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# A Note on Blackett's Hypothesis and the $A_p$-Stars

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In 1947 Blackett (1947) assumed that the magnetic moment of the celestial bodies is proportional to their rotational moment. This suggestion was put into a quite elegant formula and was confirmed by the observations of the magnetic fields and the rotation of some celestial bodies: the Earth, Sun, the $A_p$-star, γ Vir and the Gallaxy. The formula presenting Blackett's hypothesis is as follows:

$$U = A\frac{\sqrt{G}}{c}V. \tag{1}$$

Here $U$ is the magnetic moment of the celestial body, $V$ its rotational moment, $G$ the gravitation constant and $c$ the light velocity. $A$ is a coefficient of the order of a unit. This formula is obtained by the analysis of the dimension of the quantities, characterizing the rotation and the magnetic field of the celestial bodies, but following Dibaj and Kaplan (1976), after many discussions it has been abandoned. The reason to abandon it is that there was no physical sense, i. e. no physical phenomena was discovered which would lead to a proportionality of the two pointed out moments.

In the present work we shall discuss Blackett's hypothesis from the point of view of the processes and configurations of the distribution of a given astrophysical quantity of the $A_p$ type stars.

According to our opinion, there is no physical reason for such a relation between the magnetic moment and the rotational one, since in the formula there is only one electromagnetic quantity. We think that, in principle, in the theory of the dimension analysis the requirement must be observed that, on obtaining of formulae after this theory, at least two quantities of the group of quantities characterizing a new independent property, must always be present. This requirement will automatically be fulfilled at a respective system of measuring units, including the independent properties having particular measuring units. Thus, for instance, in the absolute system the electromagnetic quantities are measured in units, including only basic mechanical measuring units. This must be examined formally, because we do not know a way according to which we may obtain an electromagnetic phenomenon by a combination of mechanical phenomena only. Taking into consideration the new property of the gravitation and thermal phenomena is simplified as a consequence of the fact that the new properties in the case of the gravitation mass and statistical characteristics, called temperature, are proportional to certain mechanical quantities in the case of the inertial mass and the energy of the particles, respectively. Taking into consideration these qualitatively different quantities of the mechanical quantities in the dimension theory takes place on including only two constants — the gravitation constant and the Boltzmann's one, respectively. Essentially different is the situation of taking into consideration quantities, characterizing the electromagnetic processes. In this case no electromagnetic quantity is proportional to a mechanical quantity. For example let us take the electric charge $q$ of a given body. It cannot be obtained after the formula $q=km$, where $k$ is random, but not the same for all bodies dimension coefficient — constant, and $m$ is a certain mechanic quantity. That is why, on analysing the dimensions of a given group of quantities in systems of measuring units, without including separately an electromagnetic dimension, the electromagnetic quantities must be at least two.

Following what is pointed out in Dibaj's and Kaplan's (1976) monograph as a confirmation of the above-mentioned assumption, we shall point out that in the field of the elementary particles there is a dependence between the two pointed out moments and it is:

$$U=A\frac{e}{m_p c}V, \tag{2}$$

where $e$ is the electric charge of the electron, and $m_p$ is the mass of the proton. There is the same dependence between the magnetic moment and the moment of rotation of the "black holes":

$$U=A\frac{q}{Mc}V, \tag{3}$$

where $q$ and $M$ are the electric charge and the mass of the "black hole," respectively. We may call the pointed out formulae Blackett's formulae of the elementary particles and the "black holes." Here, there are no difficulties, which we have pointed out for the application of Blackett's formula for the celestial bodies, according to the already mentioned reasons: both in (2) and (3) each there are two electromagnetic quantities and the explanation of the phenomenon is analogical to the explanation of the magnetic field of a rotating sphere charged with electricity.

In formula (1) figures the constant $G$. It is not decisive for the relation between the mechanical and the electromagnetic phenomena, except if it is necessary to take into consideration the gravitation, which in this case is not necessary. The dimension of the root from it in the absolute system is:

$$[\sqrt{G}]=\mathrm{g}^{-\frac{1}{2}}\,\mathrm{cm}^{\frac{3}{2}}\,\mathrm{s}^{-1}. \tag{4}$$

Quantities with the same dimension, including the electromagnetic quantities of the celestial bodies and their basic characteristics such as mass $M$, radius $R$ and rotational period around the axis $P$ are: $q/M$, $iP/M$, $\nu_m/MR$ etc. Here $q$ is the electric charge of a given body, $i$ is the electric current rubbing through a definite area, $\nu_m$ is the coefficient of the magnetic viscosity. We may define also other such combinations, but we think that the above are sufficient in our case. With their help we obtain the following formulae from (1):

$$U=A\frac{q}{Mc}V; \tag{5}$$

(6) $$U = A\frac{iP}{Mc}V;$$

(7) $$U = A\frac{\nu_m}{c\sqrt{MR}}V.$$

In the formulae thus obtained we substitute the respective quantities with their equals from the following equations:

(8) $$U = \frac{1}{2}B_0R^3;$$

(9) $$V = I\Omega;$$

(10) $$I = \zeta_n MR^2;$$

(11) $$\Omega = \frac{2\pi}{P}.$$

In these equations $B_0$ is the homogeneous magnetic induction in a star with radius $R$, mass $M$, period of rotation around the axis $P$, inertial moment $I$ and angular velocity $\Omega$. $\zeta_n$ is the coefficient characterizing the substance distribution in the star and it is considerably smaller than 1. After substituting the equations (8)—(11) in the formulae (5)–(7), we obtain respectively:

(12) $$B_0RP = A\frac{4\pi\zeta_n q}{c};$$

(13) $$B_0R = A\frac{4\pi\,\zeta_n\,i}{c};$$

(14) $$\frac{B_0P}{\sqrt{\rho}} = A\frac{8\pi^{3/2}\,\zeta_n\,\nu_m}{c\sqrt{3}}.$$

From the obtained three formulae under equal other conditions (for example the same mechanism for obtaining the magnetic field and equal distribution of density in the interior of the star), most generally expressed, follow the relations:

(15) $$B_0 \sim \frac{1}{R};$$

(16) $$B_0 \sim \frac{1}{P};$$

(17) $$B_0 \sim \sqrt{\rho}.$$

Considering the fact that, according to their radii, as well as according to the rest of their astrophysical parameters such as mass, effective temperature on the surface, etc., the peculiar A stars differ a little from the normal A stars, the relation (15) does not explain the difference in the magnetic fields on the surface of the peculiar and the normal A stars. If the normal stars from class A ever have a magnetic field on their surface, it should be smaller than the error with which the magnetic fields of the $A_p$ stars are defined, i. e. they must be at least tens of times weaker [see for example Babcock (1958)]. It is clear that the relation between such a strongly changing quantity as $B_0$ and a weakly changing one in the considered spectral interval of $R$ [see for example Allen (1977)] cannot be of the pointed type, if ever such a relation exists. This can be valid, if we also assume a different structure between the normal and the peculiar A stars, and essentially different values of the electromagnetic quantities on the right side of the equation in the formulae (12), (13) and (14), since the radii of the stars in the whole interval of the spectral class, in which the $A_p$ stars are, do not change more than three times. At the same time the observed amplitudes of their magnetic fields are from several hundreds to several thousands of gauss.

Let us consider the following relations (16). We think that the slower rotation of the $A_p$ stars is a result of the transmission of the rotation moment of the circumstellar medium through the magnetic field. It immediately follows that a star with a stronger magnetic field will lose a greater part of its rotation moment and, therefore, will have a longer period of a circumaxis rotation. This is just the opposite of the considered correlation. Besides, if we assume that the mechanism of the formation of the magnetic field is the same for both the normal and peculiar A stars, the normal A stars must then have stronger magnetic fields than the peculiar ones, since they rotate around their axis for a shorter period.

Now let us dwell on the third correlation (17). It also does not give an explanation either of the magnetic fields between the separate $A_p$ stars, or about the fact that the normal A stars have a unnoticeble magnetic field. It follows from this that both the normal and peculiar stars do not differ essentially in mass and radius [see for example Allen (1977)] and hence in average density.

The examination of the problem of the orientation of the magnetic field according to the rotation axis will lead us to the same conclusions. The angles between the axes of the $A_p$ stars magnetic field, considered as a dipole one, and the rotation axis are as a rule large [see Böhm-Vitense (1966)] and according to Blackett's hypothesis they should be small and even equal to zero.

At the same time we must take into consideration the common properties of matter characterized by the correlations of the type of the three examined above. They are as follows: stronger magnetic fields are observed in smaller spaces in the presence of quicker movements and greater densities [see for example Muradjan (1978)]. This is confirmed namely on comparing the magnetic moment with the rotational one for the different celestial bodies, reduced to Blackett's work. As a conclusion we may summarize that Blackett's hypothesis cannot explain the availability of magnetic fields of the $A_p$ stars. The reason is not in the fact that we cannot show its physical sense, as the above mentioned shows, this is easily overcome. The reason lies in the fact that the most common mechanism of existence of the magnetic field, which it describes and which generally manifests itself when the scales of the phenomena are large, does not manifest itself either as it is explained for the difference of the observed magnetic fields of the normal and peculiar A stars, or in the explanation of the magnetic fields between the separate $A_p$ stars.

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# Заметка относительно гипотезы Блякета и $A_p$-звезд

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(Резюме)

В этой работе обсуждается предложенная в 1947 г. гипотеза Блякета о связи между вращательным и магнитным моментами небесных тел с точки зрения анализа размерностей физических величин. Показывается, что с ее помощью нельзя объяснить различий между нормальными и пекулярными звездами спектрального класса А, но что она отражает самые общие свойства материи относительно ее магнитного поля.

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