Bianchi Type–VI_0 Cosmological Model with Electromagnetic Field

P. Pavan Kumar[1,3], R. L. Naidu[2], Shaik Kalesha Vali[3]
[1] Department of Mathematics, Sri GCSR College, Rajam, Andhra Pradesh, India
[2] Department of Mathematics, GMRIT Deemed to be University, Rajam, Andhra Pradesh, India
[3] Department of Mathematics, JNTU-GV, Vizianagaram, Andhra Pradesh, India
lakshumnaidu.r@gmrit.edu.in
(Submitted on 29.12.2025; Accepted on 14.05.2026)

Abstract. In this work, we examine a spatially homogeneous and anisotropic Bianchi type–VI_0 cosmological model filled with a perfect fluid and an electromagnetic field within the framework of $f(R,T)$ gravity, where R denotes the Ricci scalar and T represents the trace of the energy–momentum tensor. We restrict our analysis to the functional form $f(R,T) = R+2\lambda T$, with λ being a coupling constant that characterizes the matter–geometry interaction. Assuming that the electric current is aligned along the z–axis, the magnetic field is described by a single non–zero component of the electromagnetic field tensor, namely F_{12}. Exact solutions of the modified field equations are derived by adopting a time–dependent deceleration parameter expressed as a linear function of cosmic time, $q = -1 + m - kt$, where m and k are constants. The resulting model is analysed in detail by exploring its kinematical and dynamical properties, which are illustrated through graphical plots of the relevant cosmological parameters. Additionally, the energy conditions are tested to assess the physical viability of the model. Also, to enhance the physical understanding of the model, we analyse the associated thermodynamic parameters (Entropy, Enthalpy, Helmholtz free energy and Gibbs free energy). Our results indicate that the presence of an electromagnetic field in intergalactic regions can play a significant role in generating the observed accelerated expansion of the Universe within the considered modified gravity scenario.
Key words: $f(R,T)$ gravity; Bianchi type–VI_0 cosmology; Electromagnetic field; Deceleration parameter.

1 Introduction

Observational evidence accumulated over the past few decades has firmly established that the present Universe is undergoing a phase of accelerated expansion. This remarkable behaviour was first revealed through high-precision measurements of Type Ia Supernovae (Perlmutter et al. 1999, Riess et al. 1998, Schmidt et al. 1998) and later confirmed by independent observations of the large-scale structure of the Universe and anisotropies in the cosmic microwave background radiation (Spergel et al. 2003, Tegmark et al. 2004, de Bernardis et al. 2000, Hanany et al. 2000). These observational milestones collectively indicate that the cosmic expansion is dominated by an unknown component with repulsive gravitational effects. The accelerated expansion is commonly attributed to an exotic cosmic fluid known as dark energy (DE), whose physical nature remains one of the most profound open problems in modern cosmology. Together with dark matter (DM), dark energy constitutes the major fraction of the cosmic energy budget, with current observational estimates suggesting approximately 73% DE, 23% DM, and only about 4% ordinary baryonic matter. Considerable efforts have been devoted to identifying suitable candidates capable of explaining this phenomenon (Nojiri and Odintsov 2007, Sahni and Starobinsky 2000). Although the cosmological constant Λ provides the simplest explanation for late-time acceleration, it suffers from serious theoretical difficulties, most notably the finetuning and coincidence problems (Peebles and Ratra 2003). Numerous alternative dark energy models have been proposed; yet no direct experimental evidence for such exotic components has yet been obtained so far.

An alternative and widely explored approach to address cosmic acceleration is to modify the underlying theory of gravitation itself. This idea has motivated extensive studies of modified theories of gravity, including Brans–Dicke theory (Brans and Dicke 1961), Saez–Ballester theory (Saez and Ballester 1986), and various forms of $f(R)$ gravity (Nojiri and Odintsov 2003). In this context, Harko et al. (2011) proposed the $f(R,T)$ theory of gravitation, in which the gravitational Lagrangian is constructed as a general function of the Ricci scalar R and the trace T of the energy momentum tensor. The explicit dependence on T leads to a non-minimal coupling between matter and geometry, offering new possibilities to explain the late-time cosmic acceleration. Several cosmological models within this framework have been examined, particularly in homogeneous and isotropic Friedmann–Robertson–Walker (FRW) space-times.

Magnetic fields are believed to have played a crucial role during the early stages of the Universe when matter was highly ionized. As pointed out by Melvin (1975), the presence of primordial magnetic fields is physically plausible due to the strong coupling between charged matter and electromagnetic fields in the earlu Universe. As cosmic expansion and cooling proceeded, ions gradually recombined to form neutral matter; however, magnetic fields persisted and are now observed on galactic and intergalactic scales. Several studies (Singh and Singh 1999, Banerjee et al. 1990, Tikekar and Patel 1994, Chakraborty 1990) provide compelling evidence for the cosmological relevance of magnetic fields, motivating the investigation of cosmological models involving matter coupled with electromagnetic fields.

Anisotropic cosmological models, particularly the Bianchi space-times, play a significant role in understanding the early Universe. These models are spatially homogeneous but anisotropic and thus provide a natural generalization of the standard isotropic cosmologies. The nine Bianchi types (I–IX), classified by Taub (1951), have been extensively explored within both general relativity and modified theories, as they offer valuable insights into anisotropic phases of cosmic evolution.

In recent years, several authors have examined Bianchi cosmological models in the context of $f(R)$ and $f(R,T)$ gravity. Notable contributions include studies by Yilmaz et al. (2012), Sharif and Zubair (2012, 2014), Adhav (2012), Chaubey and Shukla (2013), and Reddy et al. (2012a,b), who analyzed various anisotropic models with different matter sources. In parrticular Ahmad and Pradhan (2014) investigated a Bianchi type-V model in $f(R,T)$ gravity for the functional choice $f(R,T)$, while Mohanta (2014) discussed an LRS Bianchi type-I bulk viscous model within the same theoretical framework. Furthermore, Sahoo and Sivakumar (2015) demonstrated the occurrence of a big-rip singularity in an LRS Bianchi type-I Universe using a linearly varying deceleration parameter. More recently, Sahoo et al. (2016) explored a Bianchi type-III cosmological model filled with a perfect fluid for the choices $f(R,T) = R + 2f(T)$ and $f(R,T) = f_1(R) + f_2(T)$.

The Bianchi type–VI_0 (BT–VI_0) space-time represents a spatially homogeneous yet anisotropic generalization of the standard FRW geometry. Owing to its rich geometric structure and relevance to early-Universe anisotropies, this model has attracted considerable attention within various gravitational frameworks. In the context of general relativity, BT–VI_0 cosmological models have been studied by Priyanka et al. (2012) and Adhav et al. (2011). Extensions of these studies to modified gravity theories include analysis of BT–VI_0 spacetimes in $f(R)$ gravity by Shaikh and Katore (2016) and in Brans–Dicke theory by Vidyasagar et al. (2014). Furthermore, Reddy et al. (2016) explored a BT–VI_0 Universe filled with matter and dark energy within the Saez–Ballester scalar–tensor theory.

The role of electromagnetic fields in anisotropic cosmologies has also been widely investigated. Lorenz (1982) derived exact BT–VI_0 solutions for matter distributions coupled with electromagnetic fields, while Hegazy and Rahman (2020) studied such models in general relativity by adopting a deceleration parameter that varies linearly with cosmic time. More recently, cosmological models involving electromagnetic fieldshave been discussed within alternative modified gravity frameworks, such as $f(Q)$ gravity by Shekh et al. (2023).

Motivated by these developments, we investigate a Bianchi type–VI_0 cosmological model containing a perfect fluid and an electromagnetic field within the framework of $f(R,T)$ gravity. In particular, we restrict our analysis to the functional form with $f(R,T) = R + 2f(T)$ with $f(T) = \lambda T$, where λ is a constant parameter governing the matter–geometry coupling.

The organisation of the paper is as follows. In Section 2, we briefly review the fundamentals of $f(R,T)$ gravity in the presence of an electromagnetic field and derive the corresponding field equations using the Hilbert–Einstein action principle. In Section 3, the explicit field equations are formulated for the BT–VI_0 metric. Section 4 is devoted to obtaining exact solutions of the field equations by assuming a linearly time-dependent deceleration parameter. The physical and kinematical properties of the resukting cosmological model are analysed and discussed in Section 5. Finally, the main conclusions of the study are summarized in Section 6.

2 Basics of $f(R,T)$ Gravity with Electromagnetic Field

The field equations of $f(R,T)$ gravity are obtained by applying the Hilbert–Einstein variational principle to the gravitational action. The action functional for $f(R,T)$ gravity in the presence of an electromagnetic field is given by

$$S = \int \sqrt{-g} \left(\frac{1}{16\pi} f(R,T) + \mathcal{L}_m + \mathcal{L}_e \right) d^4x \,, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the energy–momentum tensor T_{ij} associated with the matter distribution. Here, $\mathcal{L}_m$ represents the Lagrangin density of the matter field,while $\mathcal{L}_e$ corresponds to the Lagrangian of the electromagnetic fields. Throughout this work, we adopt geometrized units by setting $G = c = 1$.

The energy–momentum tensor of the matter field is defined in standars way as

$$T_{ij}^{(m)} = -\frac{2}{\sqrt{-g}} \frac{\delta\left(\mathcal{L}_m \sqrt{-g}\right)}{\delta g^{ij}} \,, \tag{2}$$

and

$$E_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_e\sqrt{-g}\right)}{\delta g^{ij}}\,. \tag{3}$$

Accordingly, the total energy–momentum tensor of the system is expressed as the sum of the matter and electromagnetic contributions, namely

$$T_{ij} = T_{ij}^{(m)} + E_{ij}\,. \tag{4}$$

By performing the variation of the action given in Eq. (1) with respect to the metric tensor g_{ij}, one obtaines the modified field equations of $f(R,T)$ gravity.

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_R(R,T) \\ = 8\pi T_{ij} - f_T(R,T)\left(T_{ij} + \theta_{ij}\right), \end{aligned} \tag{5}$$

where $f_R(R,T) = \partial f/\partial R$ and $f_T(R,T) = \partial f/\partial T$ denote the derivatives of $f(R,T)$ with respect to R anfd T respectively. As usual ∇_i is covariant derivative and $\Box \equiv \nabla_i\nabla^i$ is the d'Alembert operator.

The tensor θ_{ij} is defined as

$$\theta_{ij} \equiv g^{kl}\frac{\delta T_{kl}}{\delta g^{ij}} = -2T_{ij} + pg_{ij}\,. \tag{6}$$

In the present analysis, we restrict ourselves to one of the simplest and most widely used functional forms of $f(R,T)$ gravity, namely

$$f(R,T) = R + 2f(T)\,, \tag{7}$$

with the specific choice

$$f(T) = \lambda T\,, \tag{8}$$

where λ is a constant coupling parameter governing the matter–geometry interaction.

The matter content of the Universe is assumed to be a perfect fluid, whose energy–momentum tensor is given by

$$T_{ij}^{(m)} = (\rho + p)u_iu_j - pg_{ij}\,, \tag{9}$$

where ρ denotes the energy density, p represents the isotropic pressure, and u^i is the four-velocity vector satisfying $u^iu_i = 1$.

Following Lichnerowicz (1967), the energy–momentum tensor associated with the electromagnetic field is taken as

$$E_{ij} = \bar{\mu}\left[h_lh^l\left(u_iu_j - \frac{1}{2}g_{ij}\right) + h_ih_j\right], \tag{10}$$

where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \epsilon_{ijkl} F^{kl} u^j \,, \tag{11}$$

with ϵ_{ijkl} being the Levi–Civita tensor density and F_{ij} the electromagnetic field tensor.

Adopting comoving coordinates such that $u^1 = u^2 = u^3 = 0$ and $u^0 = 1$, and assuming that the electric current flows along the z–axis, the electromagnetic field tensor reduces to a single non-vanishing component F_{12}. Under these assumptions, the Maxwell field equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \,, \qquad \left(\frac{1}{\bar{\mu}} F^{ij}\right)_{;j} = 0 \,, \tag{12}$$

are automatically satisfied, with F_{12} being a constant, denoted by K.

3 Metric and Field Equations

We consider the Bianchi type–VI$_0$ (BT–VI$_0$) metric as

$$ds^2 = dt^2 - A^2(t) e^{-2\alpha z} dx^2 - B^2(t) e^{2\alpha z} dy^2 - C^2(t) dz^2, \tag{13}$$

where α is a constant.

For the metric (13), Eq. (11) given us

$$h_3 = \frac{CK}{\bar{\mu} AB} \,, \qquad h^l h_l = h_3 h^3 = g^{33} h_3^2 = \frac{K^2}{\bar{\mu} A^2 B^2} \,. \tag{14}$$

Now from Eqs. (10) and (14), we obtain

$$E_1^1 = E_2^2 = -\frac{K^2}{2\bar{\mu} A^2 B^2} \,, \quad E_3^3 = \frac{K^2}{2\bar{\mu} A^2 B^2} \,, \quad E_0^0 = \frac{K^2}{2\bar{\mu} A^2 B^2} \,. \tag{15}$$

For the metric (13), we have from (9)

$$T_1^{1(m)} = T_2^{2(m)} = T_3^{3(m)} = -p \,, \qquad T_0^{0(m)} = \rho \,. \tag{16}$$

Using Eqs. (7), (15) and (16), the field equations (5) reduce to the following system of independent equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{C^2}=\lambda\rho-(8\pi+7\lambda)p+\frac{4\pi K^2}{\bar{\mu}A^2B^2}\,, \tag{17}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{C}}{AC}-\frac{\alpha^2}{C^2}=\lambda\rho-(8\pi+7\lambda)p+\frac{4\pi K^2}{\bar{\mu}A^2B^2}\,, \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\alpha^2}{C^2}=\lambda\rho-(8\pi+7\lambda)p-\frac{4\pi K^2}{\bar{\mu}A^2B^2}\,, \tag{19}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{C^2}=(8\pi+3\lambda)\rho-5\lambda p-\frac{4\pi K^2}{\bar{\mu}A^2B^2}\,, \tag{20}$$

$$\alpha\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0\,. \tag{21}$$

Conservation Equation Discussion:

Using Eqs. (6), (7) and (8), the field equation Eq. (5) reduces to the following equation

$$R_{ij}-\frac{1}{2}Rg_{ij}=(8\pi+2\lambda)T_{ij}+\lambda Tg_{ij}+2\lambda pg_{ij}\,. \tag{22}$$

Using the Bianchi identity,

$$\nabla^i(G_{ij})=0 \quad \text{i.e.,} \quad \nabla^i\left(R_{ij}-\frac{1}{2}Rg_{ij}\right)=0\,,$$

and considering divergence on both sides of Eq. (22), gives

$$\nabla^i T_{ij}=-\frac{\lambda}{8\pi+2\lambda}\,\nabla^i\left[(\rho-p)g_{ij}\right]. \tag{23}$$

For co-moving coordinates $u^i=(0,0,0,1)$, Eq. (23), gives

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=-\frac{\lambda}{8\pi+2\lambda}(\dot{\rho}-\dot{p})\,. \tag{24}$$

Due to the electromagnetic field contribution, the density and pressure become

$$\rho=\rho+\rho_B=\frac{K^2}{2\bar{\mu}A^2B^2}\,, \qquad p=p+\rho_B=\frac{K^2}{2\bar{\mu}A^2B^2}\,, \tag{25}$$

where

$$\rho_B=\frac{K^2}{2\bar{\mu}A^2B^2}$$

is the electromagnetic energy density.

Using Eq. (25), Eq. (24) becomes

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\dot{\rho}_B+2\rho_B\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=-\frac{\lambda}{8\pi+2\lambda}(\dot{\rho}-\dot{p})\,. \quad (26)$$

Verification of the Conservation Equation

To verify consistence of the model, we express the field equations (17)–(20) in terms of the directional Hubble parameters H_x, H_y and H_z. Combining these equations appropriately and differentiating the constraint equation (20), we obtain

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\dot{\rho}_B+2\rho_B\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=-\frac{\lambda}{8\pi+2\lambda}(\dot{\rho}-\dot{p}). \quad (27)$$

Equations (26) and (27) show that the conservation relation is identically satisfied by the field equations, confirming the internal consistency of the model. This coincides exactly with the conservation equation Eq. (26) derived above.

Hence, the conservation equation of the energy-momentum tensor is identically satisfied for the obtained solution. This confirms the internal consistency and physical validity of the present Bianchi type–VI$_0$ cosmological model with electromagnetic field in $f(R,T)$ gravity.

4 Solutions and the Model

From Eq. (21), we obtain

$$A(t)=c_1 B(t)\,, \quad (28)$$

where c_1 is a constant of integration.

We use the deceleration parameter of the model as a linear function of cosmic time t in the form (Akarsu and Dereli 2012) given by

$$q=-1+m-kt\,, \quad (29)$$

where m and k are non-zero positive constants.

Using the definition of the deceleration parameter, we have

$$q=-\frac{R\ddot{R}}{\dot{R}^2}\,, \quad (30)$$

where R is the average scale factor of the Universe given by

$$R^3(t)=ABC\,. \quad (31)$$

Solving Eqs. (29) and (30), we have

$$R(t) = c_2 \exp\left[\frac{2}{m}\tanh^{-1}\left(\frac{kt}{m} - 1\right)\right] = c_2\left(\frac{kt}{2m - kt}\right)^{\frac{1}{m}}, \tag{32}$$

where c_2 is a constant.

Now using Eqs. (28), (31) and (32), we have

$$A(t) = c_1 B(t) = c_1\left(\frac{\frac{kt}{m}}{2 - \frac{kt}{m}}\right)^{\frac{1}{m}}, \quad C(t) = c_3\left(\frac{\frac{kt}{m}}{2 - \frac{kt}{m}}\right)^{\frac{1}{m}}, \tag{33}$$

where c_3 is a constant and $c_1 c_3 = c_2^3$.

Using Eqs. (33) and the metric (13) can be written in the form (after a suitable choice of constants $c_1 = c_2 = c_3 = 1$)

$$ds^2 = dt^2 - \left(\frac{\frac{kt}{m}}{2 - \frac{kt}{m}}\right)^{\frac{2}{m}}\left(e^{-2\alpha z}dx^2 + e^{2\alpha z}dy^2 + dz^2\right), \tag{34}$$

which represents the BT–VI_0 cosmological model in $f(R,T)$ gravity.

5 Kinematical and Physical Parameters

We now present the following physical and kinematical parameters of our model given by Eqs. (34) and their discussion.

The volume element is

$$V = \sqrt{-g} = ABC = \left(\frac{\frac{kt}{m}}{2 - \frac{kt}{m}}\right)^{\frac{3}{m}}. \tag{35}$$

The average scale factor is

$$R = V^{1/3} = \left(\frac{\frac{kt}{m}}{2 - \frac{kt}{m}}\right)^{\frac{1}{m}}. \tag{36}$$

The expansion scalar is

$$\theta = \frac{6}{2mt - kt^2}. \tag{37}$$

The mean Hubble parameter is given by

$$H = \frac{\dot{R}}{R} = \frac{2}{2mt - kt^2}. \tag{38}$$

The deceleration parameter is defined as

$$q = -1 + m - kt\,. \tag{39}$$

The magnetic permeability of the model is expressed as

$$\bar{\mu} = -\frac{4\pi}{\alpha^2}\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}}. \tag{40}$$

The isotropic pressure is given by

$$p = -\frac{1}{64\pi^2+80\pi\lambda+16\lambda^2}\left[2\lambda\alpha^2\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}} + \frac{(64\pi+24\lambda)(kt-m+1)+32\pi}{(2mt-kt^2)^2}\right]. \tag{41}$$

The energy density of the cosmic fluid is obtained as

$$\rho = \frac{1}{64\pi^2+80\pi\lambda+16\lambda^2}\left[-(14\lambda+16\pi)\alpha^2\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}} + \frac{4(24\pi+6\lambda-10\lambda kt+10\lambda m)}{(2mt-kt^2)^2}\right]. \tag{42}$$

The equation-of-state (EoS) parameter is defined as

$$\omega = \frac{p}{\rho} = -\left[\frac{2\lambda\alpha^2\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}} + \frac{(64\pi+24\lambda)(kt-m+1)+32\pi}{(2mt-kt^2)^2}}{(14\lambda+16\pi)\alpha^2\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}} - \frac{4(24\pi+6\lambda-10\lambda kt+10\lambda m)}{(2mt-kt^2)^2}}\right]. \tag{43}$$

The combinations required to test the energy conditions are obtained as

$$\rho + p = \frac{1}{4\pi+\lambda}\left[-\alpha^2\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}} + \frac{4(m-kt)}{(2mt-kt^2)^2}\right], \tag{44}$$

$$\rho - p = \frac{1}{4(4\pi+\lambda)(\pi+\lambda)}\left[-\alpha^2(4\pi+3\lambda)\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}} + \frac{4[(4\pi-\lambda)(kt-m)+(12\pi+3\lambda)]}{(2mt-kt^2)^2}\right], \tag{45}$$

$$\rho + 3p = \frac{1}{4(4\pi+\lambda)(\pi+\lambda)}\Bigg[-\alpha^2(4\pi+5\lambda)\left(\frac{2m-kt}{kt}\right)^{\frac{2}{m}} + \frac{4[(12\pi+7\lambda)(m-kt)-(12\pi+3\lambda)]}{(2mt-kt^2)^2}\Bigg]. \tag{46}$$

We now focus our attention on behaviour of the physical and kinematical parameters of our model through their graphical representtation. For all figures, we fixed the model parameters as

$$\alpha = 0.01, \quad m = 6, \quad k = 0.05, \quad \lambda = 8.$$

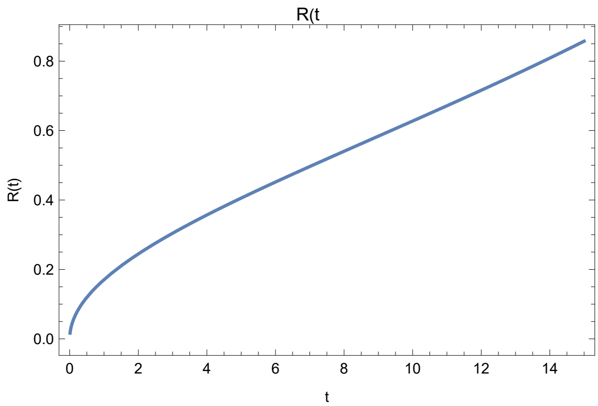

Fig. 1. Variation of the scale factor $R(t)$ with cosmic time t for $0 \leq t \leq 15$.

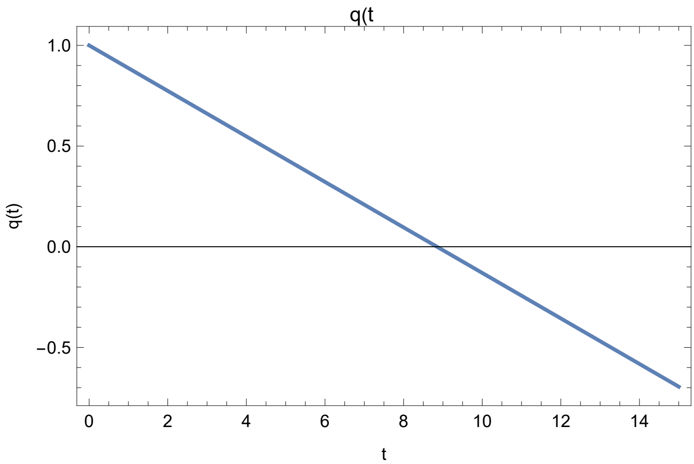

Fig. 2. Variation of the deceleration parameter $q(t)$ with cosmic time t for $0 \leq t \leq 15$. The linear decrease of $q(t)$ demonstrates a smooth dynamical transition from an initially decelerating phase to a late-time accelerating regime, indicating the dominance of repulsive effects at sufficiently large t.

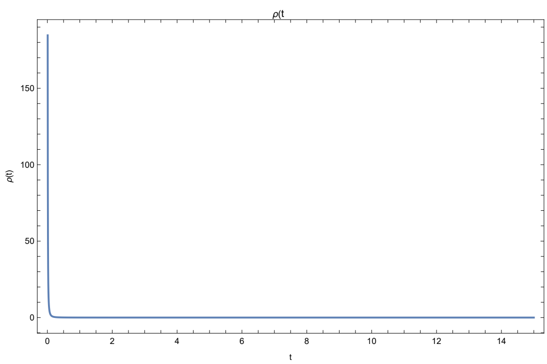

Fig. 3. Evolution of the energy density $\rho(t)$ with cosmic time t for $0 \leq t \leq 15$.
The decreasing behaviour of $\rho(t)$ is consistent with the dilution of matter and electromagnetic energy in an expanding anisotropic universe and reflects the gradual weakening of gravitational attraction at late times.

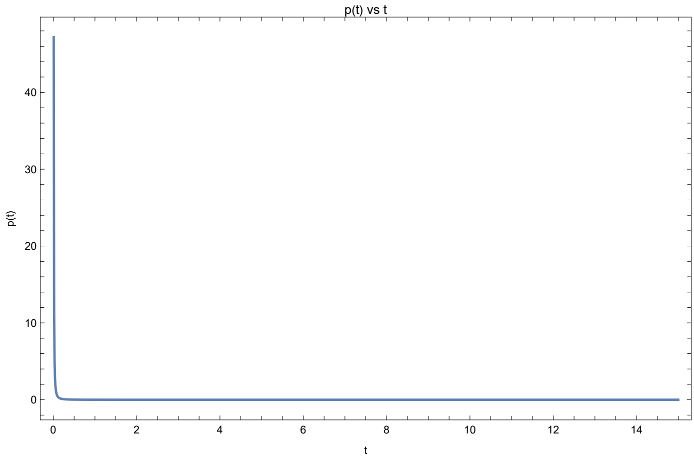

Fig. 4. Variation of the isotropic pressure $p(t)$ with cosmic time t for $0 \leq t \leq 15$. The pressure decreses monotonically as the universe expands, supporting the emergence of negative effect pressure and the accelerated expansion indicated by the behaviour of $q(t)$.

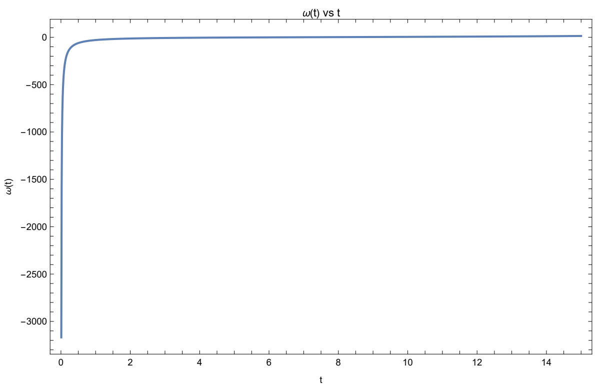

Fig. 5. Evolution of the equation-of-state parameter $\omega = \frac{p}{\rho}$ with cosmic time t for $0 \leq t \leq 15$. In the physical domain $0 < t < 240$, $\omega(t)$ increases monotonically from an early-time phantom-like value $\omega(t \to 0) \approx -1.30$ toward positive values, approaching $\omega(t \to 240) \approx 0.85$ as the model approaches a finite-time singularity at $t = 240$. This evaluation implies a transition from a strongly negative effective pressure (accelerated, phantom-like expansion) at early times to a pressure-dominated, decelerating-like regime near $t \approx 240$. This evaluation indicates a transition from accelerated(photom-like) expansion to a pressure-dominated regime and signals the breakdown of the model at the pole $12 - 0.05t = 0$.

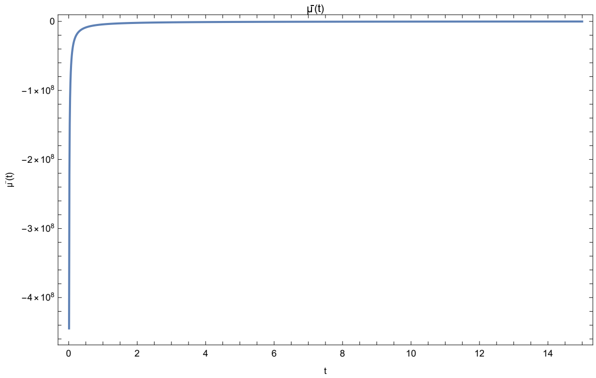

Fig. 6. Variation of the magnetic permeability $\bar{\mu}(t)$ with cosmic time t for $0 \leq t \leq 15$. The sharp decrease of $\bar{\mu}(t)$,reflecting a rapid dilution of the electromagnetic field strength and the suppression of anisotropic effects in the late time Universe.

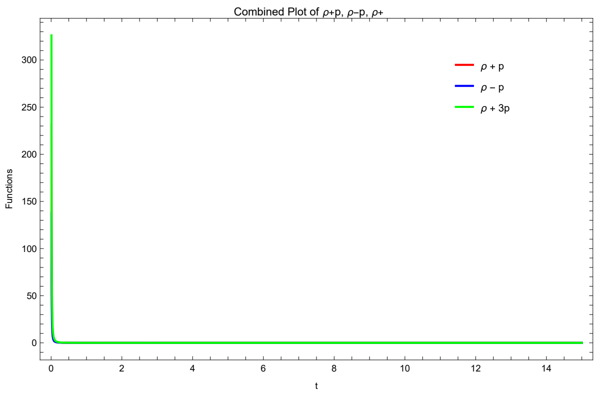

Fig. 7. Evolution of the energy condition combinations $\rho{+}p$, $\rho{-}p$, and $\rho{+}3p$ with cosmic time t for $0 \leq t \leq 15$. These quantities initially remain positive and then decrease monotonically, crossing zero at approximately $t \approx 10.42$ for $\rho - p$, $t \approx 29.06$ for $\rho + p$, and $t \approx 37.02$ for $\rho + 3p$. Accordingly, the dominant energy condition (DEC) holds up to $t \approx 10.4$, the null energy condition (NEC) holds up to $t \approx 29$, and the strong energy condition (SEC) holds up to $t \approx 37$. The divergence of these quantities is governed by the factor $(2mt - kt^2)^{-2}$, indicating the presence of a finite-time singularity and the eventual breakdown of classical energy conditions.

Thermodynamic Parameters discussion:

To enhance the physical understanding of the model, we analyse the associated thermodynamic parameters. Hegazy (2019) introduced a new class of Bianchi type-I cosmological models within the framework self-creation theory and investigated the behaviour of thermodynamic functions in the presence of a scalar field ϕ. In subsequent studies, Hegazy examined Bianchi type-VI spacetimes in Barber's second self-creation theory (Barber 1982, Barber 2010), where the influence of the scalar field ϕ on the entropy S of the Universe was explicitly analysed. Motivated by these studies, we extend the investigation of thermodynamic quantities to the context of $f(R,T)$ gravity in the presence of an electromagnetic field, and the corresponding thermodynamics functions of the Universe are formulated and discussed as follows.

Basic Thermodynamic Relations

The internal energy of the system is defined as

$$U = \rho V \, ,$$

where V is the volume of the Universe.

From the second law of thermodynamics,

$$T\, dS = dU + p\, dV = \frac{V}{T} d\rho + \frac{\rho + p}{T}\, dV \, ,$$

where T denotes the temperature of the Universe. In cosmology, the temperature is often related to the Hubble parameter (Cai and Kim 2005, Ebadi and Moradpour 2015) as

$$T \approx \frac{H}{2\pi} \, .$$

The average scale factor is given by

$$a = V^{1/3} \, .$$

Thermodynamic Potentials

Enthalpy:

$$H = U + pV = (\rho + p)V \, .$$

Entropy: From the thermodynamic relation,

$$T\, dS = d(\rho V) + p\, dV \, ,$$

for quasi-equilibrium cosmology, the entropy is given by

$$S = \frac{(\rho + p)V}{T} \, .$$

Helmholtz Free Energy:

$$F = U - TS = \rho V - T\left(\frac{\rho + p}{T}V\right) = -pV\,.$$

Thus, the Helmholtz free energy becomes negative for positive pressure fluids.
Gibbs Free Energy:

$$G = H - TS = (\rho + p)V - T\left(\frac{\rho + p}{T}V\right) = 0\,.$$

This implies that the cosmic fluid evolves in a state of thermodynamic equilibrium. The vanishing Gibbs energy suggests that the expansion process does not produce additional usable thermodynamic work, which is consistent with the large-scale equilibrium behaviour of the Universe.

Explicit Forms of Thermodynamic Quantities

Using Eqs. (35), (41) and (42), we obtain the following expressions:

$$H = -\frac{\lambda + \pi}{4\pi^2 + 5\pi\lambda + \lambda^2}\left[\alpha^2\left(\frac{2m - kt}{kt}\right)^{-\frac{1}{m}} - \frac{4(kt - m)}{(2mt - kt^2)^2}\left(\frac{2m - kt}{kt}\right)^{-\frac{3}{m}}\right],$$

$$S = -\frac{\lambda + \pi}{4\pi^2 + 5\pi\lambda + \lambda^2}\frac{1}{T}\left[\alpha^2\left(\frac{2m - kt}{kt}\right)^{-\frac{1}{m}} - \frac{(kt - m)}{(2mt - kt^2)^2}\left(\frac{2m - kt}{kt}\right)^{-\frac{3}{m}}\right],$$

$$F = -\frac{1}{8\left(4\pi^2 + 5\pi\lambda + \lambda^2\right)}\left[\lambda\alpha^2\left(\frac{2m - kt}{kt}\right)^{-\frac{1}{m}} + \frac{(32\pi + 12)(kt - m + 1) + 16\pi}{(2mt - kt^2)^2}\left(\frac{2m - kt}{kt}\right)^{-\frac{3}{m}}\right],$$

$$G = 0\,.$$

Physical Interpretation of Thermodynamic Potentials

In the present model, the energy density gradually decreases as the Universe expands and the cosmic volume grows with time. Consequently, the enthalpy initially evolves dynamically and later stabilizes, indicating that the thermodynamic system approaches a quasi-equilibrium state during cosmic evolution. In the present model, the temperature T decreases as the Universe expands.

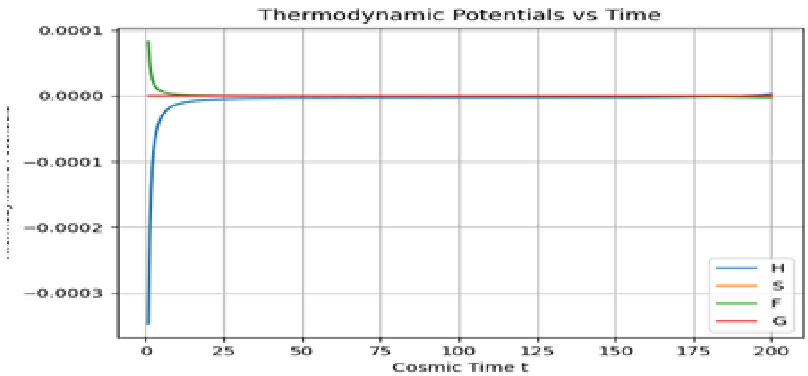

Fig. 8. The graphical representation of the thermodynamic potentials for $\alpha = 0.01$, $m = 6$, $k = 0.05$, and $\lambda = 8$.

The cosmic volume increases with time. Since entropy is proportional to V/T, the entropy increases with cosmic time. This behaviour indicates that the Universe evolves from a highly ordered early state to a more disordered late-time state. Thus, the entropy growth confirms that the thermodynamic evolution of the model is physically consistent with the second law of thermodynamics. For the present cosmological model, the pressure remains negative during expansion. The volume increases with time. Therefore, the Helmholtz free energy gradually decreases (becomes more negative), indicating that the Universe naturally evolves towards a stable thermodynamic configuration.

$G = 0$ implies that the cosmic fluid evolves in a state of thermodynamic equilibrium. The vanishing Gibbs energy suggests that the expansion process does not produce additional usable thermodynamic work, which is consistent with the large-scale equilibrium behaviour of the Universe.

Overall, the analysis of thermodynamic potentials shows that:

- Entropy increases with cosmic time, satisfying the second law of thermodynamics.
- Enthalpy evolves smoothly, reflecting the balance between energy density decay and cosmic expansion.
- Helmholtz free energy becomes increasingly negative, indicating thermodynamic stability.
- Gibbs free energy remains zero, implying that the Universe evolves in a quasi-equilibrium thermodynamic state.

These results confirm that the cosmological model remains thermodynamically viable throughout cosmic evolution.

6 Conclusion

Sahoo et al. (2016) analysed a Bianchi type-III cosmological model within the framework of $f(R,T)$ gravity. Motivated by their work, the present study investigates a Bianchi type-VI_0 cosmological model incorporating an electromagnetic field in the $f(R,T)$ gravity theory proposed by Harko et al. (2011). The modified field equations are solved by assuming a linearly time-dependent deceleration parameter expressed as a function of cosmic time t.

As a result, the key kinematical quantities of the model, including the average scale factor, Hubble parameter, expansion scalar, and deceleration parameter, are explicitly derived and analysed. In addition, important physical parameters such as isotropic pressure, energy density of matter distribution and Magnetic permeability have been evaluated. The graphical representations of these quantities provide a clear picture of the dynamical evolution of the model.

The evolution of the Bianchi type-VI_0 magnetized cosmological model indicates that the deceleration parameter q decreases linearly with time, signalling a smooth transition from an early decelerating epoch to a late-time accelerated phase of expansion. The scale factor $R(t)$ increases monotonically, conforming continuous anisotropic expansion driven by the combined effects of perfect-fluid and electromagnetic field.

Furthermore, the energy density $\rho(t)$ exhibits a progressive decrease with cosmic time, consistent with the dilution of matter in an expanding universe, while the pressure $p(t)$ decreases correspondingly, supporting the emergence of negative effective pressure required for cosmic acceleration. The magnetic permeability $\bar{\mu}(t)$ is also found to decrease sharply with time, reflecting the weakening influence of electromagnetic fields and contributing to the suppression of anisotropy in the late-time cosmic evolution.

From a thermodynamic perspective, the model exhibits physically consistent behaviour. The entropy increases with cosmic time, satisfying the second law of thermodynamics. The enthalpy evolves smoothly, indicating a balance between the decay of energy density and cosmic expansion. The Helmholtz free energy becomes increasingly negative, suggesting thermodynamic stability of the system, while the Gibbs free energy remains zero, implying that the Universe evolves in a quasi-equilibrium thermodynamic state.

Overall, the present model demonstrates that the inclusion of an electromagnetic field within the $f(R,T)$ gravity framework can play a significant role on explaining the observed accelerated expansion of the Universe, while also providing a viable anisotropic scenario consistent with current observational trends.

Acknowledgements

The authors are grateful to the referees for their valuable suggestions and insightful comments, which have significantly improved the quality of the paper. The authors also thank Prof. D. R. K. Reddy, Retd. Professor of Andhra University, for his helpful discussions in the preparation of the revised manuscript.

References

Adhav, K.S., Astrophys. Space Sci. **339**, 365 (2012).
Adhav, K.S., *et al.*, Astrophys. Space Sci. **332**, 497 (2011).
Ahmad, N., Pradhan, A., Int. J. Theor. Phys. **53**, 289 (2014).
Akarsu, O., Dereli, T., Int. J. Theor. Phys. **51**, 612 (2012).

Banerjee A., Sanyal, A.K., Chakraborty, S., Pramana **34**, 1 (1990).
Barber, G.A., Gen. Relativ. Gravit. **14**, 117 (1982).
Barber, G.A., arXiv:1009.5862 (2010).
Brans, C.H., Dicke R.H., Phys. Rev. **124**, 925 (1961).
Cai, R.G., Kim, S.P., J. High Energy Phys. **2005**, 050 (2005).
Chakraborty, S., Indian J. Phys. **29**, 31 (1990).
Chaubey, R., Shukla, A.K., Astrophys. Space Sci. **343**, 415 (2013).
de Bernardis, P., *et al.*, Nature **404**, 955 (2000).
Ebadi, H., Moradpour, H., Int. J. Mod. Phys. D **24**, 1550098 (2015).
Hanany, S., *et al.*., Astrophys. J. **545**, L5 (2000).
Harko, T., Lobo, F.S.N., Nojiri S and Odintsov S.D., Phys. Rev. D **84**, 024020 (2011).
Hegazy, E.A., Iran. J. Sci. Technol. Trans. A Sci. **43**, 2069 (2019).
Hegazy, E.A., Rahaman, F., Indian J. Phys. **94**, 1847 (2020).
Lichnerowicz, A., *Relativistic Hydrodynamics and Magnetohydrodynamics* (W.A. Benjamin, New York, 1967), p. 93.
Lorentz, D., Astrophys. Space Sci. **85**, 69 (1982).
Melvin, M.A., Ann. N.Y. Acad. Sci. **262**, 253 (1975).
Mohanta, K.L., Astrophys. Space Sci. **353**, 683 (2014).
Nojiri, S., Odintsov, S.D., Phys. Rev. D **68**, 123512 (2003).
Nojiri, S., Odintsov, S.D., Int. J. Geom. Methods Mod. Phys. **4**, 115 (2007).
Peebles, P.J.E., Ratra, B., Rev. Mod. Phys. **75**, 559 (2003).
Perlmutter, S., *et al.*, Astrophys. J. **517**, 565 (1999).
Priyanka, S.C., Singh, M.K., Ram, S., Glob. J. Sci. Front. Res. **12**, 83 (2012).
Reddy, D.R.K., *et al.*, Int. J. Theor. Phys. **51**, 249 (2012).
Reddy, D.R.K., *et al.*, Astrophys. Space Sci. **342**, 249 (2012).
Reddy, D.R.K., *et al.*, Astrophys. Space Sci. **361**, 349 (2016).
Riess, A.J., *et al.*, Astron. J. **116**, 1009 (1998).
Sahoo, P.K., Sivakumar, M., Astrophys. Space Sci. **357**, 60 (2015).
Sahoo, P.K., Sahu, S.K., Nath A., Eur. Phys. J. Plus **131**, 18 (2016).
Saez, D., Ballester, V.J., Phys. Lett. A **113**, 467 (1986).
Sahni, V., Starobinsky, A., Int. J. Mod. Phys. D **9**, 37 (2000).
Schmidt, B.P., *et al.*, Astrophys. J. **507**, 46 (1998).
Shaikh, A.Y., Katore, S.D., Bulg. J. Phys. **43**, 184 (2016).
Sharif, M., Zubair, M., J. Phys. Soc. Jpn. **81**, 114005 (2012).
Sharif, M., Zubair, M., Astrophys. Space Sci. **349**, 457 (2014).
Shekh, S.H., *et al.*, arXiv:2309.15853 [gr-qc] (2023).
Singh, G.P., Singh, T., Gen. Relativ. Gravit. **31**, 377 (1999).
Spergel, D.N., *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).
Taub, A.H., Ann. Math. **53**, 472 (1951).
Tegmark, M., *et al.*, Phys. Rev. D **69**, 103501 (2004).
Tikekar, R., Patel, L.K., Gen. Relativ. Gravit. **42**, 483 (1994).
Vidyasagar, T., *et al.*, Eur. Phys. J. Plus **129**, 36 (2014).
Yilmaz, I., Baysal, H., Aktar, C., Gen. Relativ. Gravit. **44**, 9 (2012).