Evolution of the magnetic field reversals in galaxies

Evgeny Mikhailov[1], Tatiana Khasaeva[1]
M.V.Lomonosov Moscow State Universuty, Leninskie gori, 1, 119991, Moscow, Russia
ea.mikhajlov@physics.msu.ru

(Submitted on 07.11.2018. Accepted on 25.11.2018)

Abstract. Some of the galaxies have regular magnetic fields which are generated by the dynamo mechanism. It is based on joint action of differential rotation and alpha-effect. For some objects it is possible to have the field reversal, when it has opposite directions in different regions of galaxy. The reversals can be described by the nonlinear equations of the galaxy dynamo theory. We have studied this process using no-z approximation for the magnetic field. At first we took the simple one-dimensional model, which allows us to make some asymptotic estimates. Some analytical expressions for small values of the turbulent diffusivity coefficient are given. After that we model the field numerically, and describe the reversal evolution for realistic values of parameters. According to all these approaches, the reversal can be generated during the first period of the magnetic field evolution. After that the reversal moves with some velocity of order of 1 km/s.

Key words: magnetic fields, galaxies, dynamo, reversals

Introduction

Magnetic field in spiral galaxies have been studied for several tens of years. First estimates for the field were based on the spectrum of the synchrotron emission (Ginzburg 1959). Different observations, mostly connected with measuring the Faraday rotation of the polarization plane of the radiowaves, strongly proved that several objects have magnetic fields of several micro-gauss (Beck et al. 1996; Arshakian et al. 2009). The magnetic field consists of two parts: regular one, which has lengthscale comparable with the size of the whole galaxy, and random magnetic field which has the value of the same order of magnitude, but it is associated with small domains comparable with the turbulent cells.

From the theoretical point of view, the regular field evolution is described by the mean field dynamo theory (Donner & Brandenburg 1990). The magnetic field of most of the galaxies lies in the equatorial plane, and it has two main components: radial and angular one. The field generation is based on joint action of alpha-effect, characterizing the vorticity of the turbulent motions, and the differential rotation (connected with the non-solid rotation of the galaxy). The alpha-effect transforms the angular field to the radial one, and the differential rotation transforms the radial component to the angular one. Working jointly, these effects also make the field value grow. They compete with the turbulent diffusion process which tries to make the field decay. So the field generation is a threshold process: it can grow only for some values of the parameters, when differential rotation and alpha-effect are stronger than the differential rotation (Arshakian et al. 2009).

Usually the magnetic field is mainly angular. The dynamo equations allows the field to have each of two opposite directions (”clockwise” and ”counterclockwise”), and the structure of the field in each galaxy depends on initial effects. Sometimes it is possible to have two different regions in one galaxy, where the field can have different directions. Such structures are called magnetic field reversals (Moss et al. 2012, Moss & Sokoloff 2013). There is a thin transition layer that connects the parts of the galaxy with different magnetic fields. Nowadays there is a strong evidence that the magnetic field in the Milky Way has such reversals: the observations of the rotation measure show that the field in the Sagittarius arm is opposite to the field in the Carina arm (Han et al. 2006, Van Eck et al. 2011, Beck 2011, Andreasyan et al. 2018). It is quite possible that the transition layer is quite close to the Sun, but the observations through the disc are much more difficult than the ones for the external objects.

Mathematically, the reversals of the magnetic field can be described as contrast structures which are well-known in mathematical physics, especially for the nonlinear parabolic equations (Butuzov et al. 1995, Bozhevol'nov & Nefedov 2011, Nefedov & Davydova 2013). The saturation of the magnetic field growth is described by nonlinear terms in the dynamo equations (Moss & Sokoloff 2011), the equations have two stable solutions and there can be a transition layer connecting them. According to the contrast structure theory, the transition layer can move with some small velocity connected with the parameters of the interstellar turbulence (Bozhevol'nov & Nefedov 2011).

The equations of the magnetic fields of galaxies are usually written using the no-z approximation (Mestel & Subramanian 1993, Moss 1995). It is based on the fact that the galaxy disc is very thin, so we can change the z-derivatives of the magnetic field by algebraic expressions or reconstruct them from the solenoidality condition. These assumptions allow us to reduce compicated three-dimensional Steenbeck – Krause – Rädler equation to a system of two equations which are much more convenient for analysis.

The reversals of the magnetic fields in galaxies have been studied numerically in some previous works (Moss & Sokoloff 2013, Moss et al. 2012). However, the numerical results usually cannot help to understand in details the mechanism of generation. Also the possibility of the movement of the transition layer is not completely clear basing on these works. The wavefronts of the field have been studied analytically (Vasil'eva et al. 1995, Moss et al. 2000, Mikhailov 2015), but the models were very approximate. The initial system of equations is reduced to simple problems, and the velocity of the movement of the contrast structure strongly depends on the way of reducing. Unfortunately, now we do not have any analytical formulae which could describe the velocity of the transition layer for the system of equations (even approximate).

Here we present a semi-numerical approach for studying the magnetic field reversals in the galaxies. At first we give ideas that allow us to suppose some principal laws for main parameters of the transition layer. It is based on the spectrum of the linear operator describing our problem for small values of the field, and the projections of the solutions on the eigenvectors are studied. We also explain that the magnetic field reversal moves with the velocity proportional to the coefficient of the turbulent diffusivity.

After that we study the magnetic field structures numerically. We discuss the initial conditions that allow us to obtain the magnetic field with changing direction. We study the magnetic field reversal evolution for a wide range of the parameters, and after that we give an approximate formula that describes the connection between the velocity of the transition layer and the kinematic parameters, such as angular velocity, turbulent diffusion coefficient and the alpha-effect.

The velocity of the wavefront is proportional to the turbulent diffusion for weak dissipation effects. As for another parameters, the magnetic field growth depends on the square root of product of angular velocity and the alpha-effect coefficient. It lead us to the idea that the reversal can move with the velocity connected with the field growth which contains the same combination (Mikhailov & Modyaev 2015).

1. Basic equations

The magnetic field of galaxies demonstrate two different scales. Firstly, there is a small-scale magnetic field which has the typical length of changing of 50–100 pc. It is thought to be generated with its own specific mechanism (Subramanian 1998) connected with the turbulent motions of the interstellar medium. There is also the large-scale component of the field, which has the characteristic lengthscale comparable with the radius of the galaxy and it is described by mean-field dynamo mechanism (Ruzmaikin et al. 1988; Brandenburg 2018) based on joint action of alpha-effect and the differential rotation.

The evolution of the magnetic field is described by Steenbeck – Krause – Rädler equation (Steenbeck et al. 1966):

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\alpha \mathbf{B}) + \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \Delta \mathbf{B}, \tag{1}$$

where $\mathbf{B}$ is the large-scale magnetic field, t is time, $\mathbf{V}$ is the large-scale velocity (as for galaxies, it is usually connected with rotation), η is the turbulent diffusivity coefficient, α characterizes the vorticity of the turbulent motions (alpha-effect).

It is quite convenient to assume that the disc is quite thin (Mestel & Subramanian 1993; Moss 1995). The field will lie in the equatorial plane, so it will be possible to describe only radial magnetic field B_r and the angular one B_φ. They can be described as:

$$B_{r,\varphi}(r,\varphi,z,t) = B_{r,\varphi}(r,\varphi,0,t)\cos\left(\frac{\pi z}{2h}\right); \tag{2}$$

where h is the half-thickness of the galaxy (here we take the values $|z| < h$). As for the alpha-effect we take:

$$\alpha = \alpha_0 \frac{z}{h}, \tag{3}$$

where α_0 is some typical value. As for the large-scale velocity we take $\mathbf{V} = r\Omega \mathbf{e}_\varphi$. Using the solenoidality condition for the field and assuming the field axisymmetric, we can obtain the following equations (Moss & Sokoloff 2011, Moss & Sokoloff 2013, Mikhailov et al. 2014):

$$\frac{\partial B_r}{\partial t} = -\frac{\alpha_0 B_\varphi}{h} - \eta \frac{\pi^2 B_r}{4h^2} + \eta \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_r)\right); \tag{4}$$

$$\frac{\partial B_\varphi}{\partial t} = r\frac{d\Omega}{dr}B_r - \eta \frac{\pi^2 B_\varphi}{4h^2} + \eta \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_\varphi)\right) \tag{5}$$

The magnetic field generation is connected with transition of energy of turbulent motion to magnetic field energy. So the field growth should be restricted by equipartition value $B_{max} = 2v\sqrt{\pi\rho}$ (Arshakian et al. 2009). This can be included to the magnetic field equations as:

$$\frac{\partial B_r}{\partial t} = -\frac{\alpha_0 B_\varphi}{h}\left(1 - \frac{B_r^2 + B_\varphi^2}{B_{max}^2}\right) - \eta \frac{\pi^2 B_r}{4h^2} + \eta \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_r)\right); \tag{6}$$

$$\frac{\partial B_\varphi}{\partial t} = r\frac{d\Omega}{dr}B_r - \eta \frac{\pi^2 B_\varphi}{4h^2} + \eta \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_\varphi)\right) \tag{7}$$

So the generation of the field will lower if the field will become comparable with the equipartition value. There are also some different models for the saturation of the growth (Shukurov et al. 2006; Sur et al. 2007; Mikhailov 2013) but for our aims this parametrization is quite sufficient. Most of the following results were obtained using $B_{max} = 3\ \mu$G.

We used the zero boundary conditions (Moss 1995; Phillips 2001):

$$B_r|_{r=0} = B_\varphi|_{r=0} = B_\varphi|_{r=R} = B_\varphi|_{r=R} = 0; \tag{8}$$

where R is the radius of the galaxy. We usually took the value $R = 10$ kpc.

The initial conditions were the following:

$$B_r|_{t=0} = 0;\ B_\varphi|_{t=0} = \pm B_0 \sin\left(\frac{2\pi r}{R}\right). \tag{9}$$

For the kinematic parameters we used the models:

$$\alpha_0 = A\frac{r_0}{r}; \tag{10}$$

$$\Omega = \frac{\Omega_0}{\sqrt{1 + (r/r_0)^2}}. \tag{11}$$

where A and Ω_0 varied. The values of the turbulent diffusivity η varied, too.

2. Qualitative analytical model of the reversal

Here we will describe qualitative model of the magnetic field reversal for the cases when the turbulent diffusion can be used as a small parameter. It is based on some approximate assumptions that allow us reduce system (6)–(7) to one equation and use the methods of the contrast structures theory (Bozhevol'nov & Nefedov 2011). In this section all results assume the turbulent diffusivity η as a small parameter. For larger values it is necessary to use numerical methods.

Firstly, the linear analysis of the system (6)–(7) shows that for small values of the magnetic fields and weak diffusivity ($\eta \to 0$) the magnetic field will grow exponentially

$$B \sim \exp(\gamma t) \tag{12}$$

where $\gamma = \sqrt{\alpha_0 hr d\Omega/dr}$ and the ratio between field components is the following:

$$B_r/B_\varphi = -\left(\frac{\alpha_0}{hr d\Omega/dr}\right)^{1/2}. \tag{13}$$

If we assume that for a quite wide range of the magnetic field values (even for modestly nonlinear case) the ratio between the field components is the same, and taking into account that $B_r \ll B_{varphi}$, we will obtain the following approximate equation for B_φ :

$$\frac{\partial B_\varphi}{\partial t} = \gamma B_\varphi \left(1 - \frac{B_\varphi^2}{B_{max}^2}\right) + \eta \left(\frac{\partial^2 B_\varphi}{\partial r^2} - \frac{\partial B_\varphi}{r \partial r} + \frac{B_\varphi}{r^2}\right). \tag{14}$$

If we take $\eta = 0$, the equation will have three stationary solutions:

$$B_\varphi = 0;\ B_\varphi = \pm B_{max}. \tag{15}$$

If there is a magnetic field reversal, there should be a region where $B_\varphi = B_{max}$ and a region where $B_\varphi = -B_{max}$. They will be connected with a transition layer located near some point r_0. For the stationary case, the solution in small vicinity of the r_0 will be approximately the following:

$$B_\varphi = u(r) = B_{max} \tanh\left(\sqrt{\frac{\gamma}{2\eta}}(r - r_0)\right). \tag{16}$$

So, we will have the transition layer with typical width:

$$\Delta = \sqrt{\frac{2\eta}{\gamma}} = \frac{(2\eta)^{1/2}}{(\alpha_0 hr d\Omega/dr)^{1/4}}. \tag{17}$$

According to the contrast structure theory (Bozhevol'nov & Nefedov 2011, Mikhailov 2015) the transition layers can move with some velocity. So we can try to find the non-stationary solution, assuming $u = u(r - ct)$ and taking into account the fact that η is a small parameter (some of the terms are omitted):

$$-cu' = \gamma(r_0)u(1-u^2/B_{max}^2)+\gamma'(r_0)(r-r_0)u(1-u^2/B_{max}^2)+\eta u''-\eta\frac{u'}{r}. \quad (18)$$

If we multiply both of the parts of the equations on u':

$$-c(u')^2 = \gamma(r_0)u'u(1-u^2/B_{max}^2)+$$

$$\gamma'(r_0)(r-r_0)u'u(1-u^2/B_{max}^2)+\eta u'u''-\eta\frac{(u')^2}{r_0}. \quad (19)$$

and integrate for a wide range (much larger than Δ) of r, we will have the following formulae for the velocity ("-" is connected with the solution $-u$:

$$c = \pm\frac{\eta\gamma'}{2\gamma} - \frac{\eta}{r_0}. \quad (20)$$

3. Numerical modelling for qualitative equation

Here we use our approximate equation for B_φ (14), where we consider $\gamma = \sqrt{\alpha_0 hrd\Omega/dr}$. This model is one-dimensional, therefore it is more suitable for analytic solution than any two-dimensional model. Thus we need it to verify the accuracy of our analytical vision of the magnetic field evolution. The numerical solution with respect to r is provided on Fig. 1.

A reversal point (the point where magnetic field equals zero) emerges over time and starts to shift. The reversal point changes location according to the dependence presented on Fig. 2. We show large values of t because at the beginning of the evolution the reversal is not completely formed.

The derivative of this function equals to the instantaneous velocity of the reversal point. The function is partly linear, so to find the average velocity we need to know the inclination angle of the linear part of the function. The dependence of the average velocity from η is provided on Fig. 3.

According to asymptotic model (see previous section) the transitional layer should move with velocity $c = -0.226\eta$ and the velocity obtained numerically is $c = -0.230\eta$. Thus we can conclude that the expression (20) gives proper results at least for small values of turbulent diffusivity coefficient. For larger diffusivity it is necesary to use full system (6) – (7), which can be solved only numerically.

4. Full model of the reversal

Now, let us go back to the the equations (6) – (7), which were described in section 1. In this case we use so called no-z approximation, as we consider the galaxy as a flat disk, with radius much bigger than its thickness. As we

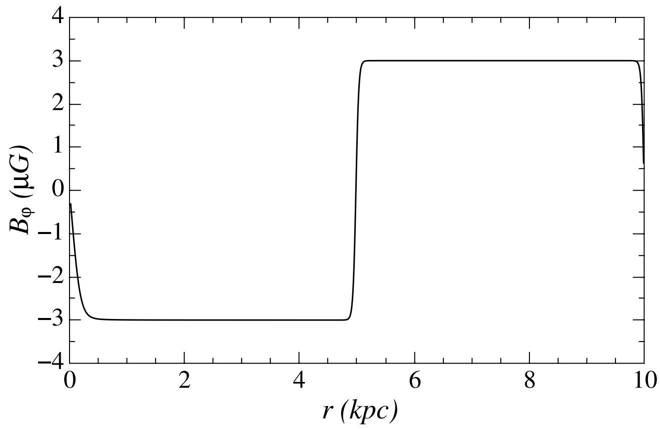

Fig. 1. Structure of the magnetic field in the qualitative model for $t = 10$ Gyr.

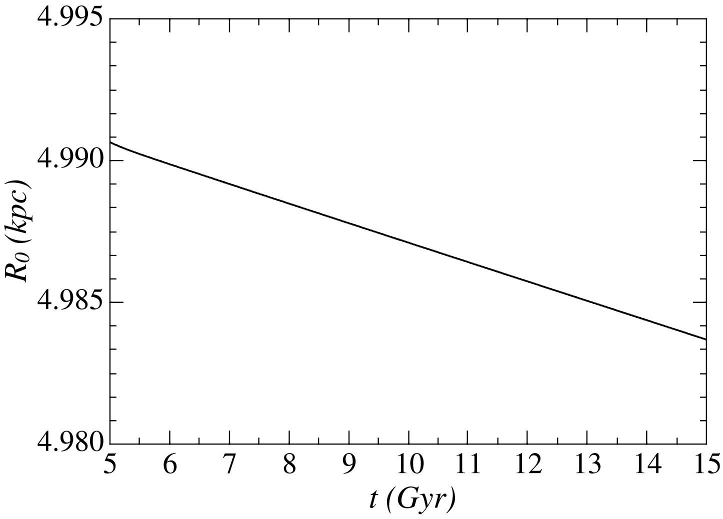

Fig. 2. The movement of the reversal point in the qualitative model

have mentioned, these equations can be solved only numerically, and the solution is provided on Fig. 4.

Here, for small values of η the reversal point changes location in time according to a similar function, which is also partly linear and $c = -0.230\eta$. We provide it on the Fig. 5.

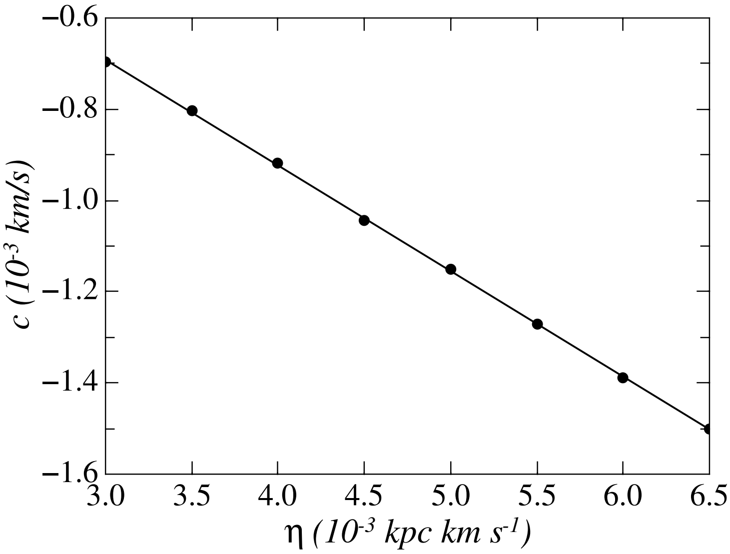

Fig. 3. Dependence of the velocity of transition layer from η in the qualitative model.

However for bigger values of η (over 0.07 kpc km s^{-1}) the situation is significantly different. The reversal point moves towards the edge of the galaxy according to the dependence given on Fig. 6, while in case of smaller η it merges towards its center. As in this case reversals form rapidly, we consider the reversal point movement from the very beginning of the magnetic field evolution (from $t = 0$).

In this case we expect the average velocity to depend from the input parameters namely α and Ω. After conducting a numerical experiment it turned out that the average velocity is proportional to the square root of the composition of these values.

Table 1. Dependence of the velocity from η

η, 10^{-3} kpc km s^{-1}	v, 10^{-4}km s^{-1}
3.0	-6,97
3.5	-8,02
4.0	-9,19
4.5	-10,45
5.0	-11,52
5.5	-12,70
6.0	-13,90
6.5	-15,00

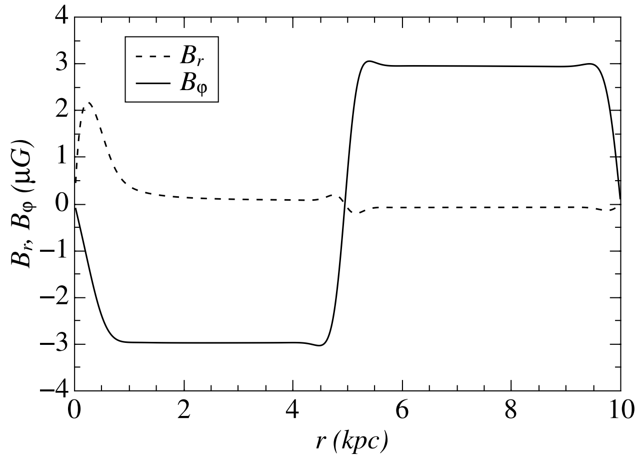

Fig. 4. Structure of the magnetic field for $t = 10$ Gyr in full model. Solid line shows angular component of the field, and dashed line shows radial one.

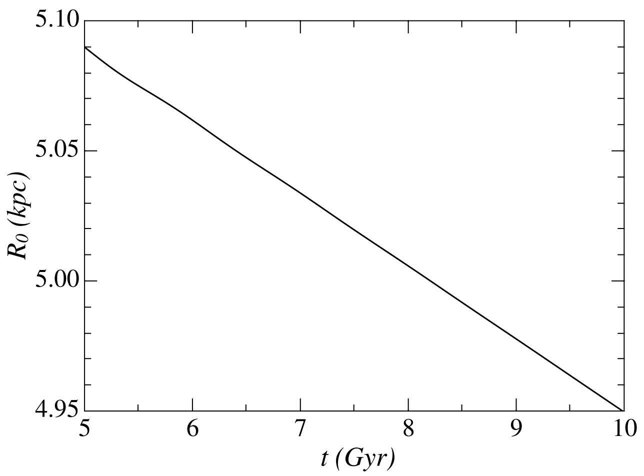

Fig. 5. The movement of the reversal point in full model

Finally we obtain the approximate expression for the average velocity:

$$c = A_1 \sqrt{\alpha \Omega} + A_2 \tag{21}$$

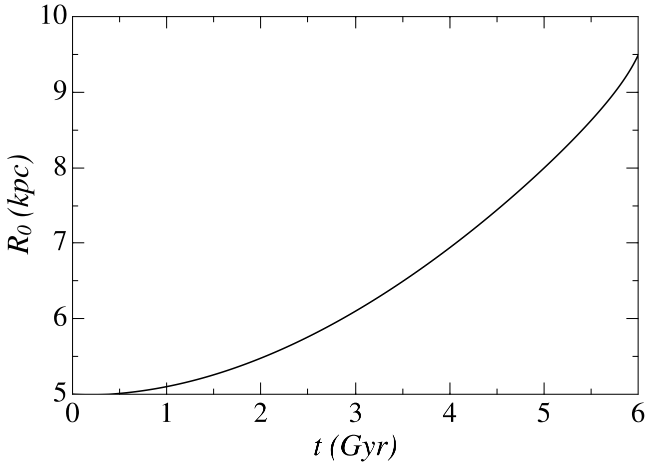

Fig. 6. Movement of the transition layer for case of bigger values of η (η = 0.4 kpc km s^{-1}) in full model

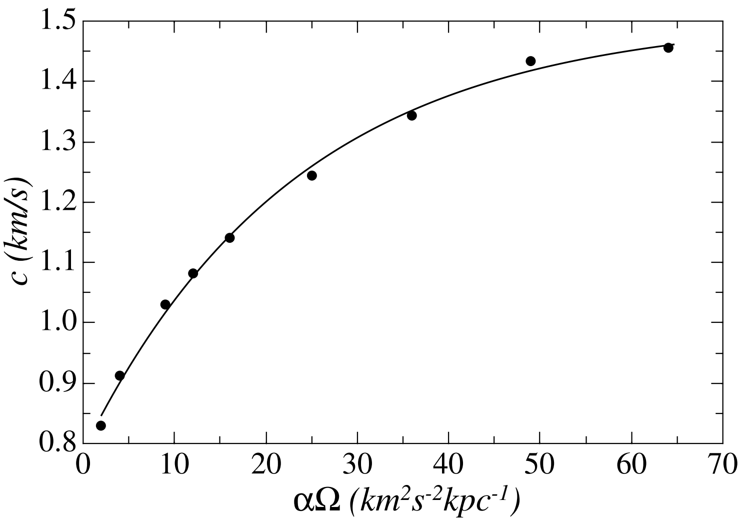

Fig. 7. Dependence of the velocity of the transition layer from $\alpha\Omega$ in full model.

Here for $\eta = 0.45$ kpc km/s, which is close to the real values of turbulent diffusivity, $A_1 = 0.108\ kpc^{1/2}$ and $A_2 = 0.699\ km/s$.

Conclusion

We have studied the magnetic field reversals using different modifications of no-z approximation for the magnetic field. Firstly, we have described the magnetic field evolution using one-dimensional model, which is simple, but allows us to give some analytical expressions for the typical structure of the field and its evolution. The results were obtained using the asymptotic contrast structure theory. They are nearly the same as the ones given by numerical simulations. However, we can be sure that this model gives proper results only for asymptotically small values of the turbulent diffusivity coefficient, so we also used the full system of equations of no-z approximation. For smaller values of the turbulent diffusivity coefficient the results were very close to the simple model, so we can conclude that it is quite possible to use it for low intensive turbulence. As for larger values of the turbulent diffusivity coefficient, we have constructed an approximate expression for the realistic values of the parameters, which includes the alpha-effect coefficient and the rotation angular velocity.

Using all these models, it was shown that the reversal can be generated during several Gyr. To generate such structure we should have specific initial conditions. Maybe they can be caused by some random factors, for example connected with supernovae explosions, active star formation and another effects during first period of the galaxy formation (Moss 2014, Mikhailov & Modyaev 2015, Mikhailov & Pushkarev 2018). So the magnetic field reversal can be rare, but quite possible configuration of the field. Another important feature is that the magnetic field reversal is not a static structure, and it moves with some small velocity (several km/s) to the galaxy center for small turbulent diffusion and from the galaxy center for large turbulent diffusion.

From the observational point of view, nowadays it is possible to say that in the Milky Way the magnetic field has the reversal (Van Eck et al. 2011). As for another galaxies, the magnetic fields seem to have the same direction in all of the galaxy. This can be connected with random factors, which influence the process of the field generation and the probability to have the reversal is quite low. So we should study a larger list of galaxies to observe the magnetic field reversals.

This work is supported by the Russian Science Foundation under grant 18-11-00042.

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