}.(\frac{\partial \boldsymbol{v_1}}{\partial t}) = -4\pi G\rho - \boldsymbol{\nabla}.(\frac{dp}{d\rho_1}\frac{1}{\rho_0}\frac{\partial \rho_1}{\partial r}, 0, 0) - \boldsymbol{\nabla}.(\frac{B_0}{\rho_0}\frac{\partial B_1}{\partial r}, 0, 0) + \boldsymbol{\nabla}.(f(r), 0, 0)$$
$$+2\Omega\boldsymbol{\nabla}.(v_1, 0, 0) - 2\Omega\boldsymbol{\nabla}.(0, u_1, 0) + \Omega^2.$$

After substituting equation (18) in the above equation, we equate the real parts from both sides which gives

$$-\frac{1}{\rho_0}\frac{\partial^2 \rho_1}{\partial t^2} = -4\pi G(\rho_0 + \rho_1) - \frac{\partial}{\partial r}(\frac{R\bar{B}(1-\beta)(\rho_0 + \rho_1)^{-\beta}}{\rho_0}\frac{\partial \rho_1}{\partial r}) - \frac{B_0}{\rho_0}\frac{\partial^2 B_1}{\partial r^2}$$
$$+f'(r) + \Omega^2.$$

As $0 < \beta < 1$ then for small perturbations $\frac{\rho_1}{\rho_0} < 1$, $0 < 1 - \beta\frac{\rho_1}{\rho_0} < 1$ and $\frac{B_1}{B_0} < 1$. Now substituting perturbation form of ρ_1 and B_1 (from equation 10) in the above equation, we get the dispersion relation as

$$w^2 = \frac{4\pi^2}{\lambda^2}[\frac{B_0}{\rho_1}B_1 + R\bar{B}(1-\beta)\rho_0^{-\beta} - R\bar{B}\beta(1-\beta)\rho_0^{-\beta}\frac{\rho_1}{\rho_0}] + \frac{\rho_0}{\rho_1}[f'(r) - 4\pi G\rho_0 + \Omega^2]. \tag{22}$$

For real wave propagation $w^2 > 0$. Hence equation (22) gives rise to a critical length (Jeans length) for gravitational contraction as

$$\lambda_J^2 = \frac{\pi R\bar{B}(1-\beta)\rho_0^{-\beta-1} + \pi\frac{B_0^2}{\rho_0^2}}{G(1 - \frac{f'(r)}{4\pi G\rho_0} - \frac{\Omega^2}{4\pi G\rho_0})}.$$

Therefore the Jeans mass for gravitational collapse of molecular clouds at a distance r from the Galactic center is given by

$$M_J = \frac{\pi}{6}\rho_0\{\frac{\pi R\bar{B}(1-\beta)\rho_0^{-\beta-1} + \pi\frac{B_0^2}{\rho_0^2}}{G(1 - \frac{f'(r)}{4\pi G\rho_0} - \frac{\Omega^2}{4\pi G\rho_0})}\}^{\frac{3}{2}}.$$

Table 2. Jeans masses in presence of a constant magnetic field along with constant rotation of molecular clouds, for various values of parameters

r (pc)	B_0 (μG)	E (erg)	Ω (km s^{-1} kpc^{-1})	l (pc)	β	T (K)	M_J ($M_\odot$)
20	1	10^{54}	4	2.4	0.2	13.26	4.24
					0.4	2.20	0.33
					0.8	0.06	0.0013
20	10	10^{54}	4	2.4	0.2	13.26	4.80
					0.4	2.20	0.60
					0.8	0.06	0.12
20	1	10^{56}	4	2.4	0.2	13.26	3.82
					0.4	2.20	0.30
					0.8	0.06	0.0012
20	10	10^{56}	4	2.4	0.2	13.26	4.33
					0.4	2.20	0.54
					0.8	0.06	0.10
50	1	10^{54}	4	6	0.2	18.44	15.93
					0.4	4.25	2.08
					0.8	0.22	0.02
50	10	10^{54}	4	6	0.2	18.44	24.24
					0.4	4.25	7.05
					0.8	0.22	3.10
50	1	10^{56}	4	6	0.2	18.44	15.46
					0.4	4.25	2.02
					0.8	0.22	0.02
50	10	10^{56}	4	6	0.2	18.44	23.54
					0.4	4.25	6.84
					0.8	0.22	3.02
100	1	10^{54}	4	12	0.2	23.66	43.58
					0.4	7.00	8.39
					0.8	0.61	0.22
100	10	10^{54}	4	12	0.2	23.66	111.62
					0.4	7.00	57.67
					0.8	0.61	37.29
100	1	10^{56}	4	12	0.2	23.66	43.10
					0.4	7.00	8.29
					0.8	0.61	0.22
100	10	10^{56}	4	12	0.2	23.66	110.37
					0.4	7.00	57.02
					0.8	0.61	36.88
150	1	10^{54}	4	18	0.2	27.38	78.96
					0.4	9.37	19.13
					0.8	1.10	0.82
150	10	10^{54}	4	18	0.2	27.38	322.01
					0.4	9.37	213.55
					0.8	1.10	159.79
150	1	10^{56}	4	18	0.2	27.38	78.46
					0.4	9.37	19.01
					0.8	1.10	0.81
150	10	10^{56}	4	18	0.2	27.38	319.97
					0.4	9.37	212.20
					0.8	1.10	158.78
500	1	10^{54}	4	60	0.2	42.24	497.00
					0.4	22.30	241.94
					0.8	6.22	41.57
500	10	10^{54}	4	60	0.2	42.24	1.38×10^4
					0.4	22.30	1.30×10^4
					0.8	6.22	1.21×10^4
500	1	10^{56}	4	60	0.2	42.24	496.41
					0.4	22.30	241.65
					0.8	6.22	41.52
500	10	10^{56}	4	60	0.2	42.24	1.38×10^4
					0.4	22.30	1.30×10^4
					0.8	6.22	1.20×10^4
1000	1	10^{54}	4	119	0.2	54.21	1.64×10^3
					0.4	36.73	1.18×10^3
					0.8	16.87	412.34
1000	10	10^{54}	4	119	0.2	54.21	1.52×10^5
					0.4	36.73	1.50×10^5
					0.8	16.87	1.46×10^5
1000	1	10^{56}	4	119	0.2	54.21	1.64×10^3
					0.4	36.73	1.17×10^3
					0.8	16.87	412.15
1000	10	10^{56}	4	119	0.2	54.21	1.52×10^5
					0.4	36.73	1.50×10^5
					0.8	16.87	1.46×10^5

3 Mathematical Model: Presence of a varying magnetic field along with constant rotation of molecular clouds

Here basic assumptions are same (except the nature of magnetic field) as in Section 2. Here we consider a density dependent magnetic field in the form of a power law, $B \equiv B(\rho) = B_0(\frac{\rho}{\rho_0})^\kappa$ (Boss, 1999), where B_0 and ρ_0 are the initial values of magnetic field strength and density respectively. This dependence of magnetic field strength on density has fairly robust outcome for the asymptotic value, $\kappa = \frac{1}{2}$ (Tomisaka et al., 1990; Ciolek & Mouschovias, 1995). After doing similar kind of calculations as in Section 2, we derived the Jeans mass for gravitational collapse of molecular clouds at a distance r from the Galactic center as

$$M_J = \frac{\pi}{6}\rho_0\{\frac{\pi R\bar{B}(1-\beta)\rho_0^{-\beta-1} + \pi\frac{B_0^2}{2\rho_0^2}}{G(1-\frac{f'(r)}{4\pi G\rho_0}-\frac{\Omega^2}{4\pi G\rho_0})}\}^{\frac{3}{2}}.$$

Table 3. Jeans masses in presence of a varying magnetic field along with constant rotation of molecular clouds, for various values of parameters

r (pc)	B_0 (μG)	E (erg)	Ω (km s^{-1} kpc^{-1})	l (pc)	β	T (K)	M_J ($M_\odot$)	r (pc)	B_0 (μG)	E (erg)	Ω (km s^{-1} kpc^{-1})	l (pc)	β	T (K)	M_J ($M_\odot$)
20	1	10^{54}	4	2.4	0.2	13.26	4.24	50	1	10^{54}	4	6	0.2	18.44	15.89
					0.4	2.20	0.33						0.4	4.25	2.06
					0.8	0.06	0.0011						0.8	0.22	0.02
20	10	10^{54}	4	2.4	0.2	13.26	4.51	50	10	10^{54}	4	6	0.2	18.44	19.90
					0.4	2.20	0.46						0.4	4.25	4.30
					0.8	0.06	0.04						0.8	0.22	1.15
20	1	10^{56}	4	2.4	0.2	13.26	3.82	50	1	10^{56}	4	6	0.2	18.44	15.43
					0.4	2.20	0.30						0.4	4.25	2.00
					0.8	0.06	0.0010						0.8	0.22	0.02
20	10	10^{56}	4	2.4	0.2	13.26	4.07	50	10	10^{56}	4	6	0.2	18.44	19.32
					0.4	2.20	0.41						0.4	4.25	4.17
					0.8	0.06	0.04						0.8	0.22	1.11
100	1	10^{54}	4	12	0.2	23.66	43.29	150	1	10^{54}	4	18	0.2	27.38	78.04
					0.4	7.00	8.22						0.4	9.37	18.55
					0.8	0.61	0.17						0.8	1.10	0.62
100	10	10^{54}	4	12	0.2	23.66	74.65	150	10	10^{54}	4	18	0.2	27.38	185.69
					0.4	7.00	29.16						0.4	9.37	98.27
					0.8	0.61	13.64						0.8	1.10	58.19
100	1	10^{56}	4	12	0.2	23.66	42.81	150	1	10^{56}	4	18	0.2	27.38	77.54
					0.4	7.00	8.13						0.4	9.37	18.44
					0.8	0.61	0.17						0.8	1.10	0.62
100	10	10^{56}	4	12	0.2	23.66	73.82	150	10	10^{56}	4	18	0.2	27.38	184.52
					0.4	7.00	28.83						0.4	9.37	97.65
					0.8	0.61	13.49						0.8	1.10	57.82
500	1	10^{54}	4	60	0.2	42.24	466.48	1000	1	10^{54}	4	119	0.2	54.21	1.41×10^3
					0.4	22.30	218.09						0.4	36.73	965.39
					0.8	6.22	28.84						0.8	16.87	269.27
500	10	10^{54}	4	60	0.2	42.24	5.65×10^3	1000	10	10^{54}	4	119	0.2	54.21	5.70×10^4
					0.4	22.30	5.02×10^3						0.4	36.73	5.54×10^4
					0.8	6.22	4.35×10^3						0.8	16.87	5.23×10^4
500	1	10^{56}	4	60	0.2	42.24	465.93	1000	1	10^{56}	4	119	0.2	54.21	1.41×10^3
					0.4	22.30	217.83						0.4	36.73	964.96
					0.8	6.22	28.80						0.8	16.87	269.15
500	10	10^{56}	4	60	0.2	42.24	5.64×10^3	1000	10	10^{56}	4	119	0.2	54.21	5.70×10^4
					0.4	22.30	5.01×10^3						0.4	36.73	5.54×10^4
					0.8	6.22	4.34×10^3						0.8	16.87	5.22×10^4

4 Results and conclusion

In the present work we have developed a model of star formation in the central region of Our Galaxy under an explosive event taking place at its center. We have considered three situations e.g. (i) presence of constant magnetic field along the azimuthal direction, (ii) constant magnetic field along with fixed rotation of molecular clouds and (iii) density varying magnetic field with fixed rotation of molecular clouds. When such big explosion occurs at the center of a galaxy, shock waves are generated and during its propagation through the ambient medium it compresses the gas behind it in a thin shell. Such cooled and compressed high density medium when undergoes gravitational instability, produces fragments, called molecular clouds. Stars are formed out of these clouds as a result of gravitational collapse. We have derived minimum Jeans mass for gravitational collapse under the above three situations and they are shown in Tables 1, 2 and 3 respectively. Analysis of the results in this paper lead us to the following conclusions.

(i) Very close to the Galactic center ($r \sim 20$ pc) field star formation is preferred (viz. Tabs. 1, 2 and 3). Formation of massive stars or starbursts are preferred at larger distances. This is consistent with the radio and infrared observations near the Galactic center (Yusef Zadeh et al., 2015a; Yusef Zadeh et al., 2015b; Yusef Zadeh et al., 2015c). Thus closer to the center, formation of star cluster is very unlikely.

(ii) As temperature and density relation becomes steeper (i.e. β is higher), temperature rapidly falls with the increase of density hence Jeans mass becomes smaller. Very small Jeans mass ($\sim 10^{-2} M_{\odot}$) cannot lead to the formation of stars but Jupiter like objects. Thus star formation is preferred for a moderate rate of change of temperature with density.

(iii) The scaling relation of the magnetic fields in interstellar clouds from Zeeman observations – with exponent $\kappa \sim 0$ at densities below 10^3 cm^{-3} has been revised by Crutcher et al. (2010). In the present work – with exponent $\kappa \sim \frac{1}{2}$, we have shown that weak magnetic field (viz. $B_0 \sim 1\ \mu$G) favours formation of field stars whereas strong magnetic field (viz. $B_0 \sim 10\ \mu$G) is suitable for bursts of stars and the effect is enhanced at a larger distance (viz. r $\sim$ 1 kpc).

(iv) High explosion energy can lead to first generation star formation and hierarchy of fragmentation is not always necessary (Hoyle, 1953). Moderate explosion energy is suitable (viz. $E \sim 10^{54}$ erg, Fujita et al., 2017) for star formation rather than a strongest one (viz. $E \sim 10^{56}$ erg, Hartmann, 1995). This is because stronger shock pressure results into enhanced compression which somewhat resists radiative energy transport thus opposing a smooth and continuous collapse.

(v) Rotation always tends to stabilize a system (Fall & Efstathiou, 1980). Hence growth of instability is only possible for a slow rotation i.e. supersonic radial shock should predominate over rotation for creation of a consistent environment of star formation. This is clear from Tables 2 and 3, where rotation of molecular cloud is of the order of few km s^{-1} kpc^{-1} (Turner, 1984) for the growth of instability. For the same reason Jeans mass increases in presence of rotation. This rotation of molecular clouds is something different from differential Galactic rotation which can be observed clearly at somewhat larger distance > 2 kpc (Chemin et al., 2015). The kinematics as well as star formation mechanism is very complex close to the Galactic center and is still not clearly understood (Morris, 1993; Kauffmann, 2017).

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