# Theoretical Model of Magnetic Braking in Contact Binary Stars

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**Abstract.** Magnetic braking is a significant cause of angular momentum loss in contact binary stars. The effect of differential rotation in binary stars can be very important in case of generation of magnetic field in companion stars. However, interestingly, the generated magnetic field usually puts a brake in the existing rotation rate. The changing profiles of magnetic field strength, braking in rotation rate due to the magnetic field strength as well as effective angular velocity at different latitudes for each of the components have been demonstrated separately for a synthetic contact binary system.
**Key words:** Magnetic Braking; Contact Binary Stars ; Angular momentum

## 1 Introduction

The origin of magnetic fields of binary stars on the main sequence is still debated. Possibilities are that the fields are fossil from pre main-sequence phases of evolution perhaps dating back to the time of star formation which is coined as primordial magnetic field or/and the magnetic field is generated by the dynamo process due to differential rotation. The compact degenerate donor star can be the strongest source of magnetism. White dwarf source fields can be in excess of $10^7 G$, while those on neutron stars are about $10^{12} G$. The field influence is determined by the stellar magnetic moment. So white dwarfs can have stronger effects than neutron stars due to their much larger radius. Such degenerate star fields may be of fossil origin. The gainer star and accretion disc fields can be $10^2$ *to* $10^4 G$, and dynamo mechanisms are necessary to generate and maintain them. It is also well known that the dynamo mechanism is likely cause of large-scale magnetic field production in the stars with convective layers (Parker, 1955). The Sun is known to rotate more rapidly at the equator than at the poles. The speed of this differential rotational and convection are important for the production of magnetic fields in the convective zone. The differential rotation between the radiative core and convective envelope winds up the field and causes a deformation of the poloidal field which in turn generates an additional toroidal field component and thus creates a Lorentz force which counteracts the shear (deformation) due to poloidal field. Again in convective stellar bodies the principal role of this magnetic field is the redistribution of angular velocity and so the angular momentum. As a consequence, the generated magnetic field later puts a brake on the existing rotation rate and this phenomenon is termed as 'magnetic braking'. Magnetic braking is a common phenomenon to all the contact binary stars (Bradsreet and Guinan, 1994; Huang et al., 2007). According to Bradstreet and Guinan (1994) and Stepien (1995,

2006) the loss of angular velocity and angular momentum plays an important role in the formation and evolution of low -mass contact binary stars. The effect of magnetic fields on the mass and angular momentum transfer and ( or) loss of both the companions are quite appreciable. In a close binary where the spin and orbital angular momentum are strongly coupled this stellar spin down forces a decrease in the orbital period of the system even without mass transfer. However, at high rotation rates the angular momentum loss rate grows more slowly even with the same underlying dynamo model (Keppens et al., 1995) The coupling constant depends on the magnetic field strength and can be important if the field is strong enough (on the order of few MG). Another effect of magnetic field is to alter the spin through torqueing of the star by mass overflow. The magnetized star allows winds move outward from the active star, but are twisted due to rapid rotation of the star. Charged particles in the star wind get trapped in the magnetic field of the star and are dragged along the field lines. The result is angular momentum transfer from the star by magnetic field to the charged particles. As the winds leave the star surface they are dragged by the magnetic field which, in turn, slows down the rotation of the star. For close binaries in which synchronization of rotational and orbital period is expected, loss of rotational angular momentum occurs at the expense of orbital angular momentum. As a result, the period decreases and the components spin up and approach one another to form a single rapid rotating star (Stepien, 1995; Skumanich, 1972). As stated by Stepien (1995, 2006) contact binary stars are magnetically very active and they lose mass and angular momentum mainly via magnetized winds. Moreover the separation of the components is relatively low. Therefore one expects the magnetic field interactions between the two components to be intensified and consequently its effect on the angular momentum loss to be enhanced, due to the formation of magnetic loops between the surface magnetic fields of the components. This statement is consistent with that of Bradstreet and Guinan (1994) that the magnetic torque produced by magnetic field in the wind depends on the strength of magnetic field. The magnetic field lines are bent due to rapid rotation of the star . Their curvature causes counter force on the surrounding stellar plasma. If we assume that magnetic poles coincide with the rotation poles, then the dissipated angular momentum is very small and braking is almost negligible but if field is anchored at or near the equatorial plane then the braking is the strongest and therefore maximum angular momentum is removed. It is also observed that low-mass x-ray binaries undergo rotational braking by a magnetically coupled stellar wind, in a similar way as single main-sequence stars. Because the low- mass components of close binaries are forced by tidal forces to remain in co-rotation, this leads to a loss of orbital angular momentum from the systems, and to an enhanced mass transfer rate. Also the rotation rate of main-sequence stars with convective envelopes is observed to decrease rapidly with the age (Kraft, 1967). Historically an angular momentum loss rate was derived from average rotational velocities (vsini) of stars in populations of different ages. An angular momentum loss rate is assumed to have some functional dependence of mass, radius, effective temperature and rotational rate of a star, which is motivated by theoretical considerations and then calibrated using rotational data. We here consider a theoretical model of a contact binary star where the two components possess strong magnetic fields generated in a dynamo process for their very fast and differential rotation apart from the primordial magnetic fields already in those (if any) and an attempt is made to model the reciprocated relationship between the differential rotations at different latitudes in the surface, the magnetic field generated as a consequence, braking in the rotation rate due to the magnetic field strength and finally the effective rotation rate at different latitudes in highly rotating components of contact binary stars. We have taken into consideration the model of differential rotation at the surface with respect to latitude of Scheiner (1630). Here we produce a synthetic model of a contact binary system and applied the present theory over this system to demonstrate the changing profiles of magnetic field strength and braking in rotation rate due to the magnetic field strength and the final effective rotation rate at different latitudes on the surface for each of these two components separately

## 2 Theory

Reiners and Mohanty (2012) identified that the magnetic field strength ($B$) on the stellar surface is related to the angular velocity ($|\omega|$) at all instants by the following power law:

$$B \propto \omega^a \tag{1}$$

where $a$ is likely to vary between 1 and 2 for unsaturated fields and can drop to 0 by definition when field strength saturates as can be expected in $\alpha^2$-dynamo that may govern in case of fully convective very low- mass stars (Chabrier and Kuker, 2006). So in the stellar bodies for which angular velocity at the surface is supposed to vary with the latitude (Scheiner, 1630) we can have in view of (1) the following relation at the surface at all instants

$$B(\varphi) = k_1\{\omega(\varphi)\}^a, \tag{2}$$

where $\varphi$ represents the latitude and $k_1$ is a proportionality constant characterizing an individual stellar body.

This generated magnetic field influences the rotation altogether creating a drop in angular velocity or a brake on the rotation ($\delta\omega$). Cohen and Drake (2014) showed that at any / time the kinetic energy density of the escaping wind at the Alfven radius, responsible for the angular momentum loss and in turn the loss in angular velocity, is proportional to the square of magnetic field strength. This observation is homologous to

$$\delta\omega(\varphi) \propto [B_0(\varphi) + B(\varphi)]^2, \tag{3}$$

where $B_0(\varphi)$ is the primordial magnetic field strength distributed at the latitude $\varphi$. This leads to the following conclusion:

$$\delta\omega(\varphi) = k_2[B_0(\varphi) + B(\varphi)]^2, \tag{4}$$

where $k_2$ is a proportionality constant portraying the characteristics of the stellar body. In view of (2) and (4) we have

$$\delta\omega(\varphi) = k_2[B_0(\varphi) + k_1\{\omega(\varphi)\}^a]^2. \tag{5}$$

The effective rotation $[\omega^*(\varphi)]$ as a consequence of braking can be given by:

$$\omega^*(\varphi) = \omega(\varphi) - \delta\omega(\varphi). \tag{6}$$

Using (5) in (6) we have

$$\omega^*(\varphi) = \omega(\varphi) - k_2[B_0(\varphi) + k_1\{\omega(\varphi)\}^a]^2. \tag{7}$$

From the above analysis it is clear that if the angular velocity $\omega$ at any time and primordial magnetic field strength at certain latitude $(\varphi)$ are known then the corresponding magnetic field generated due to rotation, consequent magnetic braking and final effective rotation can be estimated at that time and at that latitude $(\varphi)$ . So it is very important to understand the distribution of angular velocity at the surface at any time as a changing function of latitude. We follow the model prescribed by Scheiner (1630)

$$\omega(\varphi) = \omega_e - (\omega_e - \omega_p)\sin^2\varphi, \tag{8}$$

where $\omega_e$ and $\omega_p$ stand for the angular velocities at the equator and the poles respectively anticipating that the angular velocity is maximum at the equator and minimum at the poles: Now if we write

$$\omega_e = \alpha\omega_p, \tag{9}$$

where $\alpha > 1$. We can have from (8)

$$\omega(\varphi) = [\alpha - (\alpha - 1)\sin^2\varphi]\omega_p. \tag{10}$$

Now the average angular velocity $< \omega >$ distributed over entire surface at any time can be given by

$$< \omega > = \frac{\int_0^{\frac{\pi}{2}} \omega(\varphi) d\varphi}{\int_0^{\frac{\pi}{2}} d\varphi} \tag{11}$$

$$= \frac{1}{2}(\omega_e + \omega_p) \qquad [using (8)]$$

$$= \frac{1}{2}(\alpha+1)\omega_p. \qquad [using (9)] \tag{11a}$$

In practical computation for an observed stellar body the average angular velocity over the surface should be found. This will help to determine directly the angular velocity at the pole at the time of observation by means of (11a) because

$$\omega_p = \frac{2 < \omega >}{(\alpha + 1)}, \tag{12}$$

provided at that time we know the ratio $\omega_e\ \omega_p$ between the angular velocities at the equator and the pole given by $\alpha$. This allows us to estimate the angular velocity at the equator ($\omega_e$) as well as the angular velocities at different latitudes at the time of observation using (9) and (10) respectively. Again using (12) in (10) we have the following

$$\omega(\varphi, \alpha) = \frac{2[\alpha - (\alpha - 1)\sin^2\varphi] < \omega >}{(\alpha + 1)}. \tag{13}$$

The left hand side of above is so written as for a detected star the angular velocity at any latitude at any time eventually depends also on the parameter $\alpha$ in addition to the latitude $\varphi$. The reciprocated relationships between angular velocity, generated magnetic field strength and braking in the angular velocity and finally the model of effective angular velocity at different latitudes at the surface for all instants for a detected star [for which the average angular velocity at the surface and the ratio between the angular velocities at the equator and the pole are found] can be demonstrated as an association of (13), (2), (5) and (7). Now from (13) we can have some interesting results which help us to visualize the nature of distribution of angular velocity over the surface for all instants. First it can be seen from (13) that for any $\alpha$ we have

$$\omega = < \omega > \ \ for\ \varphi = \frac{\pi}{4}. \tag{14}$$

This means that for a specific stellar body at any instant all the curves for $\omega(\varphi)$ for different choices of $\alpha$ have a common point of intersection at $\varphi = \frac{\pi}{4}$. From (13) we can also have

$$\frac{\partial \omega}{\partial \varphi} = -2 < \omega > \frac{\alpha - 1}{\alpha + 1} \sin 2\varphi. \tag{15}$$

This immediately confirms a decreasing profile of angular velocity within $0 < \varphi < \frac{\pi}{2}$ for any $\alpha > 1$. Again form the present model of angular velocity we can get

$$\frac{\partial^2 \omega}{\partial \varphi^2} = -4 < \omega > \frac{\alpha - 1}{\alpha + 1} \cos 2\varphi, \tag{16}$$

which gives

$$\frac{\partial^2 \omega}{\partial \varphi^2} < 0 \ \ , \ \ 0 \leq \varphi < \frac{\pi}{4},$$

$$= 0 \ , \ \ \varphi = \frac{\pi}{4}.$$

and

$$> 0 \ , \ \ \frac{\pi}{4} < \varphi \leq \frac{\pi}{2}, \tag{17}$$

This in turn suggests that for a fixed $\alpha$ the angular velocity is convex for $0 \leq \varphi < \frac{\pi}{4}$ and it is concave for $\frac{\pi}{4} < \varphi \leq \frac{\pi}{2}$. The point $\varphi = \frac{\pi}{4}$ is the point of inflection. Again the stellar magnetic fields are produced within the convective zone of the star. The convective circulation of the conducting plasma in presence of differential rotation functions like a dynamo. This activity destroys the primordial magnetic field of the stars like Sun or below the mass of the Sun. Hence for this category we can write

$$B_0 = 0. \tag{18}$$

Using (18) we can have from (4), (5) and (7) respectively for this category of stellar bodies at all instants

$$\delta\omega(\varphi) = k_2[B(\varphi)]^2, \tag{19}$$

$$\delta\omega(\varphi) = k_2 k_1^2[\omega(\varphi)]^{2a}, \tag{20}$$

$$\omega^*(\varphi) = \omega(\varphi) - k_2 k_1^2[\omega(\varphi)]^{2a}. \tag{21}$$

However Ivanova and Taam (2003) showed, for fast rotating stars like Sun or below the mass of the Sun the angular momentum loss rate at any time due to magnetic braking is proportional to $\omega^{1.3}$ and this drives us to write

$$\delta\omega(\varphi) \propto [\omega(\varphi)]^{1.3}, \tag{22}$$

Comparing (20) and (22) we get

$$2a = 1.3 \quad so \quad a = 0.65. \tag{23}$$

So in view of (23) we can rewrite (2), (20) and (21) respectively as follows:

$$B(\varphi) = k_1[\omega(\varphi)]^{0.65}, \tag{24}$$

$$\delta\omega(\varphi) = k[\omega(\varphi)]^{1.3}, \tag{25}$$

$$\omega^*(\varphi) = \omega(\varphi) - k[\omega(\varphi)]^{1.3}, \tag{26}$$

where $k = k_2 k_1^2$.

Because B, $\delta\omega$ and $\omega^*$ are all related to $\omega$ with some power law relations as given by (24), (25) and (26) respectively the profile observed for $\omega$ in view of (17) is similar for B, $\delta\omega$ and $\omega^*$. If we write the surface magnetic field strength at the equator and the pole as $B_e$ and $B_p$ respectively then from (24) we can have,

$$B_p = k_1 \omega_p^{0.65}, \tag{27}$$

$$B_e = k_1 \omega_e^{0.65}. \tag{28}$$

Using (9) in (28) we have,

$$B_e = k_1 \alpha^{0.65} \omega_p^{0.65}. \tag{29}$$

Combining (27) and (29), we find

$$\alpha = \left(\frac{B_e}{B_p}\right)^{\frac{1}{0.65}}. \tag{30}$$

In practical computation using (30) we can derive the angular velocity at the pole $\mid \omega_p \mid$ and equator $\mid \omega_e \mid$ from (12) and (9) respectively.Using (27) or (28) we have

$$k_1 = \frac{B_p}{\omega_p^{0.65}} = \frac{B_e}{\omega_e^{0.65}}. \tag{31}$$

It is quite desirable that in a highly rotating contact binary system the values of $k_1$ and $k_2$ and as a consequence k should be identical for both the component stars.

Durney (1972) and Skumanich (1972) proposed that the rotational velocity at the stellar surface decays as the inverse square root of time for the stars with polytropic index n=2 after the age 106 years. Mestel (1984) proposed a general model of the decay of the rotational velocity at the stellar surface at given latitude with respect to time for the stars where magnetic field is generated by dynamo action depending on the angular velocity as follows:

$$\omega \propto t^{-b}. \tag{32}$$

where b can take the values 0.25, 0.5, 0.75, 1.33, 1.5 and 4 depending whether mass loss rate is independent of magnetic strength or decays with it as well as whether the wind is thermal, thermo-centrifugal or magneto-centrifugal. This leads to the following time dependent decaying profiles of generated magnetic strength and magnetic braking as well for given latitude for both the component stars in a binary system where magnetic field is generated due to the dynamo action depending on the angular rotation:

$$B \propto t^{-0.65b}. \qquad [using(24)] \tag{33}$$

$$\delta\omega \propto t^{-1.3b}. \qquad [using(25)] \tag{34}$$

## 3 Results

For practical computation we generate a synthetic contact binary system where both the components are possessing strong magnetic fields. It is also presumed that here the dynamo process destroys the primordial magnetic field and therefore the existing magnetic fields in these two companion objects are only due to the dynamo process. We consider the average rotational velocities of the donor star and the gainer star as $< \omega >= 2.5 \times 10^{-5} rads^{-1}$ and $3.9 \times 10^{-5} rads^{-1}$ [ $<>$ stands for average]. The average magnetic field strength of both the components are assumed as $< B >= 2.5 \times 10^3 G$. Here we take $\alpha = 1.1$ $k_1 = 4 \times 10^6$ and $k_2 = 1.6 \times 10^{-14}$ for both the components of the present contact binary system. We consider the magnetic braking $\delta\omega$

in the order of $10^{-7}$ in view of (19) [since B is in the order of $10^3$ ] and this gives $k = k_2 k_1^2 = 0.26$. From (11a) we can obtain the angular velocity at the pole ($\omega_p$ ) and next using (9) we can have the angular velocity at the equator ( $\omega_e$ ) for the component stars. The angular velocity, generated magnetic strength, magnetic braking and effective angular velocity at different latitudes for both the components can be estimated from (8), (24), (25) and (26) respectively. Figures 1, 2, 3 and 4 give the profile of angular velocity, generated magnetic strength, magnetic braking and effective angular velocity respectively with respect to latitude for the donor star and Figures 5, 6, 7 and 8 give the similar profiles for the gainer star.

Following the simulation strategy as incorporated for the present synthetic system the present model can be efficiently exercised over practically observed contact binary systems for which the angular velocities at the pole and the equator and average magnetic field over the surface for both the component stars are already identified.

## 4 Discussion

We have here framed a theoretical model of a contact binary star where the two components possess strong magnetic fields generated due to the dynamo process for their very fast and differential rotation apart from the primordial magnetic fields already in those (if any) and an attempt is made to model the mutual relationship between the differential rotations at different latitudes in the surface, the magnetic field generated as a consequence, braking in rotation rate due to the magnetic field strength and finally the effective rotation rate at different latitudes in highly rotating components of the contact binary star. We have followed the model of differential rotation at the surface with respect to latitude proposed by Scheiner (1630). For practical computation a synthetic model of contact binary system has been considered in which both the components are assumed to have convective envelopes owning strong magnetic fields as a result of strong rotation only (no primordial magnetic field has been considered in the system). We have exhibited the varying profiles of magnetic field strength and braking in rotation rate due to the magnetic field strength and the final effective rotation rate at different latitudes on the surface for each of these two components in the present synthetic system separately. In future studies we can also incorporate the differentiability in the internal rotation with respect to the radial distance in the study of magnetic braking in contact binaries.

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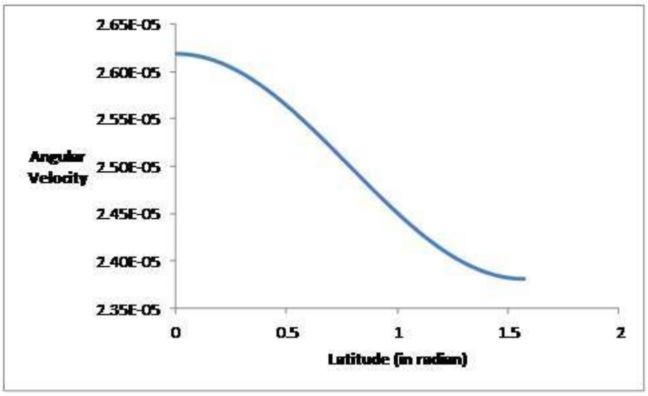


**Fig. 1.** Angular velocity vs. Latitude for the donor star.

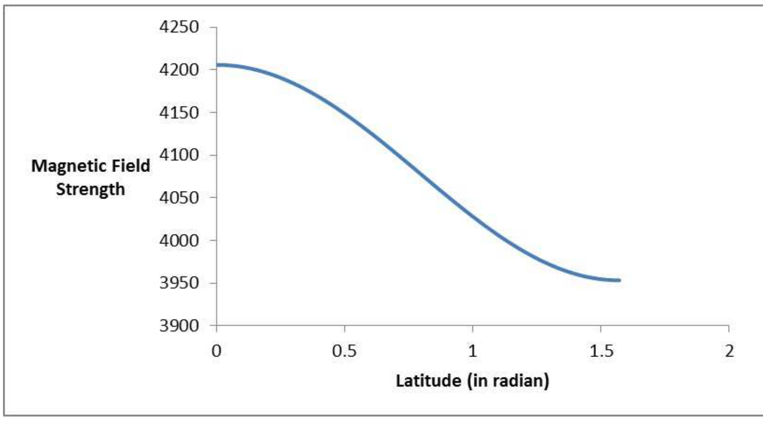


**Fig. 2.** Magnetic Field strength vs. Latitude for the donor star.

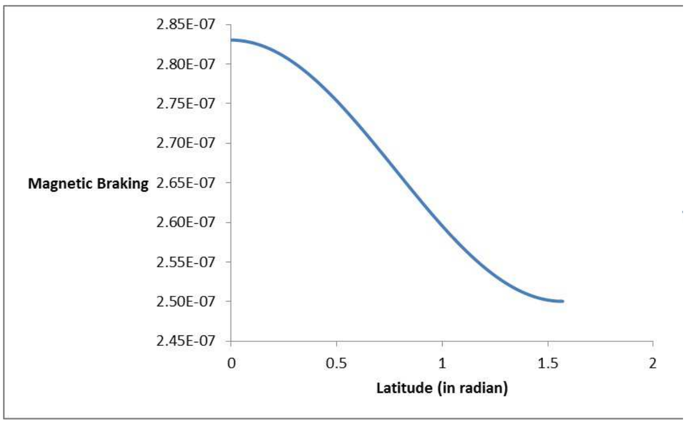


**Fig. 3.** Magnetic Braking in Angular velocity vs. Latitude for the donor star.

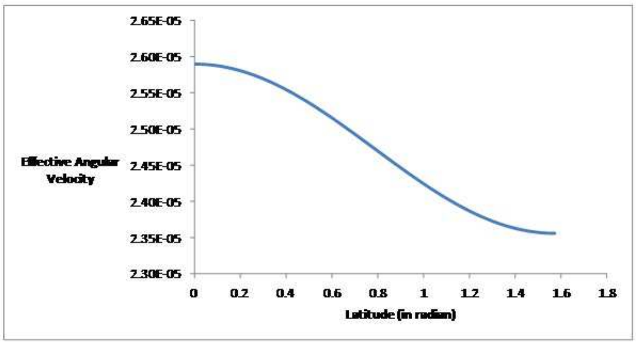


**Fig. 4.** Effective Angular velocity vs. Latitude for the donor star.

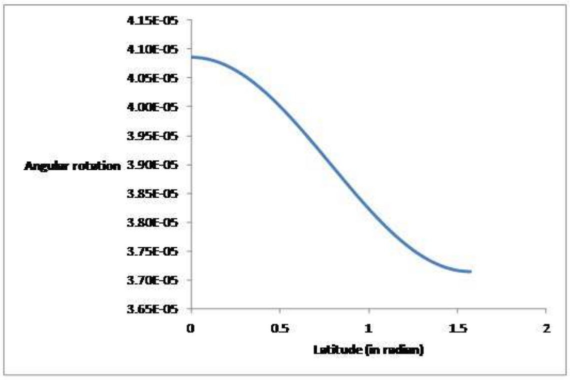


**Fig. 5.** Angular velocity vs. Latitude for the gainer star.

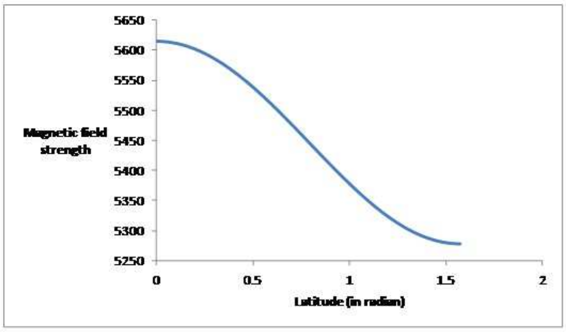


**Fig. 6.** Magnetic Field strength vs. Latitude for the gainer star.

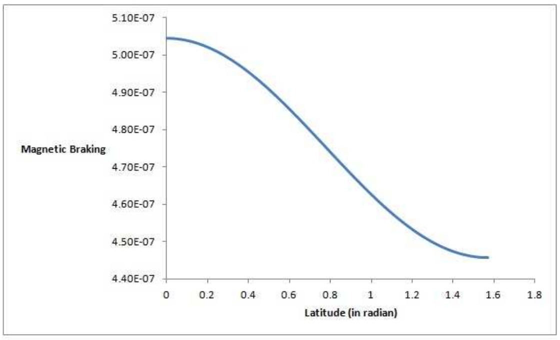


**Fig. 7.** Magnetic Braking in Angular velocity vs. Latitude for the gainer star.

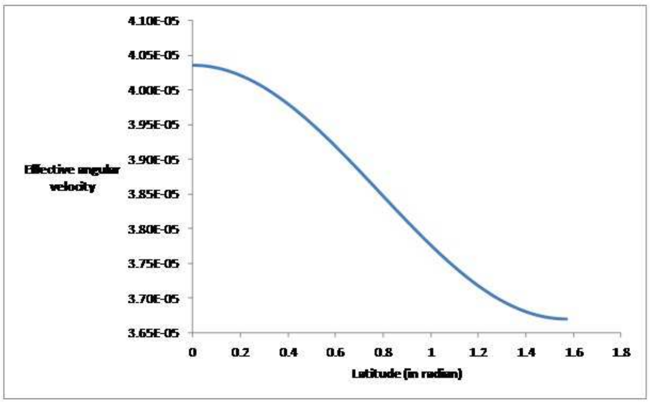


**Fig. 8.** Effective Angular velocity vs. Latitude for the gainer star.