Non-isothermal effects on the static equilibria of magnetized layers of molecular clouds

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Abstract. The molecular clouds (MCs), which are observed as Giant Molecular Clouds, Isolated Bok Globules and/or Infrared Dark Clouds, are the nurseries for forming stars and planets. Observations show that the MCs are influenced by diverse forms of magnetic field lines. The magnetic field gradients can produce the ambipolar diffusion mechanism through the MCs. Nejad-Asghar (2019) showed that considering the heating due to ambipolar diffusion in the MCs, the local thermal balance leads to a local loosely constrained power-law relation between the pressure and density $P \propto \rho^{1+\chi}$, where $-0.4 \leq \chi \leq 0.05$ depends on the functional form of the net cooling function. Physically, the value of χ depends on the power of dependence of magnetic field to the density, and also on the value of the magnetic field gradient. For a strong magnetic field and/or a large field gradient, the value of χ decreases, and vice versa. The substructures through the MCs have complex morphologies from layers to filework and again gradered. to filaments and semi-spheres. Here, for simplicity, we use stratified layer approximation to investigate the effect of the non-isothermal parameter χ on the substructure of the MCs. The results show that considering the non-isothermal equation of state with smaller χ (i.e., stronger magnetic field and/or larger field gradient) transfers the magnetic field lines to the outer cloud regions, and hence decreases the density in the central regions of the cloud. We conclude that the stronger magnetic field and/or larger field gradient can disperse the density fluctuations through the MCs.

Key words: ISM: structure – ISM: clouds – ISM: magnetic fields – stars: formation – (Galaxy:) local interstellar matter

Introduction

The molecular gases in the interstellar medium can be gathered as molecular clouds (MCs), which are usually categorized as Giant Molecular Clouds, Isolated Bok Globules (Bok & Reilly 1947), and some black regions against the mid-infrared lights that are nominated as Infrared Dark Clouds (Pérault et al. 1996). The MCs, which are uniquely distributed in some regions of our galaxy (e.g., Pineda et al. 2013, Heyer & Dame 2015), are currently known as the stellar nurseries (e.g., Stahler & Palla 2004). They have hierarchical substructures from layers to filaments and semi-spherical clumps and cores (e.g., Sawada, Koda & Hasegawa 2018, O'Dell 2018). The self-gravity is the most important mechanism for gathering the atoms, molecules and grains through the MCs. There are some physical factors that can affect the formation and evolution of the substructures through the MCs. Two most important factors in this research are the influence of the magnetic fields (e.g., Hennebelle & Inutsuka 2019), and the effect of the heating and cooling mechanisms (e.g., Nejad-Asghar 2011).

From the balance of cooling and heating, one can derive the pressure as a function of density. For example, the phase diagram (pressure vs density) of the ISM including the most relevant heating and cooling processes in the density ranges $10^{-2} - 10^5$ cm⁻³ is given in Fig. 2 of the recent review by Girichidis et al. (2020). The approximated functional form fitted to this figure, in the density range of $10^3 - 10^5$ cm⁻³ (appropriate for the MCs), is $P \propto \rho$.

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The dissipation of magnetic energy in the MCs should be considered as an important heating mechanism since this energy is not simply radiated away by atoms, molecules, and grains. Nejad-Asghar (2019) showed that considering this heating mechanism changes the phase diagram in the density range of $10^3 - 10^5 \,\mathrm{cm}^{-3}$. In this way, the local thermal balance leads to a local loosely constrained power-law relation between the pressure and density as $P \propto \rho^{1+\chi}$, where $-0.4 \leq \chi \leq 0.05$ depends on the functional form of the net cooling function. Physically, the value of χ depends on the power of dependence of magnetic field to the density, and also on the value of magnetic the field gradient. For strong magnetic field and/or large field gradient, the value of χ decreases, and vice versa.

Effects of the non-isothermal equation of state on the formation of substructures of MCs and their evolution may be crucial (e.g., Hosseinirad et al. 2018). In other words, it is important to investigate the thermal effects of the magnetic dissipation of ambipolar diffusion on the formation and evolution of the substructures through the MCs. Although the substructures of the MCs have complex morphologies (e.g., Bahmani & Nejad-Asghar 2018), the onedimensional slab modeling of clouds is theoretically the simplest way to study the formation of the substructures and their evolution (e.g., Mouschovias 1974, Shu 1983, Nejad-Asghar 2007). Note that depending on the time-scales of cooling/heating rates and dynamical effects (such as free-fall or turbulence), we can consider approximate quasi-static models, for some regions of MCs, in the theoretical approaches (Nejad-Asghar 2011). Here, to investigate the effect of the non-isothermal parameter χ , we turn our attention to the one-dimensional model of the quasi-static magnetized layers (i.e., approximate modeling for some local quasi-static regions of the Giant Molecular Clouds, Isolated Bok Globules and/or Infrared Dark Clouds).

We repeat the analysis, almost exactly, of the pioneer classical formulation of the Mouschovias (1974), for the magneto-hydrostatic equilibrium of a slab, but in a non-isothermal case. Explicitly, we study the effect of the nonisothermal parameter χ on the structure of this simplified one-dimensional model. For this purpose, the construction of the model is given in § 1. The problem is formulated in § 2, and the algorithm of the solution is presented in § 3. The results are given in § 4, and finally, the last section is devoted to the conclusions.

1. Construction of the model

The global appearance of the MCs is observed as layered and filamentary substructures, and also as semi-spherical clumps and cores (e.g., Zhou et al. 2018, Tokuda et al. 2020). To investigate the real effect of the non-isothermal parameter χ , we must consider a three dimensional model. Here, we restrict ourselves to some local regions of the substructures of the MCs (hereafter nominated as *cloud*), and approximate them with the simple layered models. The magnitude and direction of the magnetic fields through the substructures of MCs are very diverse (Crutcher 2012). Following the method of Mouschovias (1974), the magnetic field is assumed to be uniform in the outer region of the cloud (hereafter nominated as *inter-cloud medium*), while through the cloud, the magnetic field lines are non-uniform.

We consider a simplified two dimensional representation (i.e., variables depend on x and y, and independent of z) for a layered geometry of the cloud. We assume that a self-gravitational field is due to the stratified layer as a whole so that $\mathbf{g} = -\hat{j}g(y)$, where g(y) = -g(-y) is a positive constant. The magnetic field in the inter-cloud medium is aligned with the x-axis ($\mathbf{B} = \hat{i}B_0$). Through the cloud we have $\mathbf{B} = \hat{i}B_x(x,y) + \hat{j}B_y(x,y)$. The relation $\nabla \cdot \mathbf{B} = 0$ allows us to recast \mathbf{B} in terms of the magnetic potential \mathbf{A} , via $\mathbf{B} = \nabla \times \mathbf{A}$. Since we assumed that $B_z = 0$, we have $\mathbf{A} = \hat{k}A(x,y)$, and $B_x = +\frac{\partial A}{\partial y}$ and $B_y = -\frac{\partial A}{\partial x}$. In this way, $\mathbf{B} \cdot \nabla A(x,y) = 0$; thus, A is constant on a field line (i.e., each field line through the cloud retains its value of A in the inter-cloud medium with uniform magnetic fields).

The equation for force balance is

$$-\nabla P - \rho \nabla \psi + \frac{\mathbf{J}}{c} \times \mathbf{B} = 0, \tag{1}$$

where $\mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B} = \hat{k}J$, with $J \equiv -\frac{c}{4\pi} \nabla^2 A$, is the current density. The gravitational field g can be derived from the gravitational potential ψ . The pressure P and the density ρ are related via the general equation of state. In treatment of MCs, a thermal balance between heating and cooling mechanisms indicates a relation as follows

$$P \propto \rho^{1+\chi},\tag{2}$$

where the parameter χ depends on the relative importance of the net cooling and heating functions and also on the strength of the magnetic field gradients through the cloud (Nejad-Asghar 2019). $\chi = 0$ indicates the isothermal state. If we consider an isothermal stratified self-gravitating slab in the uniform

If we consider an isothermal stratified self-gravitating slab in the uniform magnetic field $\mathbf{B} = \hat{i}B_i(y)$, the equilibrium state with assumption that the ratio of the magnetic to gas pressures, $\alpha \equiv B_i^2/8\pi P_i$, is constant, has the following profiles

$$\rho_i(y) = \rho_i(0) \exp(-y/H), \tag{3}$$

$$B_i(y) = B_i(0) \exp(-y/2H),$$
 (4)

where $\rho_i(0)$ and $B_i(0) \equiv c_s \sqrt{8\pi\alpha\rho_i(0)}$ are the values of density and magnetic field at y = 0, respectively, and $H \equiv (1 + \alpha)c_s^2/g$, where c_s is the isothermal sound speed. The assumption of $\alpha \approx \text{constant}$ in the initial isothermal state is in agreement with the power-law dependence of the magnetic field to the density as $B \propto \rho^{\eta}$, with $\eta \sim 0.5$ (Crutcher 1999, Crutcher et al. 2010). The initial isothermal state with $\alpha \approx \text{constant}$ can also be deduced from the equipartition relation between the magnetic and thermal energies per unit volume, $E_{mag} = \alpha E_{th}$ (e.g., Ballesteros-Paredes & Vazquez-Semadeni 1995, Donkov, Veltchev & Klessen 2011). In any case, α is a free parameter, which can change the profiles of the initial isothermal states. The cloud is considered initially in the isothermal state, and then evolves to the non-isothermal state via small perturbations in the numerical iteration method. We use the isothermal profiles (3) and (4) as initial states for the iteration method demonstrated in Section 3. Bahmani and Nejad-Asghar

2. Formulation of the problem

We use the dimensionless quantities $\tilde{\rho} \equiv \frac{\rho}{\rho_i(0)}$, $\tilde{P} \equiv \frac{P}{c_s^2 \rho_i(0)}$, $\tilde{y} \equiv \frac{y}{c_s^2/g}$, $\tilde{\psi} \equiv \frac{\psi}{c_s^2}$, $\tilde{B} \equiv \frac{B}{B_i(0)}$, $\tilde{J} \equiv \frac{J}{cB_i(0)g/4\pi c_s^2}$, and $\tilde{A} = \frac{A}{-2HB_i(0)}$. Then the force balance (equation (1)) will be

$$\tilde{\nabla}\tilde{P} + \tilde{\rho}\tilde{\nabla}\tilde{\psi} + 4\alpha(1+\alpha)\tilde{J}\tilde{\nabla}\tilde{A} = 0,$$
(5)

and the equation of state (2) can be rewritten as

$$\tilde{P} = \kappa \tilde{\rho}^{1+\chi},\tag{6}$$

where the parameter κ depends on the relative importance of the net cooling and heating functions and also on the chosen values of the dimensional quantities (Nejad-Asghar 2019). Here, we choose the values of the parameters as $\kappa \sim 1$ and $-0.4 \leq \chi \leq 0.05$, which are suitable for some regions of Giant Molecular Clouds, Bok globules and Infrared Dark Clouds (Nejad-Asghar 2019). Hereafter, we omit for simplicity the tilde notations on the dimensionless quantities.

For investigation of the non-isothermal effects of the parameter χ on the structure of this simplified geometry of the MC, we followed the pioneer method outlined in appendix A of Mouschovias (1974). For this purpose, we reformulated the problem as follows: we define a scalar function of position, q(x, y):

$$q \equiv \exp\left(\int \frac{dP}{\rho} + \psi\right),\tag{7}$$

and rewrite equation (5) in terms of A and q as

$$\rho \nabla \ln q + 4\alpha (1+\alpha) J \nabla A = 0. \tag{8}$$

By knowing that A is constant on each field line, we decompose equation (5) in two directions of the field line: from the parallel direction we deduce that q = q(A) is constant on each field line, and from the perpendicular direction we have

$$\frac{d\ln q}{dA} = -4\alpha(1+\alpha)\frac{J}{\rho}.$$
(9)

Using the definition of current density as $J \equiv 2(1 + \alpha)\nabla^2 A$, we can rewrite equation (9) as follows

$$\nabla^2 A = -\frac{1}{8\alpha(1+\alpha)^2} \rho \frac{d\ln q}{dA}.$$
(10)

The density ρ can be evaluated by recasting equation (7) as follows

$$\rho = q(A) \left[\frac{dq}{dP} - q(A) \frac{d\psi}{dP} \right]^{-1}.$$
(11)

We obtain derivatives appearing in the right-hand sides of equation (11) in a straightforward fashion by using the chain rule

$$\frac{dq}{dP} = \frac{dq}{dA} \frac{dA}{dP}
= \frac{dq}{dA} \left(\frac{\partial A}{\partial x} \frac{\partial x}{\partial \rho} + \frac{\partial A}{\partial y} \frac{\partial y}{\partial \rho} \right) [\kappa (1+\chi) \rho^{\chi}]^{-1}$$
(12)

and

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$$\frac{d\psi}{dP} = \left(\frac{\partial\psi}{\partial x}\frac{\partial x}{\partial\rho} + \frac{\partial\psi}{\partial y}\frac{\partial y}{\partial\rho}\right) \left[\kappa(1+\chi)\rho^{\chi}\right]^{-1}.$$
(13)

In order to find the equilibrium state, we must solve equations (10) and (11); and for this task we need to calculate q(A). For calculating the function q(A), we note that the mass δm in the flux tube between A and $A + \delta A$ is

$$\delta m = \int_{-x}^{+x} dx \int_{y(x,A)}^{y(x,A)+\delta A} dy(x,A)\rho[x,y(x,A]].$$
(14)

Since the integration is done over y in equation (14), by keeping x fixed, we may write $dy = dA(\frac{\partial y}{\partial A})$. We use equation (11) for eliminating ρ in favor of A and then expand the integrand of the resulting equation in a Taylor series about A, keeping only first-order terms, we can solve for q(A) to obtain

$$q(A) = \frac{\frac{dm}{dA}}{2\int_0^x dx \frac{\partial y(x,A)}{\partial A} \left[\frac{dq}{dp} - q\frac{d\psi}{dp}\right]},\tag{15}$$

where $\frac{dm}{dA} = \lim_{\delta A \to 0} \frac{\delta m(A)}{\delta A}$ is the mass in each flux tube. In flux freezing approximation, the conservation of mass and flux implies that dm/dA is constant in any deformation of the cloud.

3. Solution algorithm

We rewrite equation (10) as

$$\nabla^2 A(x,y) = Q(y,A,\rho,q;\alpha), \tag{16}$$

where

$$Q(y, A, \rho, q; \alpha) = -\frac{1}{8\alpha(1+\alpha)^2} \rho \frac{d\ln q}{dA},$$
(17)

equation (11) as

$$\rho(x, y) \equiv F(y, A, \rho, q; \alpha), \tag{18}$$

where

$$F(y, A, \rho, q; \alpha) \equiv q(A) \left[\frac{dq}{dP} - q(A) \frac{d\psi}{dP} \right]^{-1},$$
(19)

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and equation (15) as

$$q(x,y) = L(y, A, \rho, q; \alpha), \tag{20}$$

where

$$L(y, A, \rho.q; \alpha) \equiv \frac{\frac{dm}{dA}}{2\int_0^x dx \frac{\partial y(x, A)}{\partial A} \left[\frac{dq}{dp} - q\frac{d\psi}{dp}\right]}.$$
 (21)

To solve equations (16), (18) and (20) simultaneously, we used an initial guess for ρ , A and q. Then, we iterated on these initial functions to find the best final values of ρ , A and q according to equations (16), (18) and (20). We used the isothermal stratified self-gravitating slab as an initial guess.

As mentioned in equations (3) and (4), the initial dimensionless quantities are

$$\rho_i(y) = \exp[-y/(1+\alpha)], \qquad (22)$$

$$A_i(y) = \exp[-y/(2\alpha + 2)],$$
 (23)

$$q_i(A_i) = \left[\frac{1}{A_i}\right]^{2\alpha},\tag{24}$$

$$\frac{dm}{dA} = -4X(1+\alpha)A_i,\tag{25}$$

where X is the dimensionless value of the boundary of the cloud on the x-axis. Since dm/dA is constant in any deformation of the cloud from initial to final states, q(A) for each iteration is calculated using the same dm/dA.

To solve equations (16), (18) and (20) simultaneously, we also need some boundary conditions at the boundaries $x = 0, \pm X$ and $y = 0, \pm Y$. The dimensionless forms of the boundary conditions for the vector potential A are

$$A(x, y = 0) = 1, (26)$$

$$\frac{\partial A(x,y)}{\partial x}|_{x=0,\pm x} = 0, \qquad (27)$$

$$A(x, Y) = \exp[-Y/(2\alpha + 2)],$$
(28)

and for the density ρ are

$$\rho(x, y = 0) = 1, \tag{29}$$

$$\rho(x, Y) = \exp[-Y/(1+\alpha)].$$
(30)

The algorithm for the solution is as follows: we start from the initial guess

$$A^{(0)}(x,y) = A_i(y) + \delta A = \exp[-y/(2\alpha + 2)] + \delta A,$$
(31)

$$\rho^{(0)}(x,y) = \rho_i(y) = \exp[-y/(1+\alpha)], \tag{32}$$

$$q^{(0)}(x,y) = q_i(A_i) = \exp[\alpha y/(1+\alpha)],$$
(33)

where δA is a small initial perturbation such as

$$\delta A(x,y) = -A_i(y)\mu \sin(\pi y/Y) \cos(\pi x/X); \quad 0 < \mu < 1,$$
(34)

and we use a sequence of recursive relations (n = 0, 1, 2, ...)

$$\nabla^2 A_*^{(n+1)} = Q(y, A^{(n)}, \rho^{(n)}, q^{(n)}; \alpha), \tag{35}$$

$$A^{(n+1)} = A^n \theta^{(n)} + (1 - \theta^{(n)}) A^{(n+1)}_*,$$
(36)

$$\rho_*^{(n+1)} = F(y, A^{(n)}, \rho^{(n)}, q^{(n)}; \alpha), \tag{37}$$

$$\rho^{(n+1)} = \rho^n \beta^{(n)} + (1 - \beta^{(n)}) \rho_*^{(n+1)}, \tag{38}$$

$$q_*^{(n+1)} = L(y, A^{(n)}, \rho^{(n)}, q^{(n)}; \alpha),$$
(39)

$$q^{(n+1)} = q^n \gamma^{(n)} + (1 - \gamma^{(n)}) q_*^{(n+1)}, \tag{40}$$

to find the corresponding (improved) solutions in each iteration. The quantities $A_*^{(n+1)}$, $\rho_*^{(n+1)}$ and $q_*^{(n+1)}$ are provisional, and $0 < \theta^{(n)} < 1$, $0 < \beta^{(n)} < 1$ and $0 < \gamma^{(n)} < 1$ are the relaxation parameters at the *n*th iteration. We postulate that a solution is reached if the conditions

$$\frac{A_*^{(n+1)} - A^{(n)}|}{A_*^{(n+1)}} < \epsilon_1, \tag{41}$$

$$\frac{|\rho_*^{(n+1)} - \rho^{(n)}|}{\rho_*^{(n+1)}} < \epsilon_2, \tag{42}$$

$$\frac{|q_*^{(n+1)} - q^{(n)}|}{q_*^{(n+1)}} < \epsilon_3, \tag{43}$$

are satisfied at all points (x, y), where quantities ϵ_1 , ϵ_2 and ϵ_3 are small positive numbers.

4. Results

We used the aforementioned iteration method to solve equations (16), (18) and (20) simultaneously. The results show that the variations of the parameters κ and α do not have physically important effects on the graphs, so we consider $\kappa = 1$ and $\alpha = 1$ without losing the generality of the problem. According to the algorithm mentioned in the previous section, we studied the isothermal case with $\chi = 0$ and the non-isothermal states with $\chi = -0.2$ and $\chi = -0.4$. The large densities of MCs with hard polytropic equation of sate (i.e., large positive values of the non-isothermal parameter χ) are not relevant to this research because the effect of heating due to ambipolar diffusion is negligible at large densities (Nejad-Asghar, 2019). Here, we choose the boundary values of the problem, similar to the work of Mouschovias (1974), which are X = 9and Y = 25. Using the approximate relation $g \sim 4\pi G \rho_0(i) H$ for a sheetlike cloud (Ibanez & Sigalotti 1984), and definition of H as $(1+\alpha)c_s^2/g$, the spatial dimension is

$$\frac{c_s^2}{g} \approx 0.2 \,\mathrm{pc} \left(\frac{T}{10\mathrm{K}}\right)^{1/2} \left(\frac{n_i(0)}{10^3 \mathrm{cm}^{-3}}\right)^{-1/2}.$$
(44)

In this way, the dimensionless values of X = 9 and Y = 25 correspond to (X = 1.8 pc, Y = 5 pc) and (X = 0.18 pc, Y = 0.5 pc) for typical MCs with temperature T = 10 K and central densities 10^3 cm^{-3} and 10^5 cm^{-3} , respectively.

The function q(A), which is defined by equation (7) can be expressed as

$$q = \begin{cases} e^{\ln \rho} e^{\psi}, & \text{if } \chi = 0 \& \kappa = 1, \\ e^{\frac{1+\chi}{\chi} \kappa \rho^{\chi}} e^{\psi}, & \text{if } \chi \neq 0, \end{cases}$$
(45)

for isothermal and non-isothermal states, respectively. Evidently, the behavior of the function q, in different values of the distance y, is determined by the competition between the increasing function e^{ψ} and the decreasing functions $e^{\ln\rho}$ or $e^{\frac{1+\chi}{\chi}\kappa\rho^{\chi}}$. On the other hand, increasing the distance variable y decreases the density, and consequently, decreases the magnetic field according to the flux-freezing approximation. This corresponds to decreasing of A or increasing of 1/A versus y. Therefore, we expect that the function $\log(A^{-1})$ to be an increasing function relative to the distance y. The variations of the function $\log[q(A)]$ versus $\log(A^{-1})$ over the variety of isothermal and non-isothermal states are shown in Fig. 1. As seen in Fig. 1, in the isothermal case $(\chi = 0)$, the increasing function e^{ψ} always exceeds in value the decreasing function $e^{\ln\rho}$, and so $\log[q(A)]$ is an increasing function versus $\log(A^{-1})$ at all distances y. However, in the non-isothermal states, the decrease of χ enhances the importance of the decreasing function $e^{\frac{1+\chi}{\chi}\kappa\rho^{\chi}}$, and thus, the function q decreases at large distances from the center of the cloud.

In Fig. 2, the magnetic field lines of the isothermal and non-isothermal states are plotted in one-quadrant of the x - y plane. In Fig. 2 each point of the magnetic field line is affected by the three forces: pressure gradients $\mathbf{F}_P \equiv -\nabla P$, magnetic force $\mathbf{F}_B \equiv 2\alpha \mathbf{J} \times \mathbf{B}$ and the constant gravitational force $-\rho g \hat{j}$. In the equilibrium final state, the F_{Px} and F_{Bx} components compensate each other, and the $F_{Py} + F_{By}$ components compensate the gravitational force ρg . Changing the non-isothermal parameter χ leads to change of pressure, and thus, it can change the components of the pressure gradient force, too. Decreasing the value of χ reduces both components of the pressure gradient. In this way, the force balance leads to decrease of the F_{Bx} component and increase of the F_{By} component with decreasing of χ . Therefore, as shown in Fig. 2, considering the non-isothermal equation of state with smaller χ (i.e., stronger magnetic field and/or larger field gradient) transfers the magnetic field lines to the outer cloud regions, and hence decreases the density in the central regions of the MC layer.



Fig. 1. The dependence of the final state of the function q on A in the isothermal ($\chi = 0$) and non-isothermal cases. In all cases, the value of κ is equal to 1. Both q and A are normalized to their values on the x-axis in the initial state.

Conclusion

We considered a *cloud* with a stratified layered approximation through the *inter-cloud medium* with a uniform magnetic field. We investigated the effect of the non-isothermal parameter χ on the equilibrium structure of the cloud. In this way, we arrived at equations (16), (18) and (20), and we used the iteration method to solve them simultaneously. The results show that strong magnetic fields and/or large field gradients (i.e., smaller values of the non-isothermal parameter χ) enhance the importance of the decreasing function $e^{\frac{1+\chi}{\chi}\kappa\rho\chi}$, and thus, the function q decreases at large distances from the center of the cloud. Also, considering the non-isothermal equation of state with smaller χ transfers the magnetic field lines to the outer cloud regions, and hence decreases the density in the central regions of the MC layer. In other words, the stronger magnetic field and/or larger field gradient can disperse the density fluctuations through the MCs. It is obvious that to obtain most reliable results, we must consider a most realistic geometry as filaments or spheres in the subsequent researches.

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Fig. 2. The final equilibrium states of magnetic field lines of a stratified layer of MC in the isothermal ($\chi = 0$) (solid curves) and non-isothermal cases with $\chi = -0.2$ (dash) and $\chi = -0.4$ (dot). In all cases, the value of κ is equal to 1.

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