# RAPID APPROXIMATION OF FUNDAMENTAL PARAMETERS AND SCENARIOS IN GALACTIC OPEN CLUSTER STUDIES.

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## I – Introduction and Technical notes about this code.

Since electronic sheet feel at ease over many astronomy research fields and open cluster photometric study is one of these, an Excel + VBA application that can be useful to obtain, very rapidly, cluster fundamental parameters and scenarios is presented. To provide minimal information on this application are summarized briefly some important, even if non exhaustive points, applying this code to open clusters IC 2581 and Ngc 129. Particularly are illustrated first, a photometric analysis and solution for IC 2581 and after, using Ngc 129, some arguments related to stellar variability inside open clusters.

## II - The software

The problem we will deal here with, regards the study of galactic open clusters using Excel electronic sheet particularly concerning, automatic reduction and interpretation of the photometric values obtained from *UBV* measurements upon cluster members. In dealing this argument, we will proceed giving for granted both knowledge of UBV photometric system as specified in Johnson & Morgan (1953) and the techniques of acquisition and elaboration of astronomical images that are necessary for these kinds of studies. Previous restriction even if disagreeable seems however necessary to be able to talk about other remarkable astronomical fields converging in these kinds of studies. In open cluster photometric studies, electronic sheet can give a very important contribution, especially in terms of execution speed of the numerous necessary calculations. At this purpose author has focused his activity mostly in finding various VBA (Visual Basic for Application) algorithms Giaccaglini, Harris (2000), in order to transform Excel in a code devoted to the study of open clusters. A short survey, will give an idea of the great number of information that can be obtained out of a photometric analysis using the proposed code. People who wish to know some more, can get entire package in freeware distribution from the following link: http://xoomer.virgilio.it/waphil/ftp.htm

This code, using the techniques connected to the photometric diagrams allows us to obtain the following parameters:

? Intrinsic colours
? Effective temperatures
? Distances
? Absolute and bolometric magnitudes
? Brightness
? Radii
? Ages

- ? Metallicities
- ? Luminosity functions (LFs)
- ? Present day mass functions (PDMFs)
- ? Objects lying into different HR diagram instability strips
- ? Various other data

The observational data are essentially apparent magnitudes and colours in a given photometric system. Transformations of observational quantities into effective temperatures and luminosities require various steps and are not sure a trivial work. Studying our cluster, we start from apparent magnitudes and colours, subsequently we correct them for extinction effects and if distance is known, we can translate apparent magnitudes into absolute magnitudes. Later on from absolute magnitudes we calculate bolometric magnitudes applying the so called bolometric correction (BC) Heintze (1973). This correction takes into account the fraction of flux not detected by our observing window. Further this point, we can transform colours into effective temperatures (Teff.) and finally proceed to compare resulting Hr diagram with theoretical tracks and isochrones. In order to understand on which bases this code work, in following sections we describe the various procedures for estimating cluster reddening, membership, distance and calibration scales. Since the first jobs of Johnson & Morgan (1953, 1954, 1957) we have learned that the methodology in order to obtain the fundamental parameters for any open cluster is the following:

**1)** Calculate the medium colour excess $<E(B-V)>$ for cluster.

**2)** Calculate individual colour excess for every cluster members.

**3)** Derive members intrinsic colour excesses $(B-V)o = (B-V) - E(B-V)$.

**4)** Derive members intrinsic colour excesses $(U-B)o = (U-B) - E(U-B)$.

**5)** Calculate individual apparent magnitudes without effects of due to interstellar absorption as: $Vo = V-RE(B-V)$.

**6)** Obtain individual absolute magnitudes values through application of empirical zams relation $Mv = f(B-V)o$.

**7)** Calculate individual distance modulus: $Vo-Mv = V-RE(B-V) - Mv$.

**8)** Derive cluster medium true distance modulus: $< Vo-Mv >$.

An Excel automation package related to **8** previous points, is not particularly difficult to obtain and can be an educational useful exercise since it introduce us, among other things, to programming techniques in Windows environment.

## II-a <u>How code obtains medium cluster colour excess.</u>

Using procedures normally applied to this kind of studies, we start considering intrinsic empirical calibrations that can be used on two color diagram (TCD). In our software the determination of medium color excess $<E(B-V)>$ happens on TCD, using for default, the Schmidt-Kaler (1982) empirical calibration. The code therefore arranges, where required, various other calibrations like those of: Becker & Fenkart (1971), Eggen (1965), Fitzgerald (1970), Johnson (1963) and Mermilliod (1981). The derivation of *<E(B-V)>* happens using the sliding fit technique. This technique consists in move any two color calibration according to the reddening line (***RL***) slope, until fit the open cluster distribution point's fig. 1.

To plot on TCD, the shapes of empirical calibrations the code uses tabular values from several authors, plotting them through $6^{th}$ order interpolating polynomials.

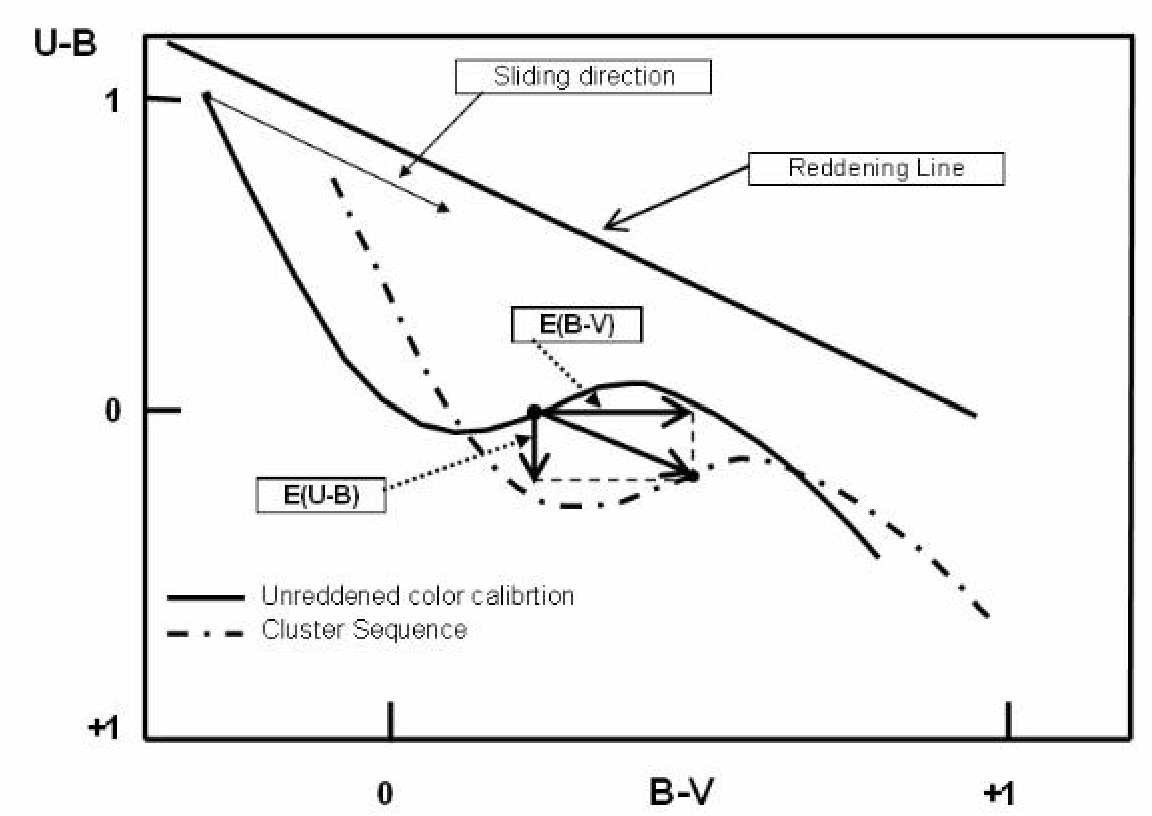


**Fig. 1 – Sliding Fit Technique.**

The general analytics shape for each empirical calibration on TCD is:

$$(U\text{-}B)o = ?\ (B\text{-}V)o^6 + ?\ (B\text{-}V)o^5 + ?\ (B\text{-}V)o^4 + ?\ (B\text{-}V)o^3 + ?\ (B\text{-}V)o^2 + ?\ (B\text{-}V)o + ? \quad (1)$$

Equation (1) represents the intrinsic colours locus. The values of polynomial regression coefficients for several authors are those of table 1, while in table 2 are shown the applicability fields for the $6^{th}$ order calibrations.

Nevertheless Fitzgerald, Johnson and Schmidt-Kaler empirical two color calibrations cannot be well represented by a $6^{th}$ order polynomial regression and a good fit with tabulated values occurs only when, at least, a $9^{th}$ order polynomial is considered for regression.

**Table 1.** Polynomial regression coefficients for intrinsic colors loci

| Coeff. (a) | Becker (b) | Eggen (c) | Fitzgerald (d) | Johnson (e) | Mermilliod (f) | Schmidt-Kaler (g) |
|---|---|---|---|---|---|---|
| α | -35,677 | 7,8538 | 3,9095 | 0,5979 | 45,118 | 0,9823 |
| β | 18,261 | -22,656 | -14,038 | -1,772 | -74,136 | -3,2903 |
| χ | 17,965 | 19,97 | 13,111 | -0,6472 | 36,194 | 1,1774 |
| δ | -3,4732 | 4,5237 | 2,5279 | 5,6951 | 5,5641 | 5,1964 |
| ε | -6,7365 | -7,6797 | -6,3924 | -4,4966 | -9,0752 | -4,6792 |
| φ | 2,0805 | 1,8142 | 1,8167 | 1,4531 | 1,8564 | 1,4482 |
| γ | -0,0602 | 0,0038 | -0,0546 | -0,1143 | -0,0051 | 0,0933 |

(a) Regession coefficients; (b) Becker values; (c) Eggen values; (d) Fitzgerald values; (e) Johnson values; (f) Mermilliod values; (g) Schmidt-Kaler values.

**Table 2.** Polynomial regression applicability field for various authors.

| *(B-V)* (a) | Becker (b) | Eggen (c) | Fitzgerald (d) | Johnson (e) | Mermilliod (f) | Schmidt-Kaler (g) |
|---|---|---|---|---|---|---|
| Min. | -0,3 | -0,25 | -0,32 | -0,32 | -0,265 | -0,33 |
| Max. | 0,63 | 1,1 | 1,68 | 2 | 0,65 | 1,93 |

(a) *(B-V)* min. max. field; (b) Becker values; (c) Eggen values; (d) Fitzgerald values; (e) Johnson values; (f) Mermilliod values; (g) Schmidt-Kaler values.

## II-b <u>Individual reddening corrections, intrinsic colours and reddening lines.</u>

The effect of interstellar reddening causes a star to move, on TCD, to the right nearly parallel to the ***RL***. Now, it can be assumed that the distance between an observed point on TCD and two color intrinsic calibration, calculated along his appropriate reddening line, yields the individual color excess *E(B-V)*. Above one ***RL*** locus we find scattered, stars of the same spectral class with the same intrinsic colours but subject to various degrees of interstellar absorption (selective). The ***RL*** fundamental property consists in crossing the two colors calibration in a point that yields individual intrinsic colors, for the stars that are found on it. Now since ***RL*** slope is known, becomes possible to determine the intrinsic colours of an observed star in the UBV system. From a rigorous point of view, the ***RL*** is not just a straight line and can be well represented by the following expressions:

$$E(U\text{-}B) = XE(B\text{-}V) + YE(B\text{-}V)^2 \quad (2)$$

or

$$E(U\text{-}B) / E(B\text{-}V) = X + YE(B\text{-}V) \quad (3)$$

In equation (3) X is the ***RL*** slope and Y its curvature.

The values commonly accepted for X and Y are: X = 0,72 and Y = 0.05 for O stars. Actually however, the curvature factor of 0,05 seems quite inappropriate for most star clusters and a value of 0,02 for curvature is considered more corrected. Turner D. G. (1989, 1994). Therefore while curvature factor Y appears to be constant along galactic plane, the slope varies from one region to another assuming a medium value closer to X = 0,75. From an operative point of view, investigate the ratio *E(U-B)/E(B-V)* directly using the available data, rather then assume the normal value, is a very important point. The ratio can be evaluated provided we know very accurate spectral types and luminosity class for a sample of cluster members. Thinking a moment over ***RL*** fundamental property we can conclude that the process to find intrinsic colors can be reduced substantially into a geometric question. However since the analytics expression of the two color calibration is not surely linear, but it can be well described by a $6^{th}$ order polynomials, it's clear that the solution of the system can involve some difficulties. Therefore we can get over such difficulties, taking in consideration that the larger quantities of stars within a young galactic open cluster are Early Type. But as we know, from MK classification Morgan (1963), the population **I** Early Type belongs to the spectral classes O, B and early A type. Considering now, for the reasons just said, only the tabular values for early type stars the previous equation (1) must be reduced from $6^{th}$ order polynomial to a regression line between spectral classes O, B, A. Hr Trace code uses for default the Schmidt-Kaler 1982 calibration on TCD and for this calibration we can express the feature concerning spectral classes O,B,A whit the following regression line:

$$(U-B)o = 3,7082(B-V)o + 0,04 \quad (4)$$

So the calculation of intrinsic color values for programmed Early Type stars in our open cluster, will consist in constructing a series of reddening lines having slope *E(U-B)/E(B-V)* equation (3) that, connecting the observed positions with the intrinsic colors line equation (4), allow us to read the coordinates of intersection point. The only attention we must lend using this technique, is to avoid its application, on TCD, where ambiguous solutions can be introduced (multiple intersections with two color calibration). In any case and more generally, to permit an accurate determination of intrinsic colors, the TCD must satisfy the requirement according to which the slope of the ***RL*** must be as different as possible from the slope of main sequence, in order to have the maximum separation between stars intrinsically similar, but affected by different degree of reddening. Observing TCD main sequence, is easy to see that previous condition is really true for stars from OV to B9V, A3V to F4V and G6V to K0V spectral classes. Unfortunately the same requirement is not satisfied on TCD, for stars from B9.5V to A2V and F5V to G5V spectral classes.

It's essentially for this last reasons that, in de-reddening Cepheids or yellow super-giants BVIc system must be preferred. At this purpose Dean, Warren and Cousins (1978) have derived the reddening free locus for Cepheids in the (V-I)c vs. (B-V) plane, calibrating the zero point from photoelectric observation of star inside cluster or associations with well known reddening values. They have found that reddening in (V-I) index can be obtained with the following relations:

$$E(V-I)/E(B-V) = X[1+ 0,06(B-V)o +0,014E(B-V)] \quad (5)$$

and still

$$E(B-V) = Eo[1-0,08(B-V)o] \quad (6)$$

Here X is given by ratio E(V-I)/E(B-V) for one star with E(B-V) ↓ 0, while Eo is the color excess which a star with (B-V) = 0 would suffer when observed through the same quantity of absorbing material.

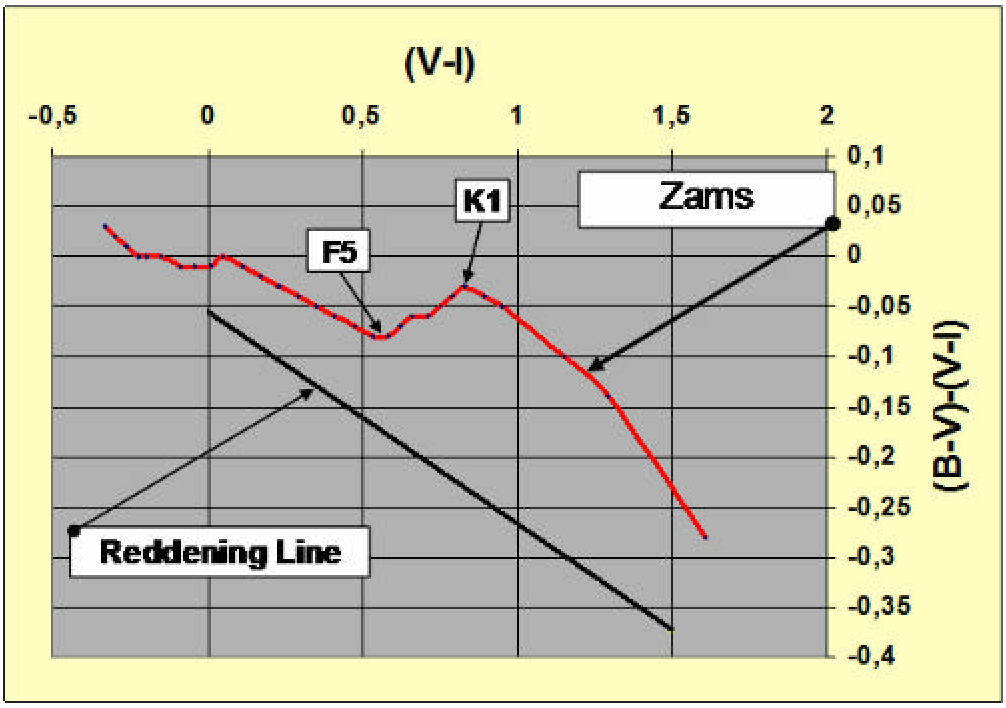


**Fig. 2 The (V-I) vs. (B-V)-(V-I) plane.**

Plotting main sequence in the (V-I) vs. (B-V)-(V-I) plane, the reddening lines are inclined with a good angle respect to main sequence itself, just in the spectral area F5 ÷ K1 that generally contains Cepheids and yellow stars see fig. (2). Data for zams in fig. (2) are from Walker R.W. (1985). For the same question and using only UBV system, the problem to find a late type reddening relative to that of an early type, can be evaluated taking in account that X = E(U-B)/E(B-V) for medium and late type super-giants is significantly different, from that of an early OB type stars. So, we can evaluate reddening for these stars following Schmidt-Kaler (1961) or Fernie (1963) with (7):

? ?= $E_{B-V}$ (Spectral Type) / $E_{B-V}$ (B0) = 0,97-0,09$(B-V)_o$ ( 7 )

**II-c <u>Computation of *R* = [Av / E(B-V)]</u>**

The total absorption in visual magnitude V, can be obtained on the base of the following quantity:

$\boldsymbol{R} = A_V / (A_B - A_V)$ ( 8 )

Were $A_V$ and $A_B$ are the absorption in V and B respectively. Observed quantities in term of magnitudes, are related to intrinsic quantities as:

$B_{Observed} = B_{Intrinsic} + A_B$
$V_{Observed} = V_{Intrinsic} + A_V$

Now because E(B-V) = $A_B$ - $A_V$, substituting in eq. (8) we have:

$$\boldsymbol{R} = A_V / E(B\text{-}V) \qquad (9)$$

If we know spectral types for a good number of objects in cluster, the ratio ***R*** of total to selective visual absorption can be evaluated Turner (1976). Practically, one uses the Johnson (1965) cluster method, under assumption that all stars within a cluster are at the same distance from us (Neglecting the depth of the cluster itself, that can be considered tiny compared with its distance). Under this condition the distance modulus *V-Mv*, where *V* is the observed visual magnitude, should be a constant except for the effects due to variable interstellar absorption within cluster volume. Value of ***R*** can be evaluated plotting *V-Mv*, versus *E(B-V)* because the slope of the straight line which best-fits observational data gives the ***R*** value. A good value of ***R*** is important because the ratio of total to selective visual absorption enters directly in the calculation of the true distance modulus as shown by the following:

$$V_O - M_V = V - \boldsymbol{R}E(B\text{-}V) - M_V \qquad (10)$$

In equation (10), $V_O$ is the visual magnitude without effects of interstellar absorption and $M_V$ is the absolute magnitude.

## III - Some consideration on automatic capture algorithm for the early type stars.

When a set of photometric UBV data is loaded the code, during user graphical search of *<E(B-V)>* parameter on TCD performs in background, the analysis and automatic capture of Early Type stars found in the photometric sequence. At the same time the code calculates, always in background and using reddening line method, the individual values for captured Early Type stars. To improve understanding of this process, maybe useful to make some clarifications. Observing the Schmidt-Kaler intrinsic color tabulation, it's easy to see, *(B-V)o* and *(U-B)o* variations through luminosity classes **V**, **III, II**, **Ib**, **Iab**, **Ia**. From a quantitatively point of view, the intrinsic colors values for the luminosity classes **V** and **III** differs very little. So, to exchange a normal giant with a main sequence star, during the intrinsic colors automatic procedure search and capture, will correspond to carry out a small and practically much negligible error for O and B Early Type stars. The same confusion between a giant or main sequence star and a bright-giant of luminosity class **II**, can involve an error ranging between 0,01 to 0,04 magnitudes for O and B Early Type. Even bigger will be the error if a normal giant or main sequence star is confused with a super-giant belonging to **Ib**, **Iab**, **Ia** luminosity classes. Such an error will be comprised in a range between 0,06 and 0,07 magnitudes for O and B Early Type. It's useless to say, that errors of this entity can lead to determine wrong intrinsic colors compared with those really observed.

### III-a Perturbing phenomena on TCD.

Perturbing phenomena on TCD, are perceived by user like evident scatter of photometric data along various reddening lines, especially along ***RL*** concerning OB stars.

In 1975 Wallenquist, using stars count technique, studied the presence of dark matter inside clusters volume and in surrounding regions. As result of his investigation he found variation in stellar densities over practically all considered clusters. On the base of Wallenquist observations seems therefore reasonable to think that the observed stars deficiency can be mainly due to presence of absorbing matter between cluster stars. It's the presence of this accidentally distributed matter inside clusters, that cause non uniform extinction phenomenon. Although the differential extinction, can be considered the main element that acts on the dispersion of photometric sequences, this is not the only one, and others can influence dispersion like: stellar evolution, stellar duplicity, stellar rotation, differences in chemical compositions, dispersion in ages, dispersion in distances, presence of non members and not last limited precision in de-reddened data Burki (1973,1975).

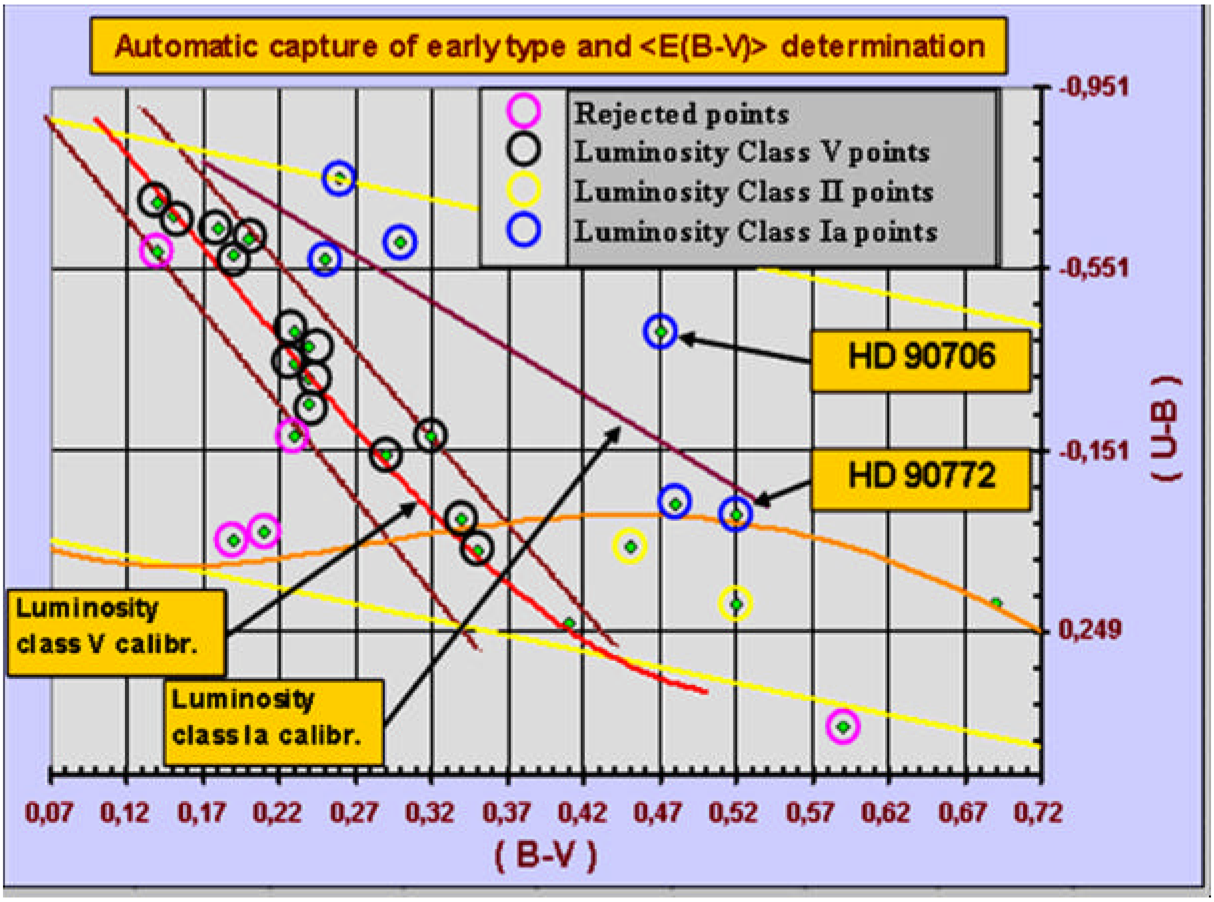


**Fig. 3 - IC 2581 Mapping of automatic early type capture by dereddening algorithm**

**III-b <u>Cases of low differential extinction.</u>**

Figure 3 show TCD for the southern cluster IC 2581 obtained plotting, the original photometry of Lloyd Evans (1969). Over this chart are also shown:

? Reached best-fit between two color Schmidt-Kaler intrinsic OB calibration and IC 2581 OB sequence.
? Luminosity class V stars within black circles.
? Luminosity class II and Ia stars respectively inside yellow and blue circles.

Naturally distinction between luminosity classes is not always therefore simple as in previous fig. 3, this because for IC 2581 cluster we are not in presence of larger differential extinction.

As far as we know from spectroscopic survey that HD 90772 is an **A7 Ia-O** Super giant, using the Schmidt-Kaler 1982 tabulation we can derive the HD 90772 intrinsic colors as follows:

$(B\text{-}V)o = 0{,}13$ and $(U\text{-}B)o = 0{,}09$ ( 11 )

Now, if tracing the HD 90772 intrinsic ***RL*** with normal slope value *E(U-B)/E(B-V)* passing through the position of this star on TCD, we try the intersection with the luminosity class **V** feature, rather than with the equation that represents the intrinsic colors for the (O,B,A) **Ia**, we will find wrongly a **B6 ÷ B7** star, rather than the more corrected **A7**. This because in the first case, (intersection of the ***RL*** with the luminosity Class **V** intrinsic sequence), we will find the following intersection point:

$(B\text{-}V)o = -\ 0{,}135$ and $(U\text{-}B)o = -\ 0{,}49$ ( 12 )

While in the second case, (intersection of the ***RL*** with the luminosity class **Ia** intrinsic sequence), we have:

$(B\text{-}V)o = 0{,}13$ e $(U\text{-}B)o = 0{,}07$ ( 13 )

It's necessary at this point to specify that Hr Trace, during calculation of the intersection point as far as the luminosity class **V** and **III** uses, in order to determine *E(B-V)* and *E(U-B)*, a linear approximation to intrinsic colors as defined by equation (4); while for luminosity classes **II**, **Ia**, **Iab**, **Ib**, uses a series of linear approximations, in order to determine the *E(B-V)* values and afterwards derives the intrinsic colors as:

$(B\text{-}V)o = (B\text{-}V) - E(B\text{-}V).$ ( 14 )

Moreover using the *(B-V)o* value, just determined, inside a $6^{th}$ order polynomial approximation to the intrinsic colors, for the considered luminosity class, obtains *(U-B)o*.

It appears clearly that the de-reddening algorithm, beyond calculating individual *E(B-V)* values must also guarantee, when differential extinction allows it, the correct interpretation of luminosity classes on TCD diagram. A misinterpretation of luminosity classes could not help us to isolate with certainty, the main sequence stars class V, that the code must capture and use to determine, not only the medium *E(B-V)* value, but also the medium distance modulus for this stars group. On the other hand observing previous fig .(3), turns out obvious like HD 90772 belongs around **Ia** luminosity class.

## III-c <u>Cases of high differential extinction.</u>

The methodology as soon as show cannot always be used, particularly in presence of high differential extinction ( clusters many young ). In this cases, user can proceed determining first, differential extinction value *?E(B-V)* and after cluster *<E(B-V)>* from this last value. The effects of differential extinction are important in young cluster, because due to their small age, interstellar nursery matter can be still present inside or in the vicinity.

A detailed discussion about differential extinction in open clusters, its effects and treatment, can be found in Burki (1975). In any case, in presence of similar scenarios, our code during

*E(B-V)* calculation, will consider that for all Early Type those Johnson **Q** method value is less than – 0,38 will exist a unique solution – intersection - between reddening line ***RL*** and two color intrinsic calibration so, following Golay (1974) will be:

$$E(B\text{-}V) = (B\text{-}V) - [(U\text{-}B) - X(B\text{-}V) - 0{,}05(B\text{-}V)^2 / 3{,}012\text{-}0{,}05(B\text{-}V)]. \qquad (15)$$

### III-d Particularly reddened sky fields.

In presence of heavily obscured regions or when particularly data dispersion on TCD gives impossible to obtain corrected value for cluster <*E(B-V)*> through sliding fit technique, user can always search spectroscopic data using Webda database connection link or ADS literature query. With this data, turn out possible to select and use the code spectroscopic de-reddening interface. This utility collecting association between spectroscopy and photometric indices allow us to obtain the variable extinction diagram. As already side, the ratio of total to selective absorption can be evaluated if we know spectral type of a good number of stars in the cluster. At this purpose the code spectral de-reddening interface uses cluster method procedure as given by Johnson (1965). So knowing spectral type and from this last ones absolute magnitudes, we can derive intrinsic colors and color excesses. Further one plots of *V-Mv* versus *E(B-V)* gives us one observational distribution whose slope value is ***R.*** Note that the above procedure, is useful only for stars that have already reached the main sequence.

## IV - Determining the true distance modulus <Vo-$M_V$>.

Star clusters are small enough compared with their distance. Thus we can assume that cluster members are at the same distance from us. If this is a reasonable idea, the apparent magnitude of cluster members V differs from their absolute magnitudes $M_V$ by the same amount V - $M_V$ and we can refer this quantity as distance modulus. Plotting now apparent magnitudes of cluster stars with respect to their spectral types or color indices, the resulting array of points has the same significance of spectrum or color index vs. absolute magnitude calibration, except for distance modulus difference. In determining the distance of a galactic cluster for photometric way, the method followed by Hr Trace, is the Zams fitting one. To hit this mark, code can use various existing empirical calibrations such as: Becker & Fenkart (1971), Blaauw (1963), Eggen (1965), Johnson (1963), Mermilliod (1981), Schmidt-Kaler (1982), Turner (1981). All this empirical calibrations put in relation the de-reddened *(B-V)o* color index, with the absolute magnitude. Now to obtain quantitatively the distance modulus of any cluster, we must only to match comparable parts of our calibration and cluster array in study on a *(B-V)o, Vo* diagram and to note the difference between apparent and absolute magnitudes. This last operation can be obtained automatically or graphically making sliding towards the bottom of our calibration until to match the cluster array. In determining distance modulus the code uses, for default, the Schmidt-Kaler 1982 empirical zams, that allows us to obtain the mathematical best-fit.

As for TCD interface, the Zams lines have been constructed with $6^{th}$ order polynomial interpolations of tabular values supplied by several authors. The zero age main sequence locus, according to general polynomial expression, can be well represented by equation (16) as follow:

$$Mv = ?\,(B\text{-}V)o^6 + ?\,(B\text{-}V)o^5 + ?\,(B\text{-}V)o^4 + ?\,(B\text{-}V)o^3 + ?\,(B\text{-}V)o^2 + ??\,(B\text{-}V)o + ?? \qquad (16)$$

While in tables 3 and 4 are shown respectively regression coefficients and applicability fields for various authors. With a mathematical best-fitting procedure the code calculates, for the captured Early Type photometric members in TCD interface, the individual distance modulus $(Vo-M_V)$ and subsequently mediating over all $(Vo-M_V)$, obtains the medium true distance modulus $<Vo-M_V>$ for the cluster. Mathematical best-fitting wizard work only with Schmidt-Kaler 1982 calibration, while for all other empirical calibrations $(B-V)o$, $M_V$ arranged by code, it's always possible to obtain a graphical fitting.

**Table 3.** Empirical zams regession coefficients for various authors.

| Coeff. (a) | Becker (b) | Blaauw (c) | Johnson (d) | Mermilliod (e) | Schmidt-Kaler (f) | Turner (g) |
|---|---|---|---|---|---|---|
| $\alpha$ | 196,54 | -1,2121 | -6,3458 | 70,886 | -6,472 | -26,249 |
| $\beta$ | -233,35 | 18,132 | 31,114 | -108,12 | 32,247 | 77,943 |
| $\chi$ | 26,918 | -46,054 | -56,193 | 14,865 | -56,115 | -90,138 |
| $\delta$ | 56,818 | 42,55 | 45,225 | 41,256 | 41,648 | 48,264 |
| $\varepsilon$ | -20,835 | -14,986 | -15,639 | -17,994 | -13,35 | -10,321 |
| $\phi$ | 6,6794 | 6,8913 | 7,113 | 6,504 | 7,7839 | 5,762 |
| $\gamma$ | 1,4838 | 1,4515 | 1,5225 | 1,5854 | 0,8925 | 1,4555 |

(a) Regession coefficients; (b) Becker values; (c) Blaauw values; (d) Johnson values; (e) Mermilliod values; (f) Schmidt-Kaler values; (g) Turner values.

**Table 4.** Polynomial regression applicability field for various authors.

| *(B-V)* (a) | Becker (b) | Blaauw (c) | Johnson (d) | Mermilliod (e) | Schmidt-Kaler (f) | Turner (g) |
|---|---|---|---|---|---|---|
| Min. | -0,3 | -0,35 | -0,25 | -0,265 | -0,33 | -0,32 |
| Max. | 0,63 | 1,2 | 1,3 | 0,65 | 1,93 | 0,9 |

(a) *(B-V)* min. max. field; (b) Becker values; (c) Blaauw values; (d) Johnson values; (e) Mermilliod values; (f) Schmidt-Kaler values; (g) Turner values.

In cases of graphical match, user will proceed searching the best-fit to the cluster array with an eye to the scatter of the observed points and try to fit the less luminous envelop, from an evolutionary point of view, for members found on the Zams.

If we superimpose, on a color – absolute magnitude diagram, several cluster as shown in fig. 4, we see that practically all arrays seems to born from an unique envelop line, under which there are only white dwarf. This line is referred as Zams (Zero age main sequence) and the various array separation points have been called Turn-off points. Note also that from an evolutionary point of view, the zams envelop can be defined as the low luminosity locus Balona (1984).

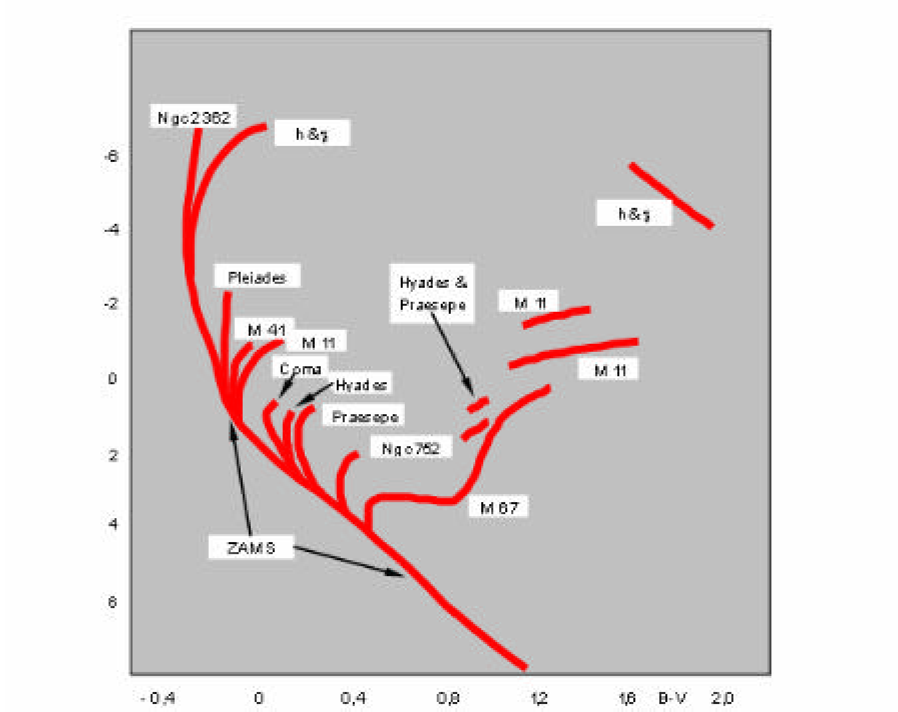


**Fig. 4 - Composite Color-Magnitude diagram for various clusters**

## IV-a <u>The IC 2581 <*Vo-$M_V$*> determination.</u>

Now we see, practically, how all we have said previous, can be obtained using the Hr Trace software. Observing the *(B-V)o, Vo* plane fig.5, we see that the IC 2581 main sequence is very well defined in the interval from *(B-V)o* = -0,25 to *(B-V)o* = 0,0 while the blue Turn-off point is found in correspondence of a *(B-V)o* value of -0,25.

The scatter in the observed points for IC 2581 is very little and this leaves us, to preview a very low displacement in the best-fit calculations (better adaptation to the points). Before starting the Fitting Wizard procedure available in Hr Trace for the automatic best-fit calculation, it will be necessary to select in the same interface the *(B-V)o* interval where we wish to find the best-fit of the empirical calibration with the IC 2581 data.

It will be also necessary to introduce, the read value of blue Turn-off, so that the code can calculate the cluster age, using A. Maeder, G. Meynet and C. Mermilliod (1993) calibration. In order to obtain this value, it will be sufficient to carry the cursor on the turn-off point and automatically read, from the diagram the *(B-V)o* value, or in lack of an obvious turn-off point, on the final part of the Zams from the blue side of the array sequence.

Introducing in Hr Trace the selected values and starting Fitting Wizard, we obtain the situation showed in fig. 6, where the true distance modulus value turns out to be: 12,35 magnitudes. Following W. Becker (1963,1966), it will be necessary to obtain the distance modulus also on *(U-B)o, Vo* plane and then average the values obtained on the two planes. We don't show best-fitting on *(U-B)o, Vo* plane because absolutely similar to previous one obtained on *(B-V)o, Vo* plane.

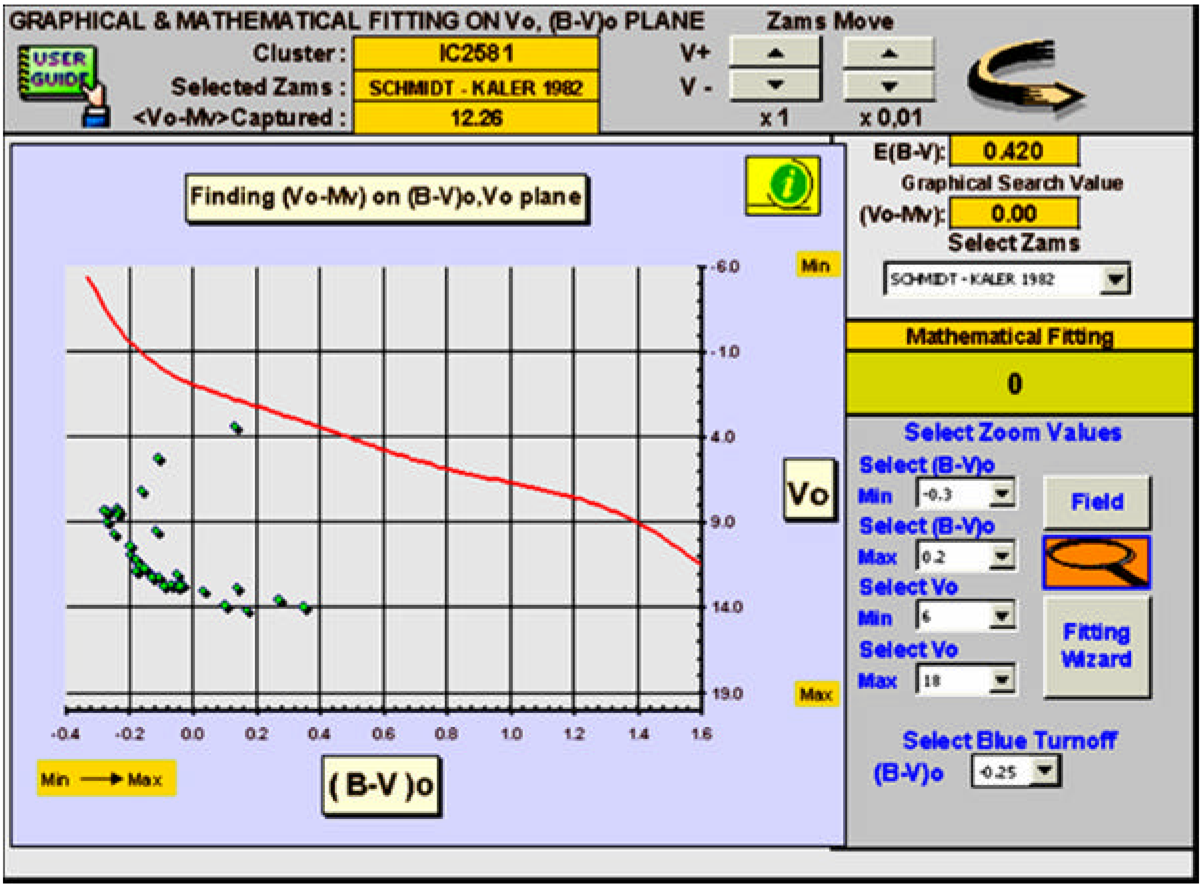


**Fig. 5 - Starting situation for distance modulus serach of IC 2582**

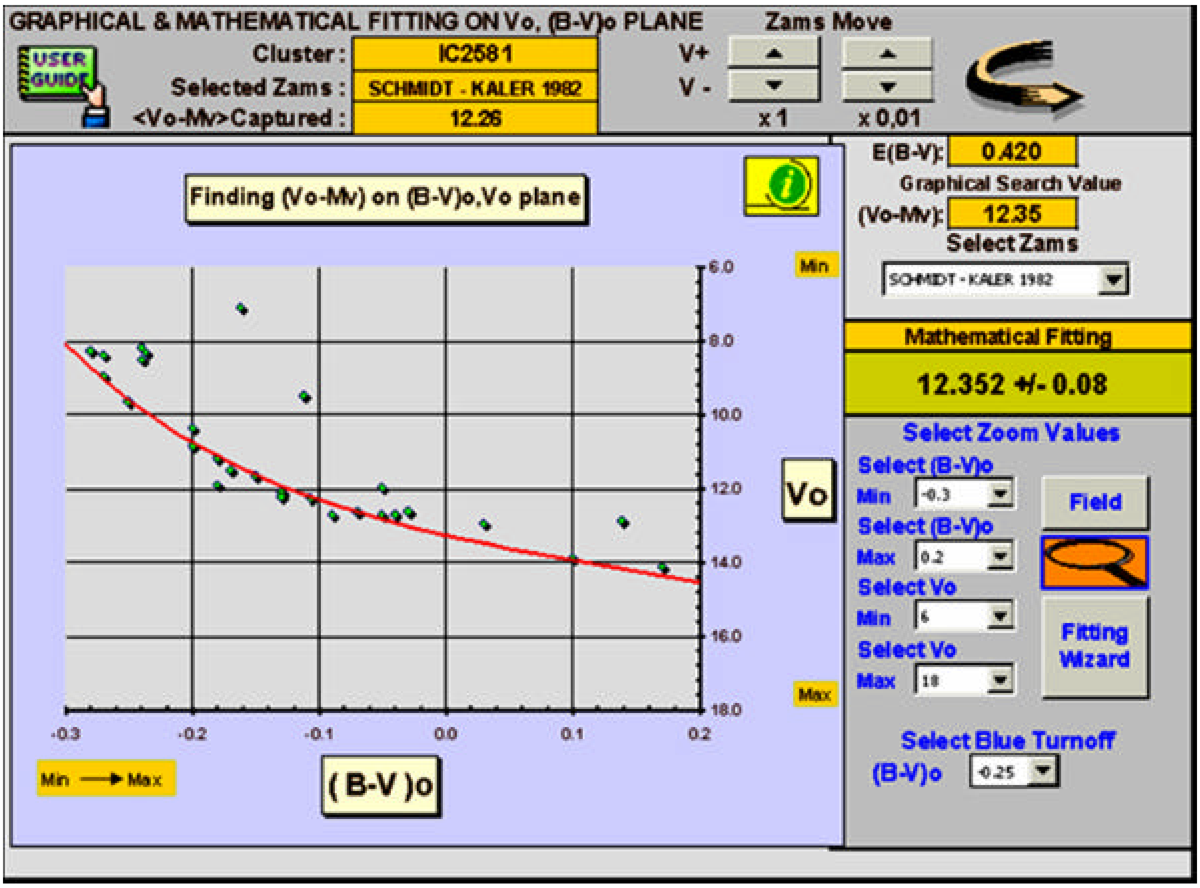


**Fig. 6 - Best fitting obtained from (B-V)o = -0,30 to (B-V)o = -0,20**

## IV-b Selection of cluster members question.

The determination of cluster membership is a difficult question and can be accomplished through the following various criteria namely: photometric, kinematics, statistical and spectroscopical. Nevertheless as often it happens, for limitation of statistical method or lack of spectroscopic data we cannot confide many in first two methods, so observers based cluster membership on the remaining two criteria. Moreover as one may note, there is no a priori way of recognizing unusual cluster members and therefore astrophysical important objects, only with photometric criterion. Hr Trace code, during distance calculations uses Walker's criteria, to select those objects with elevated membership probability. For IC 2581 case these likely stars members are those included between the dotted lines in fig. 7. Therefore Walker's criteria it's reliable only for main

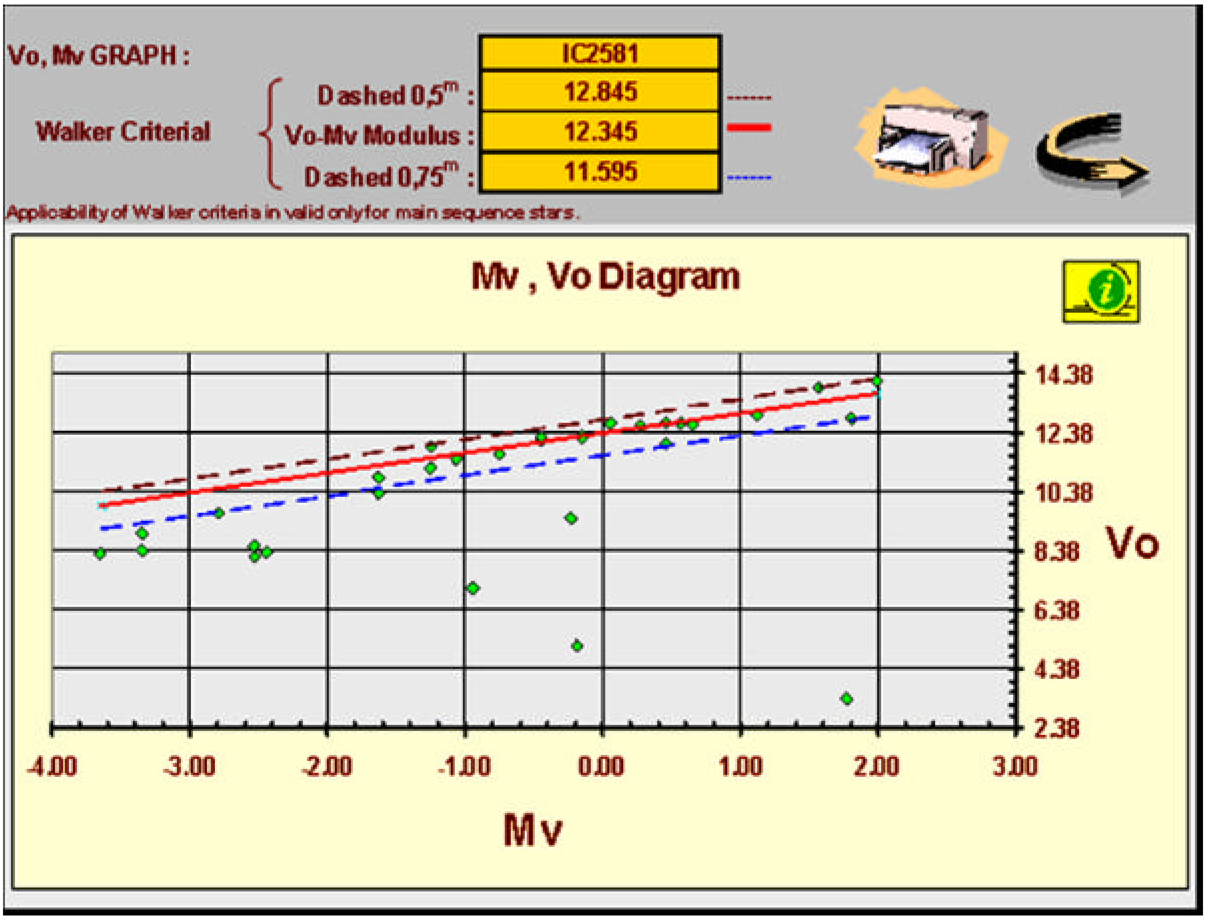


**Fig. 7 - Menbership probability according Walker's criteria**

sequence objects. Both contracting stars as well as stars slightly evolved away from main sequence, cannot be identified as members whit this criteria. Following Walker's criteria we are only in presence of probability of membership and this is judged using Vo, Mv (Zams) diagram. In this diagram Walker's limit were set arbitrarily from statistical studies of well known cluster having reliable proper motion data. These limits were set assuming that typically cluster member lie no more than 0,5 magn. above <Vo – Mv> line and that duplicity will not brighten more than 0,75 magn. away <Vo – Mv> line. Naturally it's clear that all we have just said make worse in presence of non-uniform extinction across cluster.

## IV-c The color magnitude diagram.

After determination of one distance modulus, the code can calculate the absolute magnitudes and plot the color-absolute magnitudes *(B-V)o*, $M_V$ diagram fig. 7. The simple observation of this figure, shows that IC 2581 is a young galactic cluster make by young stars of blue and white-blue color, with superficial temperatures ranging between 8000° K to 40000° K. Even the photometric radii are considerable, the values calculate by Hr Trace using Wesselink (1969) calibration ranging in the interval between $R/R_S$ = 2,2 to $R/R_S$ = 280. The maximum values calculate by Hr Trace are those of HD 90772 and HD 90706, two super giants of very high absolute magnitude *Mv* = -8,96 and *Mv* = -7,16 respectively, but with similar bolometric magnitude *Mbol* = -8,93 and -8,60.

It's also interesting to observe the relationship between the photometric radii of these two stars, that turn out to be (HD 90772 / HD 90706) = 3,67. The relationship between the radii of two super giants, must be considered according to the effective temperature of two stars, that it's calculated by Hr Trace in 8033°K for HD 90772 and 14327°K for HD 90706. Evidently the colder body HD 90772, needs a huge radiating surface of warmer body HD 90706, in order to reach the absolute magnitude of *Mv* = - 8,96. This situation is very well explained by the Stefan-Boltzmann law (17):

$$L = 4?R^{2}\ ?T^{4} \qquad (17)$$

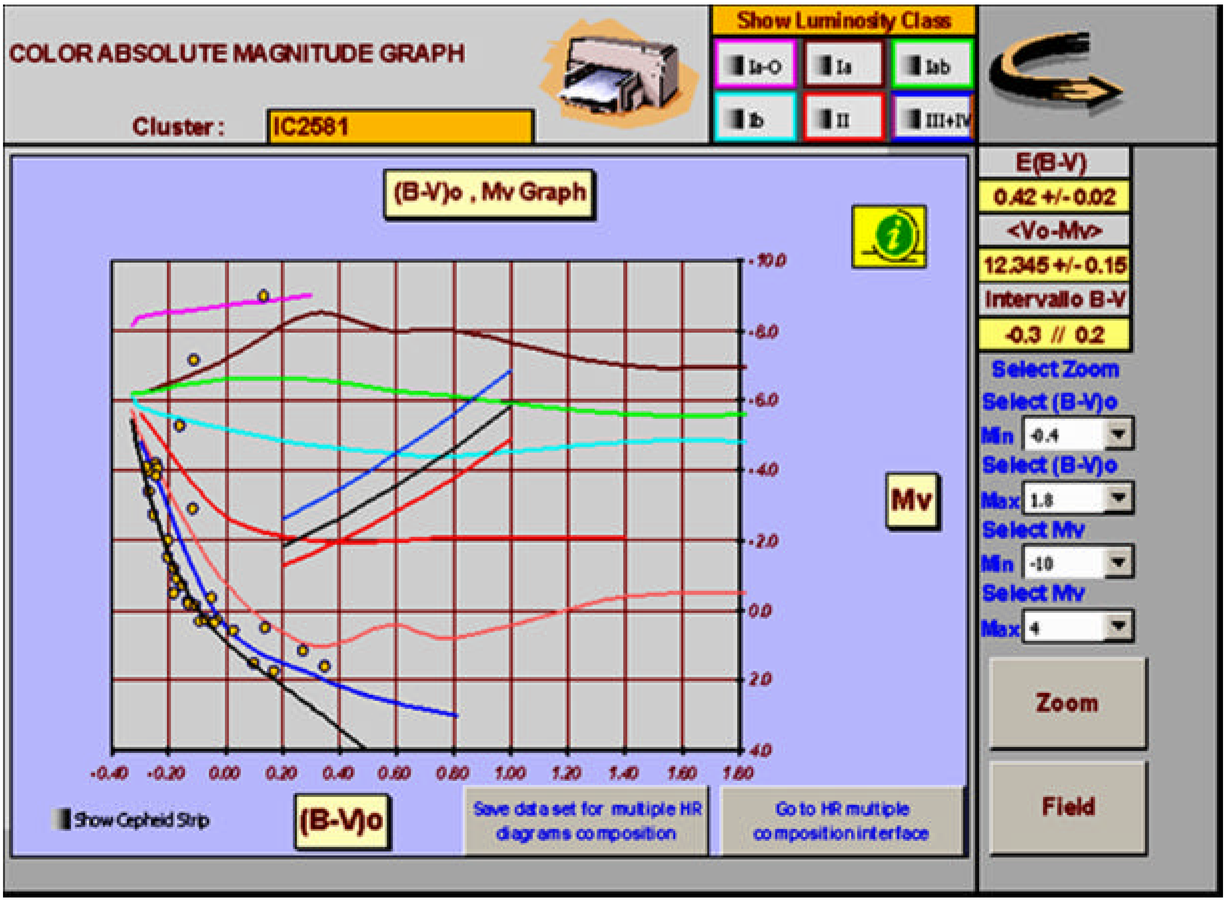


**Fig. 8 - IC 2581 color-absolute magnitude diagram.**

## V - The HR theoretical diagram log ($L/L_S$), log ($T_{eff}$) and log ($T_{eff}$), $M_{Bol}$.

For theoretical plane, we have used the effective temperature tabulations of Bohm-Vitense (1981), H.L. Johnson (1965) and P. Flower ( 1975,1977,1996) to convert the observed intrinsic color values in order to pass on theoretical plane. The polynomial equation used to obtain the Teff values from intrinsic color index is similar to the previous ones:

$$Log\ Teff = ?\ (B\text{-}V)o^6 + ?\ (B\text{-}V)o^5 + ?\ (B\text{-}V)o^4 + ??(B\text{-}V)o^3 + ?\ (B\text{-}V)o^2 + ?\ (B\text{-}V)o + ? \qquad (18)$$

In particular for the P. Flower calibration, it has been necessary to express a $7^{th}$ degree term like:

$$Log\ Teff = ?\ (B\text{-}V)o^7 + ?\ (B\text{-}V)o^6 + ??(B\text{-}V)o^5 + ?\ (B\text{-}V)o^4 + ?\ (B\text{-}V)o^3 + ?\ (B\text{-}V)o^2 + ?\ (B\text{-}V)o + ? \qquad (19)$$

while calculation of the amount $Log\ (L/L_S)$ is on the contrary obtained through (20):

$$Log\ (L / L_S) = 4{,}72 - [( Vo + BC - DM ) / 2{,}5] \qquad (20)$$

For these calibrations Hr Trace uses the polynomial coefficients showed in tab. ( 5 ).
In relation to equation (20), BC represents the bolometric correction, while DM is the true distance modulus <$Vo$-$M_V$>. To pass from color-magnitudes **CM** diagram to **HR** diagram, it's necessary to transform the color index onto effective temperature and absolute magnitude onto bolometric magnitude. In both cases, the rheology followed by the code to make calculations is always the same, therefore we will only limit ourselves to list he calibrations used for the conversion between absolute magnitudes to bolometric magnitudes.

**Table 5.** Color index versus effective temperature calibrations, polynomial regression coefficients.

| Coeff. (a) | Bhom-Vitense (b) | Flower (c) | Johnson (d) |
|---|---|---|---|
| α | 0,3501 | -0,3594 | 0,2182 |
| β | -1,8848 | 2,1929 | -1,2102 |
| χ | 3,8212 | -5,3669 | 2,572 |
| δ | -3,6782 | 6,7926 | -2,6881 |
| ε | 1,795 | -4,6088 | 1,4922 |
| φ | -0,7101 | 1,7406 | -0,6937 |
| γ | 3,9863 | -0,6544 | 3,9808 |
| η | - | 3,9791 | - |

(a) Regession coefficients; (b) Bhom-Vitense values; (c) Flower values; (d) Johnson values.

**Table 6.** Polynomial regression applicability field for various authors.

| *(B-V)* (a) | Bhom-Vitense (b) | Flower (c) | Johnson (d) |
|---|---|---|---|
| Min. | -0,31 | -0,35 | -0,3 |
| Max. | 1,6 | 1,8 | 2 |

(a) *(B-V)* min. max. field; (b) Bhom-Vitense values; (c) Flower values; (d) Johnson values.

The conversion between absolute magnitudes to bolometric magnitudes is the following:

$$M_{BOL} = Mv + BC \qquad (21)$$

For the calculation of the bolometric correction Hr Trace uses the H.L. Johnson (1966) and P. Flower (1996) $Log_{Teff.}$, *BC* tabulations. Once obtained the conversions, we go to a new interface in order to compare our calculations with a series of stellar models as it's showed in fig. 9.

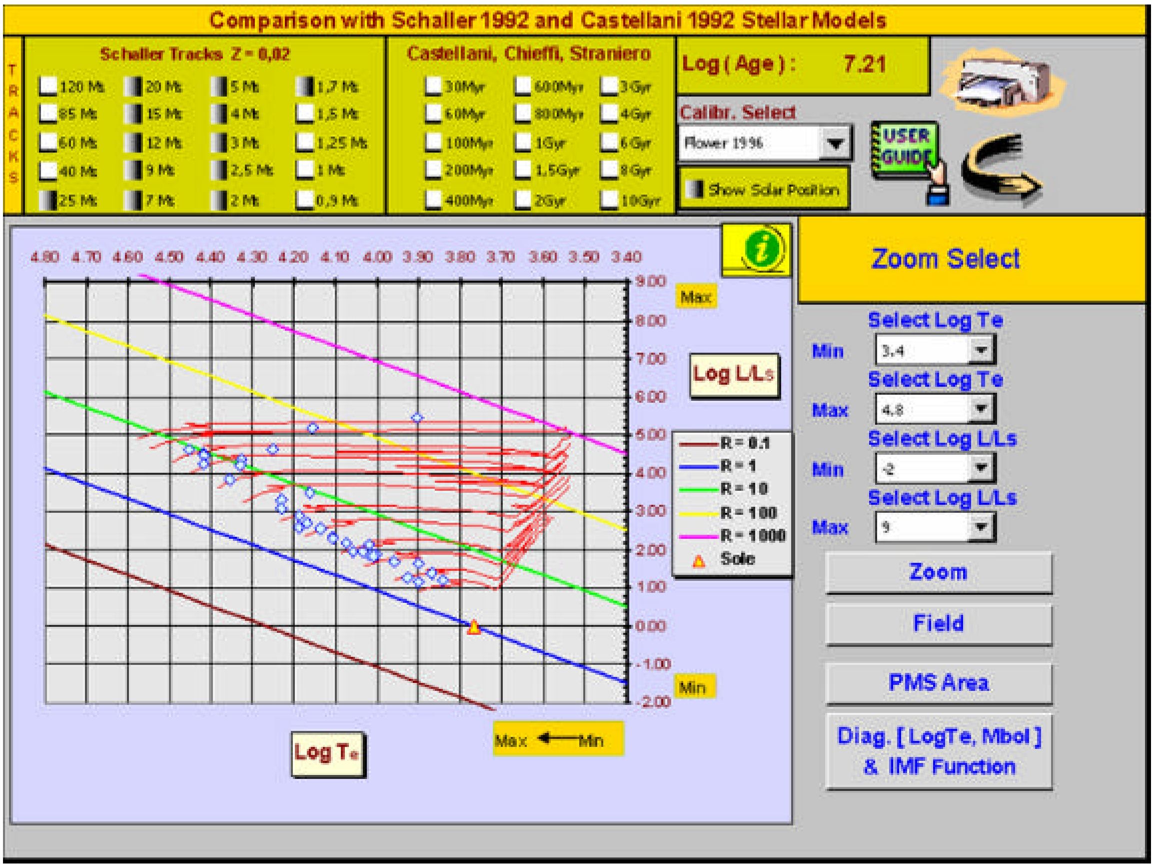


**Fig. 9 - IC 2581 theoretical HRD Log (Teff), Log(L/Ls)**

The superimposition of the evolutionary tracks from stellar models on IC 2581 array, clearly shows that the majority of the members of this cluster are formed by stars found in the range between 2 to 15 M✍. Only HD 90772 and HD 90706 have been found in the range between 30 to 40 M✍, as we could expect considering the highest absolute magnitude of these stars. The yellow-red triangle on the diagram shows for comparison, the solar position. In the same interface it's also obtained the cluster age from the blue turn-off value, using the A. Maeder, G. Meynet and C. Mermilliod (1993) calibration.

Afterwards without exit from this interface we can superimpose, on cluster sequence and for preliminary evaluations, a series of isochrones from V.Castellani, A.Cheffi and O.Straniero (1993) stellar models in the range between 30 Myr to 10 Gyr. More advanced analysis are possible using *Teff, Mbol* diagram of fig. 10, where we observe HD 90772 and HD 90706 to reach a bolometric magnitude *Mbol* = -8,96 and -8,60 respectively.

Moreover, on *Log (Teff), Mbol* interface it's possible to compare our clusters with tracks from the stellar models of Geneva group Schaller & other (1992) for metallicity Z = 0,02 and Z = 0,001 and Padova group Bertelli & other isochrones (1994), for metallicity Z = 0,02, Z = 0.008 and Z = 0.004. Always from this interface user can start calculations with the aim of obtain value for cluster PDMF ( Present Day Mass Function). The actual software release don't allow for IMF ( Initial Mass Function ), but one patch focused in solving this problem is now under development.

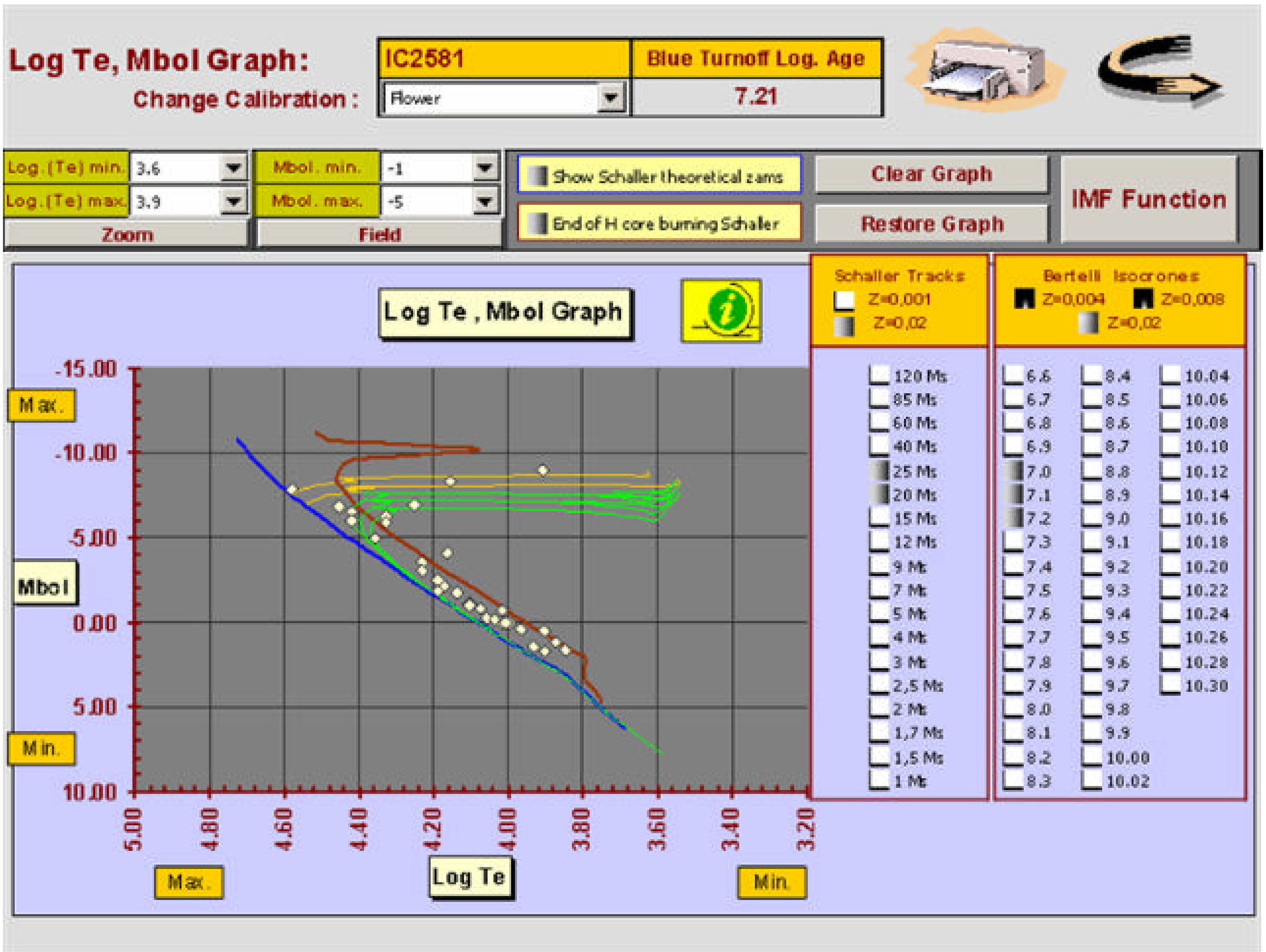


**Fig. 10 - HRD Log (Teff), Mbol for IC 2581**

## V-a <u>Variability phenomena inside Open Clusters.</u>

Stellar variability studies inside or around clusters, could disclose interesting possibilities for serious amateurs. This since, inside clusters, we can find various types of variability phenomena concerning population **I** stars. Using both Log(Teff), Mbol and Massive Regions & Instability Strips interfaces, practically all HRD instability strips can be well studied. It's possible, for instance, to take in consideration variable objects lying over, or immediately around main sequence, like ? cephei, as well as, SPB (Slowly pulsating B stars), or ? scuti. On the other hand, observing very young clusters we meet variability related to very massive objects as: LBV or S Doradus, Wolf-Rayet, OB massive stars, while inside intermediate age clusters, we can find variability connected to yellow super-giants or population **I** cepheids. Both young and intermediate age clusters, sometime can also contain very evolved objects like red super-giants with all their variability phenomena.

Finally in clusters many young exist also the possibility to study T-Tauri, another class of eruptive variable related to pre-main sequence phase or newly formed stars, those variability is mostly due to flares phenomena. Also the study of binary stars in clusters can produce other important results. This since arranging together spectroscopic and photometric data radii, spectral types, effective temperatures and bolometric magnitudes can be obtained and once known such values, turn out known also distance. Beyond this variable stars permit to compare, whenever possible, the resulting zams fitting distance with the same value calculated, for instance, with Period-Luminosity relation. A good example of those comparisons can be seen in Mochejska B.J. & Kaluzny J. (1999). In their work on intermediate age open cluster Ngc 7789 they found some W Ursae Majoris system over cluster field. To asses cluster membership for contact binary authors have applied the Rucinski & Duerbeck (1997) absolute magnitude calibration as follow:

$$Mv = -4.44 \, Log \, P + 3.02(B-V)o + 0.12 \qquad (22)$$

Where P is the contact binary period in days.

As results of calculation using equation (22) over 35 discovered contact binary in Ngc 7789 cluster field, five system seems to be cluster members with computed distance moduli within ± 0.2 of the cluster modulus. Naturally similar comparison can be done considering for instance ? scuti, and classical Cepheids whenever possible.

## VI – a Some Examples – Cepheid's & instability strip.

Ngc 129 Cepheid's member recognized at his third crossing of instability strip over $M_{Bol}$ , Log $_{Te}$ chart fig.11

( Bertelli isochrone's green curve ).

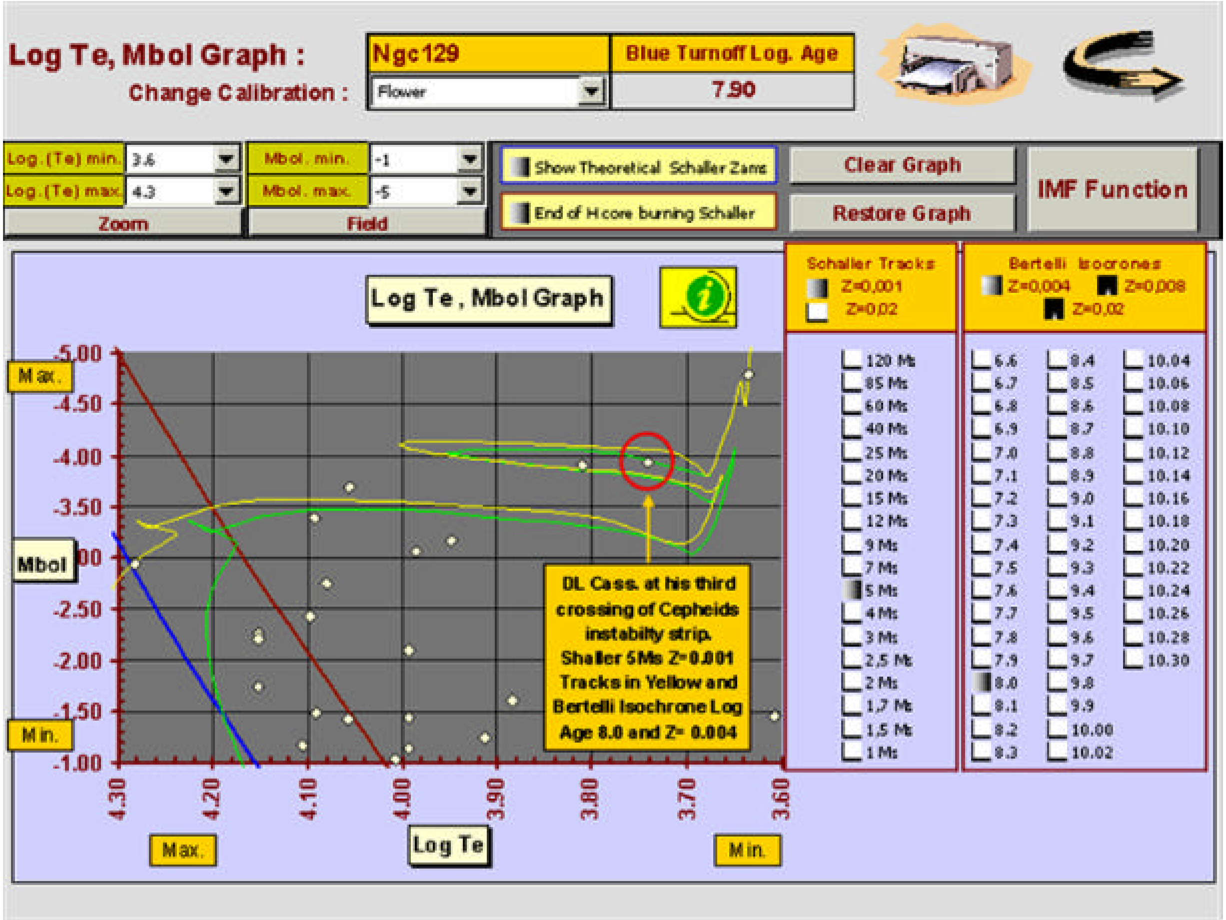


**Fig. 11 - Ngc 129 cepheid member DL Cass. On theoretical HRD**

Always DL Cass. detected inside Cepheis's instability strip over Hr Massive regions & Instability Strips inteface Fig.12.

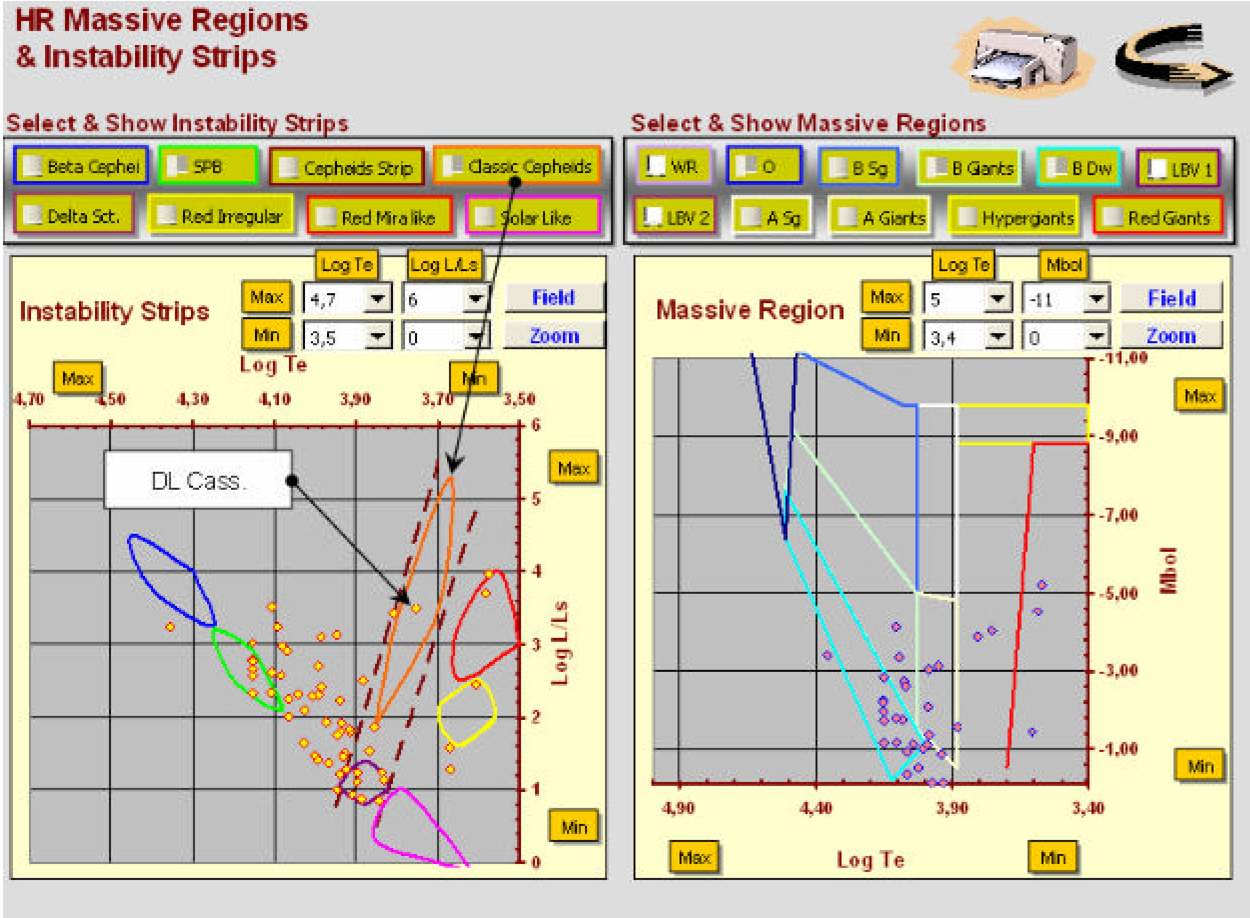


**Fig. 12 - Hr Massive Regions & Intability Strips.**

## VII - a <u>Some Examples – Comparisons between various clusters.</u>

Comparisons between various clusters are available through devoted interface for this type of analysis, see next fig.13 where IC 2581 and Ngc 457 respectively are shown in comparison.

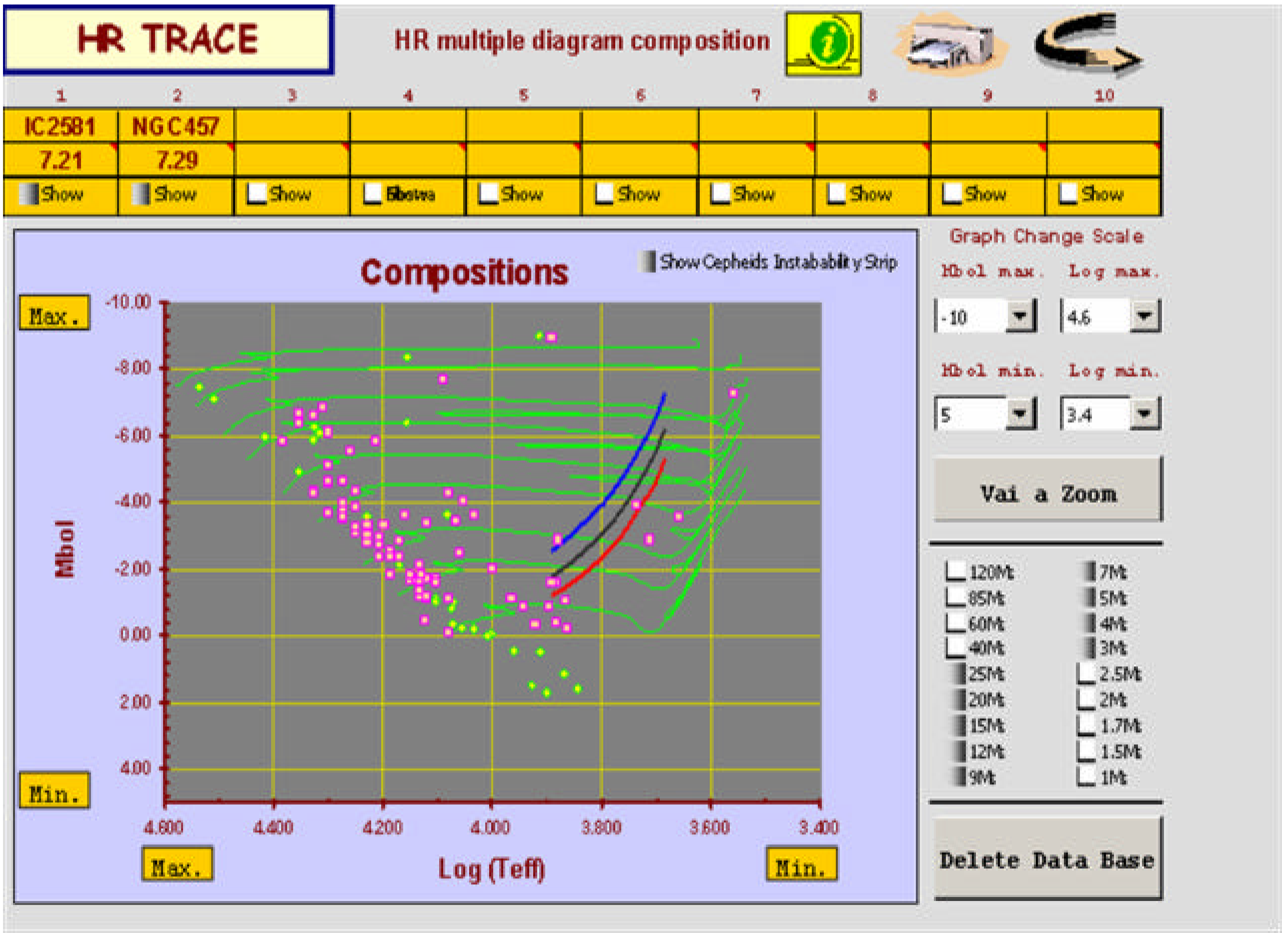


**Fig. 13 – Hr multiple composition diagram.**

Many other performances are available for the analysis of intermediate and advanced age clusters, or to obtain parameters when objects are observed through particularly reddened sky fields where it's impossible to get individual *E(B-V)* value only in the photometric way.

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